In-water SONAR shell transmitter simulation using a coupled FE-BE method

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Abstract

This article describes the application of a coupled finite element-boundary element method to obtain the steady-state response of a sonar transducer. The particular structure considered is a flooded piezoelectric spherical shell. The sonar shell is simulated to be driven by electrical charges applied onto inner and outer surfaces of the shell. It is shown that at relatively low input frequency a beam pattern which is almost close to omnidirection can be obtained. The coupled FE-BE method is described in detail.

I. Introduction

The use of software tools in the design of sonar transducers is becoming widespread. Particularly, numerical techniques, such as the finite element method (FEM) and the boundary element method (BEM) are being increasingly employed. The FEM is generally applied to the internal analysis of a structure whilst the BEM is used to study the radiated field. They have flexibility that they can be used to model any arbitrary geometry and characterize any given property of material.

Many others have reported about the in-air analysis of piezoelectric transducers using the FEM [1-4]. Modified FEMs such as the mixed FE perturbation method [5] or the mixed FE plane-wave method [6] have been developed in order to obtain efficiency in storage requirements whilst maintaining low computational costs and good accuracy. These modified FEMs are increasingly being used when an array of transducers or composite sonar transducers are being studied. Further developments have been made so as to include the effects of infinite fluid loading on transducer surface. For example, Bossut et. al. [7] and Hamonic et. al. [8] used fluid finite elements as an extension to structural finite elements with the condition that outer boundary of the fluid elements represents continued radiation. Others used 'infinite' fluid elements for infinite acoustic radiation [9,10]. The BEM is probably accepted as the most suitable method for the radiation problem,

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and if appreciably formulated to be compatible with the FEM, the two method can be coupled together [11,12].

The main aim of this paper is to simulate the structural behaviour of the flooded piezoelectric spherical shell when the sonar shell is driven by external electrical charges. The directivity pattern of the acoustic pressure is shown to be omnidirectional.

Ⅱ. Numerical Methods

1. Finite Element Method (FEM)

The following equation (1) is the integral formulation of the piezoelectric equations:

$$\{F\}+\{F_I\} = [K_{uu}]\{a\} + [K_{u\phi}]\{\phi\} - \omega^2[M]\{a\} + j\omega[R]\{a\}-\{Q\} = [K_{\phi u}]\{a\} + [K_{\phi\phi}]\{\phi\} \quad \cdots (1)$$

where

{F} Applied Mechanical Force
{F_I} Fluid Interaction Force
{Q} Applied Electrical Charge
{a} Elastic Displacement
{Φ} Electric Potential
[K_{uu}] Elastic Stiffness Matrix
[$K_{u\Phi}$] Piezoelectric Stiffness Matrix [$K_{\Phi u}$] = [$K_{u\Phi}$]'
[$K_{\Phi\Phi}$] Permittivity Matrix
[M] Mass Matrix
[R] Dissipation Matrix
ω Angular Frequency

The isoparametric formulation for 3-dimensional structural elements is well documented by Allik H. et. al. [1,2]. Each 3-dimensional finite element is composed of 20 quadratic nodes and each node has nodal displacement (a_x, a_y, a_z) and electric potential (Φ) variables. In local coordinates the finite element has 6 surface planes (±xy, ±yz, ±zx) which may be exposed to external fluid environment. The exposed surface is used as a boundary element which is composed of 8 quadratic nodes.

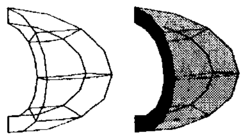

Fig. 1. A structure is discretized into finite structural elements.

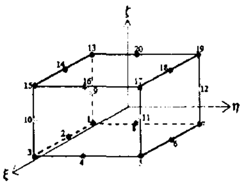

Fig. 2. Each finite element is composed of 20 quadratic nodes. Each surface boundary has 8 quadratic nodes.

2. Boundary Element Method (BEM)

For sinusoidal steady-state problems, the Helmholtz equation, $\nabla^2\Psi + k^2\Psi = 0$, represents the fluid mechanics. Ψ is the acoustic pressure with time variation, $e^{j\omega t}$, and k(=ω/c) is the wave number. In order to solve the Helmholtz equation in an infinite fluid media, a solution to the equation must not only satisfy structural surface boundary condition(BC), $\frac{\partial\Psi}{\partial n} = \rho_f\,\omega^2\,a_n$, but also the radiation condition at infinity, $\lim_{|r|\to\infty}\oint_S(\frac{\partial\Psi}{\partial r}+jk\Psi)^2dS = 0$. r is distance and $\frac{\partial}{\partial n}$ represents differentiation along the outward normal to the boundary. ρ_f and a_n are the fluid density and the normal displacement on the structural surface. The Helmholtz integral equations derived from Green's second theorem provides such a solution for radiating pressure waves;

$$\oint_S\left(\Psi(q)\frac{\partial G_k(p,q)}{\partial n_q} - G_k(p,q)\frac{\partial\Psi(q)}{\partial n_q}\right)dS_q = \beta(p)\,\Psi(p) - \Psi_{inc}(p) \quad \cdots (2)$$

where $G_k(p,q) = \frac{e^{-jkr}}{4\pi r}$, $r=|p-q|$

p is any point in either the interior or the exterior and q is the surface point of integration.

β(p) is the exterior solid angle at p.

The acoustic pressure for the i[th] global node, $\Psi(p_i)$, is expressed in discrete form [13-15]:

$$(1 \le i \le ng)$$

$$\beta(p_i)\,\Psi(p_i) - \Psi_{inc}(p_i) = \oint_S \left(\Psi(q)\frac{\partial G_k(p_i,q)}{\partial n_q} - G_k(p_i,q)\frac{\partial \Psi(q)}{\partial n_q} \right) dS_q \quad \text{(3a)}$$

$$= \sum_{m=1}^{nt} \int_{S_m} \left(\Psi(q)\frac{\partial G(p_i,q)}{\partial n_q} - G(p_i,q)\frac{\partial \Psi(q)}{\partial n_q} \right) dS_q \quad q \in S_m \quad \text{(3b)}$$

$$= \sum_{m=1}^{nt} \int_{S_m} \left(\sum_{j=1}^{8} N_j(q)\,\Psi_{m,j}\frac{\partial G(p_i,q)}{\partial n_q} - G(p_i,q)\sum_{j=1}^{8} N_j(q)\frac{\partial \Psi_{m,j}}{\partial n_q} \right) dS_q \quad q \in S_m \quad \text{(3c)}$$

$$= \sum_{m=1}^{nt} \sum_{j=1}^{8} \left(\int_{S_m} N_j(q)\frac{\partial G(p_i,q)}{\partial n_q} dS_q \right) \Psi_{m,j} - \rho_f\,\omega^2 \sum_{m=1}^{nt} \sum_{j=1}^{8} \left(\int_{S_m} N_j(q)\, G(p_i,q) \right. \quad \text{(3d)}$$

$$= \sum_{m=1}^{nt} \sum_{j=1}^{8} A^i_{m,j}\Psi_{m,j} - \rho_f\,\omega^2 \sum_{m=1}^{nt} \sum_{j=1}^{8} B^i_{m,j} a_{m,j} \quad \text{(3e)}$$

where nt is the total number of surface elements and $a_{m,j}$ are three dimensional displacements. Equation (3b) is derived from equation (3a) by discretizing integral surface. And equation (3c) is derived from equation (3b) since an acoustic pressure on an integral surface is interpolated from adjacent 8 quadratic nodal acoustic pressures corresponding the integral surface. Then equation (3d) is derived from equation (3c) by swapping integral notations with summing notations. Finally the parentheses of equation (3d) is expressed by upper capital notations for simplicity.

When equation (3e) is globally assembled, the discrete Helmholtz equation can be represented as

$$([A]-\beta[I])\{\Psi\} = +\rho_f\,\omega^2[B]\{a\} - \{\Psi_{inc}\} \quad \text{(4)}$$

where [A] and [B] are square matrices of (ng by ng) size. ng is the total number of surface nodes.

Where the impedance matrices of equation (4), [A] and [B] are computed, two types of singularity arise [16]. One is that the Green's function of the equation, $G_k(p_i, q)$, becomes infinite as q approaches to pi. This problem is solved by mapping such a rectangular local coordinates into triangular local coordinates and again into polar local coordinates [17]. The other is that at certain wavenumbers the matrices become ill-conditioned. These wavenumbers are corresponding to eigenvalues of the interior Dirichlet problem [18]. One approach to overcome the matrix singularity is that [A] and [B] of equation (4) are modified to provide a unique solution for the entire frequency range [19-22]. The modified matrix equation referred to as the modified Helmholtz gradient formulation (HGF) [22] is obtained by adding a multiple of an extra integral equation to equation (4).

$$([A]-\beta[I]\oplus\alpha[C])\{\Psi\} = +\rho_f\,\omega^2([B]\oplus\alpha[D])\{a\} - \left(\Psi_{inc}\oplus\alpha\frac{\partial \Psi_{inc}}{\partial n_p}\right)$$

$$\alpha = -\frac{\sqrt{-1}}{k\cdot(\text{Number of surface elements adjacent a surface node})} \quad \text{(5)}$$

where

[C] and [D] are rectangular matrices of (nt by ng) size. nt is the total number of surface elements. ⊕ symbol indicates that the rows of [C],[D] corresponding to surface elements adjacent a surface node are added to the row of [A],[B] corresponding to the surface node, that is,

$$\sum_{i=1}^{ng}\sum_{j=1}^{ng} A(i,j) = \sum_{i=1}^{ng}\sum_{j=1}^{ng} A(i,j) + \sum_{i=1}^{ng}\sum_{j=1}^{ng}\left(\sum_{m=1}^{S(i)} \alpha\, C(m,j)\right)$$

$$\sum_{i=1}^{ng}\sum_{j=1}^{ng} B(i,j) = \sum_{i=1}^{ng}\sum_{j=1}^{ng} B(i,j) + \sum_{i=1}^{ng}\sum_{j=1}^{ng}\left(\sum_{m=1}^{S(i)} \alpha\, D(m,j)\right) \quad \text{(6)}$$

where S(i) is the number of surface element adjacent a surface node. The derivation of the extra matrices [C],[D] are well described by Francis D.T.I.[22,23]. Equation (6) may be reduced in its formulation using superscript $\oplus$ for convenience;

$$A^{\oplus}\{\Psi\} = +\rho_f\,\omega^2 B^{\oplus}\{a\} - \Psi^{\oplus}_{inc}$$

$$([A]-\beta[I]\oplus\alpha[C]) \equiv A^{\oplus}$$

where $$([B]\oplus\alpha[D]) \equiv B^{\oplus} \quad \cdots\cdots(7)$$

$$(\Psi_{inc}\oplus\alpha\frac{\partial\Psi_{inc}}{\partial n_p}) \equiv \Psi^{\oplus}_{inc}$$

Equation (7) can be written as

$$\{\Psi\} = +\rho_f\,\omega^2(A^{\oplus})^{-1}B^{\oplus}\{a\} - (A^{\oplus})^{-1}\Psi^{\oplus}_{inc} \quad \cdots\cdots(8)$$

3. Coupled FE-BE Method

The acoustic fluid loading on the solid-fluid interface generates interaction forces. These forces can be related to the surface pressures by a coupling matrix [L] [12,15];

$$\{F_f\} = -[L]\{\widehat{\Psi}\} \quad \cdots\cdots(9)$$

where $[L] = \int N^t n N dS$. N is a matrix of surface shape functions and n is an outward normal vector at the surface element. N^t is the transposed form of N matrixss.

Equations (8) and (9) indicate that the interaction force can be expressed by functions of elastic displacement instead of acoustic pressure. This relationship can be applied to equation (1) when the sonar transducer model is submerged into the infinite fluid media:

Since the present sonar transducer is modelled as transmitter, the force matrices of equation (10), $[F]$ and $[L](A^{\oplus})^{-1}\Psi^{\oplus}_{inc}$, are removed. The

$$\{F\} + [L](A^{\oplus})^{-1}\Psi^{\oplus}_{inc} = [K_{uu}]\{a\} + [\rho_f\,\omega^2[L](A^{\oplus}) + [K_{u\phi}]\{\phi\} - \omega^2[M]\{a\} + \quad \cdots\cdots(10)$$

$$-\{Q\} = [K_{\phi u}]\{a\} + [K_{\phi\phi}]\{\phi\}$$

only applied BC for the equation is electrical charge, [Q]. The acoustic pressure in the far field is determined by β(p)=1 for given values of surface nodal pressure and surface nodal displacement; where $(A^{\oplus})^{-1}\Psi^{\oplus}_{inc} = 0$ for the transmitter modelling.

$$\Psi(p_i) = \sum_{m=1}^{nt}\sum_{j=1}^{8} A^i_{m,j}\Psi_{m,j} - \rho_f\,\omega^2\sum_{m=1}^{nt}\sum_{j=1}^{8} B^i_{m,j}a_{m,j} - (A^{\oplus})^{-1}\Psi^{\oplus}_{inc} \quad \cdots\cdots(11)$$

Ⅲ. Results

The coupled FE-BE method has been programmed with Fortran language running at SUN Ultra Workstation. Calculation is done with double precision and the program is made for three dimensional structures. Because each structural node has 4 DOF, the size of the globally assembled coefficient matrices of the matrix equation are 4*ng by 4*ng. The particular structure considered is a flooded piezoelectric (PZT4) spherical shell (Fig. 3). The structural shape is symmetrical in Z-axis. The PZT4 shell is divided into 32 isoparametric elements (4 x 8 sections). Each circumferencial section is composed of 4 elements. Global node numbers are attributed at 20 nodes of each element. Table 1 shows the material properties of the PZT4 piezoelectric ceramic. The piezoelectric ceramic is radially polarized and therefore the electrode is coated radially on inner and outer surfaces. Hence, the axially polarized property values of Table 1 is to be changed in its polling direction by the tensor theory [24].

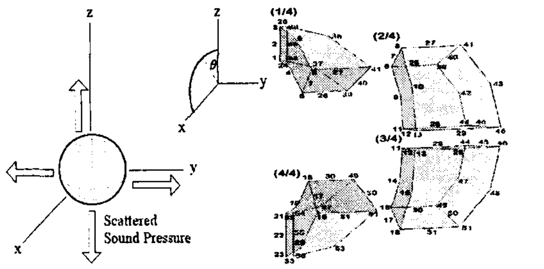

Fig. 3. A piezoelectric (PZT4) shell is divided into 32 elements(4 x 8 sections). The figure shows 4 elements of each section. Global node numbers are attributed at 20 nodes of each element.(inner radius: 3cm, outer radius: 4cm, shell thickness :1cm)

Electrical charges are applied onto the inner and the outer surfaces of the shell. This electrical energy drives the piezoelectric shell as a transmitter. The piezoelectric transmitter emitts acoustic power infinitively radially. From equation (11) the acoustic pressure in the far field is calculated along the circle with the directivity angle θ (Fig. 3). After normalizing the far field pressure, the averaged value of the pressure is calculated. This normalized and averaged value of the far filed pressure is then used as the quantitative degree of the omni-directional directivity.

Fig.(4) shows the directivity patterns of the piezoelectric shell transmitter in polar form (a),(c) and in rectangular form (b),(d) for 1KHz and 3KHz input frequencies respectively. At relatively lower frequency, the beam pattern is almost close to omnidirection. However, as the frequency is increased, the beam pattren becomes less omni-directional. The reason of this is that the number of elements for the whole shell structure should be more for higher frequency. For a higher frequency, more number of elements are required to be divided for the same shell size.

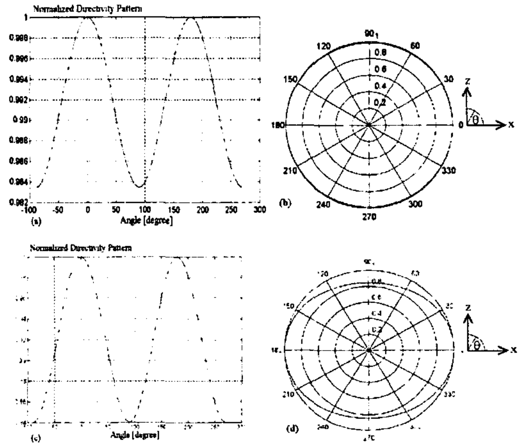

Fig. 4. The directivity patterns of the sonar Shell transmitter in polar form (a),(c) and in rectangular form (b),(d) for different input frequencies: (a),(b) 1KHz (c),(d) 3KHz.

Fig. 5 shows the vibrational mode of the spherical shell in half cross-section. The input frequency is 3KHz. (a) is the elemental structure before deformation and (b) is the deformed model after electrical loading. (c) is the overlapped picture of (a) and (b). It is remarable that the displacement of the shell is prominent in its Z

Table 18. Material Properties of PZT4 (Axially Polarized Properties, Dielectric coefficients at 100 KHz)

		Unit			Unit			Unit
ρ	7500	Kg/m^3	C_z^z	1.15E+11	N/m^2	$e_{p,z}^z$	15.1	(N/m^2)/(V/m)
C_x^x	1.39E+11	N/m^2	C_{yz}^{yz}	2.56E+10	N/m^2	$e_{p,z}^{yz}$	12.7	(N/m^2)/(V/m)
C_y^x	7.78E+10	N/m^2	C_{zx}^{zx}	2.56E+10	N/m^2	$e_{p,z}^{zx}$	12.7	(N/m^2)/(V/m)
C_z^x	7.43E+10	N/m^2	C_{xy}^{xy}	3.06E+10	N/m^2	ε_x^x	6.4605E-9	F/m
C_y^y	1.39E+11	N/m^2	$e_{p,z}^x$	-5.2	(N/m^2)/(V/m)	ε_y^y	6.4605E-9	F/m
C_z^y	7.43E+10	N/m^2	$e_{p,z}^y$	-5.2	(N/m^2)/(V/m)	ε_z^z	5.6198E-9	F/m

axis at the higher frequency. That is why the directivity pattern of the 3KHz shell model becomes less omindirectional.

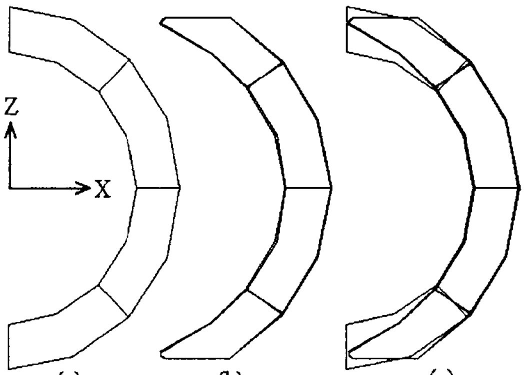

Fig. 5. Vibrational Modes (at 3KHz) (a) original structure (b) after deformation(phase=0) (c) overlapping (a) and (b)

Ⅳ. Conclusion

A coupled FE-BE method has been developed and applied to simulate a sonar transducer. The aim of the siumlation is to produce an omni-directional directivity pattern. The particular structure considered is a flooded piezoelectric shell. The resulted beam pattern of the spherical shell is omnidirectional. These present results are hardly derived in analytical approach. However, the numerical method such as the coupled FE-BE method is very useful for predicting the mechanical and the acoustical behaviour of the sonar transducer. The coupled FE-BE method could be further applied to any other shape and to any other property for sonar transducer design and applcation.

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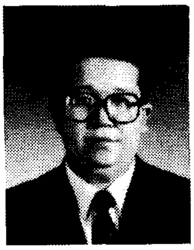

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