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Transient Response of a Stratified Thermal Storage Tank to the Variation of Inlet Temperature

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Key Words : Stratified thermal storage tank, Variable inlet temperature, Momentum-induced mixing, Analytical solution

Abstract

This paper deals with approximate analytical solutions for the two-region one-dimensional model describing the charging process of stratified thermal storage tanks at variable inlet temperature with momentum-induced mixing. An arbitrarily increasing inlet temperature is decomposed into inherent step changes and intervals of continuous change. Each continuous interval is approximated as a finite number of piecewise linear functions, which admits an analytical solution for perfectly mixed region. Using the Laplace transform, the temperature profiles in plug flow region with both the semi-infinite and adiabatic ends are successfully derived in terms of well-defined functions. The effect of end condition on the solution proves to be negligible under the practical operating conditions. For a quadratic variation of inlet temperature, the approximate solution employing a moderate number of pieces agrees excellently with the exact solution.

Nomenclature

a	: Coefficient
b	: Slope of piecewise linear function
Fo	: Fourier number, $t\alpha/H^2$
H	: Height of the storage tank or unit step function
H_m	: Depth of perfectly mixed region
H_p	: Depth of plug flow region
h_m	: Dimensionless H_m, H_m/H
h_p	: Dimensionless H_p, H_p/H
K	: Number of linear pieces
m	: Order of polynomial
N	: Number of step changes
n	: Order of integration
Pe	: Peclet number, UH/α
s, s'	: Laplace variables
T	: Temperature

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t	: Time
U	: Cross-section averaged velocity
u	: Dimensionless U, u=Pe
v	: Coefficient
x	: Distance from the interface of two regions
x'	: Distance from the tank top

Greek Symbols

α	: Thermal diffusivity of the working fluid
η_1, η_2	: Variables
λ	: Dummy variable
ψ_1, ψ_2	: Variables
ϕ_1, ϕ_2	: Variables
τ	: Dimensionless time, τ =Fo
θ	: Dimensionless temperature, $(T-T_0)/[T_i(0)-T_0]$
$\bar{\theta}$	: Laplace transformed θ
$\Delta\theta$	: Temperature difference
ξ, ξ'	: Dimensionless distance, x/H, x'/H
ζ, ζ_1, ζ_2	: Variables

Superscripts

c	: Constant
e	: Exponential function
l	: Linear function
q	: Quadratic function

Subscripts

0	: Initial state
ad	: Adiabatic end
i	: Tank inlet
j	: Representing k and n
k	: Associated with piecewise linear segment
m	: Perfectly mixed region
n	: Associated with step change

1. Introduction

One of the key ingredients affecting the performance of sensible heat storage units is thermal stratification. In this context, it is well established that the stratification characteristics are basically determined during the charging process and momentum-induced mixing between the incoming and resident fluids is a major cause of destratification (Duffie and Beckman, 1980). Therefore, a refined model which properly incorporates transport phenomena accompanying the charging process such as fluid mixing is prerequisite to the performance analysis for designing stratified thermal storage tanks. A variety of models to meet such needs have been proposed. The basic concepts and features of these have been comprehensively reviewed by Hollands and Lightstone (1989) and Zurigat et al. (1989).

Reassessing the existing models from the methodological viewpoint, most of them have relied on numerical analyses (e.g. Zurigat et al., 1991; Ghajar and Zurigat, 1991; Kleinbach et al., 1993) except a few analytical solutions based on unapproved empirical parameters (Zurigat et al., 1989). In fact, analytical approach to sophisticated models that fully account for the complicated phenomena is hard to be accomplished. If, however, analytical solutions could be derived for such models through reasonable approximations, it would be easy to use and play the role of a theoretical reference in this area. The significance of analytical solutions can be substantiated by the fact that the well-known Cabelli's solution (1977), despite oversimplifications, has been frequently cited in the literature. In the line of pursuing analytical solutions, the present author and coworkers have recently published a series of papers (Yoo and Pak, 1993; Yoo, 1995;

Yoo and Pak, 1995; Yoo et al., 1996; Yoo and Pak, 1996; Yoo et al., 1997) where new models for the charging process are developed by overcoming mathematical difficulties and deriving meaningful solutions successfully.

Nearly all the analytical studies conducted so far have focused on the case of constant inlet temperature. However, noting that the inlet temperature may vary with time in the actual systems (Abu-Hamdan et al., 1992), it is desirable that the analysis can cope with this problem. Motivated by this point, the present study is intended to derive approximate analytical solutions to a model capable of reflecting variable inlet temperature as well as fluid mixing during the charging process. Since the two-region, one-dimensional model proposed previously turned out to account for momentum-induced mixing reasonably (Yoo et al., 1997), it is adopted in this study. An arbitrary inlet temperature can be decomposed into inherent discontinuities, if exist, and intervals of continuous change, depending on its variation pattern. Our emphasis is placed on the treatments of the continuous interval which needs to be simplified. The final result is validated by comparing with the exact solution available for a specific variation of inlet temperature.

2. Modeling

2.1 Two-region one-dimensional model

The physical system considered in this study is, as depicted schematically in Fig.1, a typical stratified thermal storage tank (Zurigat et al., 1989: Zurigat et al., 1991: Ghajar and Zurigat, 1991: Yoo and Pak, 1993; Yoo and Pak, 1996). The inlet and exit ports are fixed at the top and bottom of the tank, respectively. Initially, the tank of the height H is filled with the working fluid at a uniform temperature T_0. As the caharging begins, the hot fluid at $T(t)$ flows into the tank, and the same amount of resident fluid discharges out of the tank. During the process, the flow rate (or the cross-section averaged velocity U) is kept constant, and heat loss to the surrounding is negligible.

Based on the previously reported experimental observations (Sliwinski et al., 1978; Baines et al., 1983; Pak, 1991), the mixing phenomenon due to the momentum of incoming fluid is modelled as forming a fluid layer in the upper part of the tank, the temperature and depth of the layer are uniform and invariant. Note here that the depth of this layer depends on both the inlet configuration (i.e. the type of distributor) and the flow rate (Zurigat et al., 1991; Ghajar and Zurigat, 1991; Sliwinski et al., 1978; Baines et al., 1983; Pak, 1991). Accordingly, the tank consists of two regions: perfectly mixed region of the height H_m and plug flow(or themocline) rigion of H_p. Since the multi-dimensional effects, such as no-slip condition on the inner surface of the tank and natural convection caused by conduction through the wall, are known to be secondary (Hollands and Lightstone, 1989; Zurigat et al., 1989), the model can be regarded as one-dimensional. The following two facts support the validity of this two-region, one-dimensional model. One is that a similar model has been employed in the numerical analysis (Hess and Miller, 1982). The other is that an analytical solution to the same model for constant inlet temperature has successfully resolved the thermal behavior compared with the experimental data (Yoo et al., 1997).

Referring to Fig.1, the transient temperature of perfectly mixed region $T_m(t)$ can be

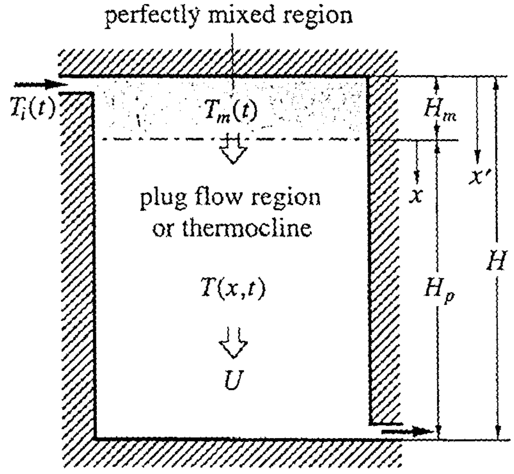

Fig.1 Schematic of the present two-region one-dimensional model.

expressed in terms of the inlet temperature $T_i(t)$ only, whereas that of plug flow region $T(x, t)$ is determined such that the interfacial condition $T(0, t)=T_m(t)$ is satisfied. If fluid mixing is prefectly suppressed ($H_m=0$) by either an ideal distributor or an extremely low flow rate, then $T_m(t)=T_i(t)$. In the analysis, we have to derive $T_m(t)$ first and $T(x, t)$ next. Hereafter, dimensionless quantities defined in Nomenclature are used instead of dimensional ones.

2.2 Piecewise linear approximation

In general, the inlet temperature is an arbitary function of time, depending on available heat sources and operating modes of the system. This study considers only the case of temperature increasing with time, so that a thermally stable stratification can be kept in the tank throughout the charging process. Otherwise, buoyancy-driven mixing makes the present model incompatible(Csordas et al., 1992). However, the variation pattern encompasses both continuous and step changes, as illustrated in Fig.2.

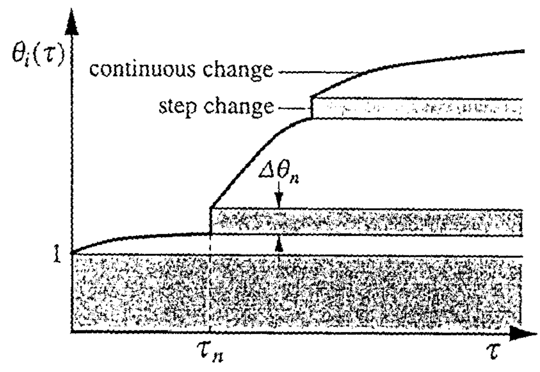

Fig.2 A typical variation pattern of the inlet temperature and the influence of step change on the thermal behavior.

In order to handle the variation of inlet temperature effectively, it is decomposed into two groups: step change and interval of continuous change (Kays and Crawford, 1993). The transient response inside the tank is different for the two changes. In view of the previous studies for constant inlet temperature (Yoo, 1995; Yoo and Pak, 1995), the step change characterized by the time of occurrence τ_n and the increment $\Delta\theta_n$ (see the shadings in Fig.2) can be analyzed easily. On the other hand, the interval of continuous change needs to be simplified because it is not an elementary function of time. Even so, analytical treatments of it seem to be nearly intractable. A first simplification would be to approximate the continuous interval as a number of finite-size steps, which is conceptually identical with the Duhamel's theorem. This approach is simple, but a considerable number of steps are needed to obtain an acceptable result (Yoo, 1995). As an alternative to improve it within the extent of analytical treatments, piecewise linear approximation is introduced. Figure 3 demonstrates its concept, and the shaded area represents the influence of a linear segment.

As a result of the approximation, the vari-

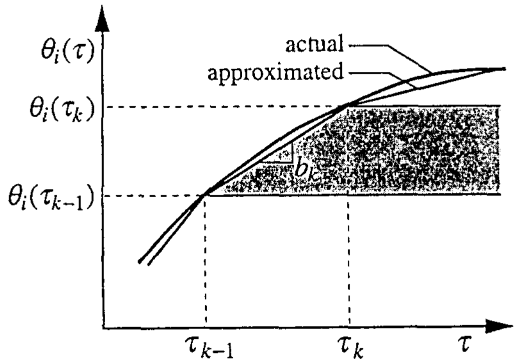

Fig.3 Piecewise linear approximation for the interval of continuous change and its influence on the thermal behavior.

ation of inlet temperature can be described by the summation of K linear segments and N step changes.

$$\theta_i(\tau) \cong \sum_{k=1}^{K} b_k[(\tau-\tau_{k-1})H(\tau-\tau_{k-1}) - (\tau-\tau_k)H(\tau-\tau_k)] + \sum_{n=0}^{N-1} \Delta\theta_n H(\tau-\tau_n) \quad (1)$$

where the Heaviside's unit function $H(\tau-\tau_j)$ is defined as

$$\begin{aligned} H(\tau-\tau_j) &= 0 \quad \text{for} \quad \tau < \tau_j \\ H(\tau-\tau_j) &= 1 \quad \text{for} \quad \tau > \tau_j \end{aligned} \quad (2)$$

Note here that $\Delta\theta_0 = 1$ and $\tau_0 = 0$ by definition. The first summation term in equation (1) expresses the influence of linearized inlet temperature confined within each segment.

3. Analysis

3.1 Perfectly mixed region

The transient temperature of prefectly mixed region in response to the variation of inlet temperature is ruled by

$$\frac{d\theta_m(\tau)}{d\tau} = a[\theta_i(\tau) - \theta_m(\tau)], \quad \theta_m(0) = 0 \quad (3)$$

where $a = u/h_m$. Equation (3) has the solution of

$$\theta_m(\tau) = a\, e^{-a\tau} \int_0^{\tau} e^{a\lambda}\theta_i(\lambda)d\lambda \quad (4)$$

for an arbitrary $\theta_i(\tau)$. Therfore, $\theta_m(\tau)$ corresponding to equation (1) is readily obtained,

$$\theta_m(\tau) = \sum_{k=1}^{K} b_k[F(\tau-\tau_{k-1}) - F(\tau-\tau_k)] + \sum_{n=0}^{N-1} \Delta\theta_n[1 - e^{-a(\tau-\tau_n)}]H(\tau-\tau_k) \quad (5)$$

where the function $F(\tau-\tau_j)$ is defined as

$$F(\tau-\tau_j) = \{(\tau-\tau_j) - [1 - e^{-a(\tau-\tau_j)}]/a\} H(\tau-\tau_j) \quad (6)$$

Note that equation (5) reduces to equation (1) in case of without mixing, i.e. $h_m = 0$ and thereby $a = \infty$.

3.2 Plug flow region

The thermal behavior in this region is described by (Cabelli, 1977; Yoo and Pak, 1993; Yoo and Pak, 1996)

$$\frac{\partial\theta}{\partial\tau} + u\frac{\partial\theta}{\partial\xi} = \frac{\partial^2\theta}{\partial\xi^2} \quad (7)$$

It is evident that the initial condition is

$$\theta(\xi, 0) = 0 \quad (8)$$

At the interface between two regions, as mentioned earlier, equation (7) must satisfy

$$\theta(0, \tau) = \theta_m(\tau) \quad (9)$$

The boundary condition at the tank exit has been imposed in two distinct ways. If the solution domain is assumed to be semi-infinite as in the work of Cabelli (1977), the condition takes the form of

$$\theta(\infty, \tau) = 0 \tag{10}$$

On the other hand, the condition for a finite domain with an adiabatic end, i.e.

$$\frac{\partial \theta(h_p, \tau)}{\partial \xi} = 0 \tag{11}$$

was also adopted in recent works (Yoo et al., 1996; Yoo and Pak, 1996). Qualitatively speaking, the former is easier to solve, whereas the latter seems to be closer to the physical reality. For the later use, let the result corresponding to each of them be the semi-infinite and adiabatic solutions, respectively. In practice, the quantitative difference between the two solutions is small enough to be neglected for the conventional operating conditions of stratified thermal storage systems (shown in Appendix B). On this basis, the present study employs the former. Since the inlet temperature is irrelevant to the end condition, the adiabatic solution may replace the semi-infinite counterpart in the final results whenever necessary.

Since equation (7) is linear and homogeneous, the principle of superposition holds. Note that $\theta_m(\tau)$ in equation (5) is composed of three types of function: constant, linear and exponential with respect to τ, though shifted in time, i.e.

$$\theta(0, \tau) = 1 \tag{12}$$

$$\theta(0, \tau) = \tau \tag{13}$$

$$\theta(0, \tau) = e^{-a\tau} \tag{14}$$

Therefore, the confronting task is to solve equation (7) subject to each type of the above interfacial conditions together with the initial and end conditions. These fundamental solutions are demarcated by the superscripts c, l and e, respectively.

The solution for the case of constant inlet temperature has already been derived via the Laplace transform (Yoo and Pak, 1993), In this study, the solution procedure is extended to cover the problem with polynomial-type interfacial conditions, the details of which are stated in Appendix A. The fundamental solutions corresponding to equation (12) and (13) are

$$\theta^c(\xi, \tau) = [\operatorname{erfc}(\zeta_1) + e^{u\xi} \operatorname{erfc}(\zeta_2)]/2 \tag{15}$$

$$\theta^1(\xi, \tau) = [(u\tau - \xi) \operatorname{erfc}(\zeta_1) + (u\tau + \xi) e^{u\xi} \operatorname{erfc}(\zeta_2)]/(2u) \tag{16}$$

respectively, where the abbreviated variables are defined as $\zeta_1 = (\xi - u\tau)/\sqrt{4\tau}$ and $\zeta_2 = (\xi + u\tau)/\sqrt{4\tau}$. Using the recurrence formula for the integrals of the complementary error function (Carslaw and Jaeger, 1959)

$$\operatorname{ierfc}(\zeta) = e^{-\zeta^2}/\sqrt{\pi} - \zeta \operatorname{erfc}(\zeta) \tag{17}$$

$$2n \, i^n \operatorname{erfc}(\zeta) = i^{n-2} \operatorname{erfc}(\zeta) - 2\zeta \, i^{n-1} \operatorname{erfc}(\zeta) \tag{18}$$

equation (16) can be cast in an alternative form

$$\theta^1(\xi, \tau) = (u\sqrt{4\tau})[\operatorname{ierfc}(\zeta_1) - e^{u\xi} \operatorname{ierfc}(\zeta_2)]/(2u^2) \tag{19}$$

The fundamental solution for the case of exponential-type interfacial condition has also been obtained (Yoo and Pak, 1995) as

$$\theta^e(\xi, \tau) = e^{-a\tau}[e^{(u-v)\xi/2} \operatorname{erfc}(\eta_1) + e^{(u+v)\xi/2} \operatorname{erfc}(\eta_2)]/2 \tag{20}$$

where $\eta_1 = (\xi - v\tau)/\sqrt{4\tau}$, $\eta_2 = (\xi + v\tau)/\sqrt{4\tau}$ and $v = 2[(u/2)^2 - a]^{1/2}$.

In consequence, the temperature profile in plug flow region is expressed as a linear combination of the fundamental solutions:

$$
\begin{aligned}
&\theta(\xi,\ \tau)\\
&=\sum_{k=1}^{K} b_k\{\theta^1(\xi, \tau-\tau_{k-1})-[\theta^{c}(\xi, \tau-\tau_{k-1})\\
&\quad-\theta^{e}(\xi, \tau-\tau_{k-1})]/a\}H(\tau-\tau_{k-1})\\
&\quad-\sum_{k=1}^{K} b_k\{\theta^1(\xi, \tau-\tau_k)-[\theta^{c}(\xi, \tau-\tau_k)\\
&\quad-\theta^{e}(\xi, \tau-\tau_k)]/a\}H(\tau-\tau_k)\\
&\quad+\sum_{n=0}^{N-1}\Delta\theta_n[\theta^{c}(\xi, \tau-\tau_n)\\
&\quad-\theta^{e}(\xi, \tau-\tau_n)]H/(\tau-\tau_n)
\end{aligned}
\tag{21}
$$

Setting $\xi' = \xi + h_m$ (see Fig.1) for convenience, the temperature in $\xi' \le h_m$ is given by equation (5), and that in $h_m < \xi' \le 1$ by equation (21).

3.3 Adiabatic solution

As pointed out earlier, the above results have been obtained for the semi-infinite domain. If the adabatic end condition, eqution (11), is imposed instead of equation (10), the procedure for deriving the fundamental solutions differs substantially from that of Appendix A (Yoo et al., 1996; Yoo and Pak, 1996). In order to meet presumable needs and to complete the analysis, the fundamental solutions for the finite domain with the adiabatic end are also dealt with here.

The adiabatic solutions for the cases of constant and exponential-type interfacial conditions are already available from the previous studies (Yoo et al., 1996; Yoo and Pak, 1996). They are rewritten in the present notation as follows:

$$
\begin{aligned}
&\theta^{c}_{ad}(\xi,\ \tau)=\theta^{c}(\xi,\ \tau)\\
&\quad+e^{uh_p}[\operatorname{erfc}(\phi_1)-(u\sqrt{\tau})\operatorname{ierfc}(\phi_1)]/2
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
&\theta^{e}_{ad}(\xi,\ \tau)=\theta^{e}(\xi,\ \tau)\\
&\quad+u^2\ e^{uh_p}\operatorname{erfc}(\phi_1)/(2a)\\
&\quad+\frac{e^{-a\tau}}{2}\left[\left(\frac{v-u}{v+u}\right)e^{(u+v)\xi/2-vh_p}\operatorname{erfc}(\psi_1)\right.\\
&\quad\left.+\left(\frac{v+u}{v-u}\right)e^{(u-v)\xi/2+vh_p}\operatorname{erfc}(\psi_2)\right]
\end{aligned}
\tag{23}
$$

where $\phi_1=(2h_p-\xi+u\tau)/\sqrt{4\tau}$, $\psi_1=(2h_p-\xi-v\tau)/\sqrt{4\tau}$ and $\psi_2=(2h_p-\xi+v\tau)/\sqrt{4\tau}$. In addition, the adiabatic solution for linear interfacial condition is sought in the same manner as equations (22) and (23). Because of highly complicated mathematical manipulations, only the result is presented here

$$
\begin{aligned}
&\theta^{1}_{ad}(\xi,\ \tau)=\theta^{1}(\xi,\ \tau)\\
&\quad+e^{uh_p}[-(u\sqrt{4\tau}\operatorname{ierfc}(\phi_1)\\
&\quad+(u\sqrt{4\tau})^2\ i^2\operatorname{erfc}(\phi_1)]/(2u^2)\\
&\quad+[e^{-u(h_p-\xi)}\operatorname{erfc}(\phi_2)\\
&\quad-e^{uh_p}\operatorname{erfc}(\phi_1)]/(2u^2)
\end{aligned}
\tag{24}
$$

where $\phi_2=(2h_p-\xi-u\tau)/\sqrt{4\tau}$.

Each adiabatic fundamental solution given by equations (22)~(24) seems to differ considerably from the corresponding semi-infinite solution. However, the additional terms appeared in the adiabatic solutions have negligible magnitudes under the practical charging conditions, so that the two sets of solutions nearly coincide. This is discussed in Appendix B. It can be deduced at this stage that the present results, equations (5) and (21), are valid as far as the end condition is concerned. The temperature profile for the adiabatic and condition, if necessary, can be readily obtained by replacing θ^c, θ^e and θ^1 in equation (21) with θ^c_{ad}, θ^e_{ad} and θ^1_{ad}, respectively.

4. Validation and Application

In order to validate the present approximation and to show the utility of the results, sample calculations for a specific case were performed. Since the published experimental data for variable inlet temperature are limited or measured in a situation unsuitable (temperature decreasing with time) to compare, selected here is the case for which an exact solution exists. In fact, the exact solution, if available, is preferable in comparison to the experimental data, since they imply uncertainties associated with measurements and data reduction to a certain degree.

As an example, consider the case that the inlet temperature is a quadratic function of time, specifically

$$\theta_i(\tau) = 1 + (u\tau)^2 \tag{25}$$

where $u\tau$ (=Pe · Fo) stands for the normalized turn-over time. Figure 4 visualizes the example. This case possesses good properties required for the present purpose, not only because it consists of a step change and continuous interval, but also because the exact

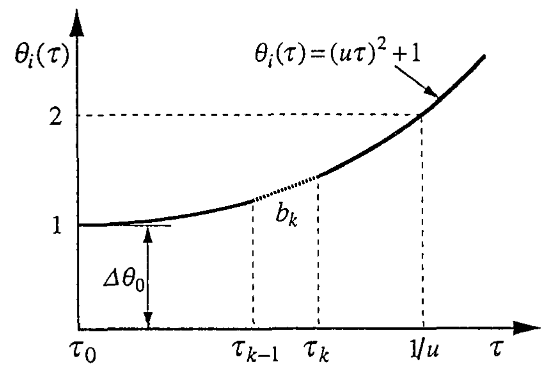

Fig.4 An example of variable inlet temperature: a quadratic function of time.

solution can be derived. Substituting equation (25) into equation (4) yields the temperature of perfectly mixed region,

$$\theta_m(\tau) = (1+2u^2/a^2)\ (1-e^{-a\tau}) - 2u^2\tau/a + (u\tau)^2 \tag{26}$$

Since the interfacial condition is composed of constant, linear, quadratic and exponential functions of time, and corresponding fundamental solutions have already been obtained (equations (15), (16) or (19), (20) and (A9)), the exact solution for plug flow region can be written immediately as follows:

$$\theta(\xi,\ \tau) = (1+2u^2/a^2)\ [\theta^c(\xi,\ \tau) - \theta^e(\xi,\ \tau)] - (2u^2/a)\theta^1(\xi,\ \tau) + u^2\ \theta^q(\xi,\ \tau) \tag{27}$$

Three different levels of piecewise linear approximation have been examined by segmenting one turn-over time into one, two and four pieces of equal spacing, i.e. K=1, 2 and 4 during $0 \le u\tau \le 1$ along with N=1. Since the comparison between the approximate and exact solutions is aimed at showing the validity and utility of this study, the mixing depth h_m, which significantly affects the thermal behavior in the tank (Yoo and Pak, 1995; Yoo et al., 1997), is also changed while the Peclet number is fixed (Pe=500). For ease of understanding, the temperature profiles are plotted as a function of the position measured from the top (ξ') instead of from the interface between the two regions (ξ).

Figures 5 and 6 illustrate the vertical temperature profiles by the approximate and exact solutions at Pe · Fo=0.6 for h_m=0 and 0.2, respectively. First of all, the approximate solutions come close to the exact solution with increasing the number of segments regardless of the mixing depth. In the present example, four-segment approximation appears to pro-

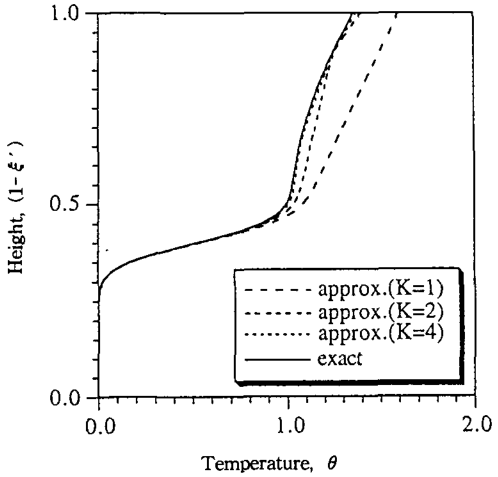

Fig.5 An example of variable inlet temperature: a quadratic function of time.

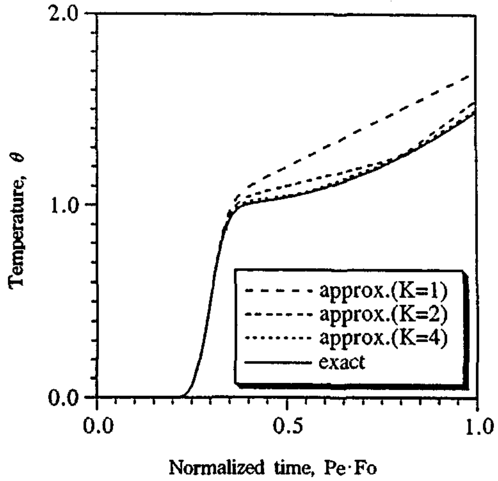

Fig.7 Comparison of the temperature variation at a fixed position(ξ'=0.3) between the exact and approximate solutions of three different levels for h_m=0 and Pe=500.

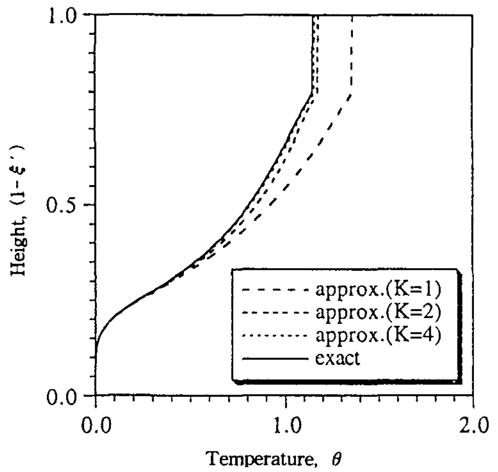

Fig.6 Comparison of the vertical temperature profile at a fixed time(Pe · Fo=0.6) between the exact and approximate solutions of three different levels for h_m=0.2 and Pe=500.

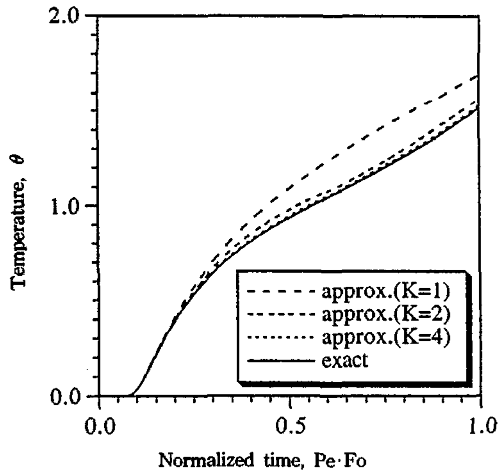

Fig.8 Comparison of the temperature variation at a fixed position(ξ'=0.3) between the exact and approximate solutions of three different levels for h_m=0.2 and Pe=500.

duce tolerable predictions. Of course, the more number of segments would provide the better result. This trend is confirmed again in Figs. 7 and 8 where the transient responses at a fixed position in the tank (ξ'=0.3) corresponding to Figs. 5 and 6, respectively, are monitored. Of particular interest is that the mixing depth alters the thermal behavior significantly, as has been known for the case of constant inlet temperature (Yoo and Pak, 1995;

Yoo et al., 1997), but does not affect the degree of discrepancy between the exact and approximate solutions appreciably. That is, the number of segments to obtain acceptable results is nearly independent of the mixing depth. It is worth noting that consistent overpredictions of the approximate solutions in all plots are not the nature of this study, but merely caused by the concave variation pattern of inlet temperature. If it were convex, underpredictions would emerge.

The foregoing discussion reveals that approximating an arbitrary variation of inlet temperature as a combination of the step change and piecewise linear function can resolve the thermal behavior in the tank to a required level of accuracy. In addition, the present approach can cover the charging process with or without momentum-induced mixing. Since the approximate solution is expressed in terms of well-defined functions, it is easy and efficient to use in comparison with the numerical analysis.

5. Summary

In order to predict the thermal behavior durig the charging process of stratified thermal storage tanks with variable inlet temperature, approximate analytical approach to a two-region one-dimensional model which accounts for the fluid mixing has been attempted. First, an arbitrarily varying inlet temperature was decomposed into a number of continuous and discontinuous changes. Next, each continuous interval was approximated as a set of piecewise linear functions. Treated in this manner, the temperature of perfectly mixed region admits an analytical solution, which constitutes the interfacial condition for the adjacent plug-flow region.

Three different types of function emerges in the transient temperature of perfectly mixed region: constant, linear and exponential with respect to time. For each of them, the governing equation of plug-flow region subject to the semi-infinite end condition yielded the fundamental solution which is compactly expressed in terms of well-defined functions. Based on the principle of superposition, the temperature profile of plug flow region could be determined as a linear combination of these solutions. In this procedure, a systematic method for solving the plug-flow problem having polynomial-type inlet conditions has been established. As supplements to the semi-infinite solutions, a set of the fundamental solutions corresponding to the adiabatic end condition have also been presented. However, the difference between them proved to be negligible.

In order to validate the present approach, the approximate solutions of different levels were compared with the exact solution for a quadratic variation of the inlet temperature. The error induced by the approximation decreases sharply with increasing number of segments regardless of the mixing depth, which suffices to show the practical utility of this study. In view of such capability and ease in use, the present solutions are expected to serve as an approximate analytical tool to predict the thermal behavior in stratified storage tanks.

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Appendix A : Generalized solution proceure for plug flow

Described here is a generalized solution procedure for equation (7) subject to the initial condition, equation (8), end condition, equation (10), and inlet condition of a polynomial function with respect to τ, i.e.

$$\theta(0, \tau) = \tau^m \ (m \geq 0) \tag{A1}$$

The Laplace transformation of equation (7) together with equation (8) results in the following subsidiary equation:

$$\frac{d^2\bar{\theta}}{d\xi^2} - u\frac{d\bar{\theta}}{d\xi} - s\bar{\theta} = 0 \tag{A2}$$

where $\bar{\theta}$ is the transformed θ. Solving equation (A2) and applying the transforms of equations (10) and (A1), we have

$$\bar{\theta}(\xi, s) = m!\ e^{u\xi/2}\ e^{-[(u/2)^2+s]^{1/2}\xi}/s^{m+1} \tag{A3}$$

Substituting equation (A3) into the inversion theorem (Carslaw and Jaeger, 1959) and converting the variable of integration (Yoo and Pak, 1993; Yoo and Pak, 1996) gives

$$\theta(\xi, s) = m!\ e^{u\xi/2-(u/2)^2\tau} L^{-1}\{\ e^{-\sqrt{s'}\xi} /[s' - (u/2)^2]^{m+1}\} \tag{A4}$$

where the converted Laplace variable is defined as $s' = s + (u/2)^2$.

In case of $m=0$, the inversion is straightforward. With the aid of the table of Laplace transfrom

$$\mathcal{L}^{-1}\{\ e^{-\sqrt{s'}\xi}/[s' - (u/2)^2]\} = e^{-u\xi/2+(u/2)^2\tau}\ [\mathrm{erfc}(\zeta_1) + e^{u\xi}\ \mathrm{erfc}(\zeta_2)]/2 \tag{A5}$$

equation (A4) readily reduces to equation (15). When $m \geq 1$, on the other hand, the convolution theorem should be invoked to accomplish the inversion. Using the inverse transform of an elementary function

$$\mathcal{L}^{-1}\{[s' - (u/2)^2]^{-m}\} = \tau^{m-1}\ e^{(u/2)^2\tau}/(m-1)! \tag{A6}$$

and equation (A5), equation (A4) can be rewritten as

$$\theta(\xi, \tau) = \frac{m}{2}\int_0^\tau (\tau-\lambda)^{m-1}\ [\mathrm{erfc}(\zeta_1) + e^{u\xi}\ \mathrm{erfc}(\zeta_2)]d\lambda \tag{A7}$$

Note that the variables in the integrand, ζ_1 and ζ_2, defined earlier should be expressed in terms of λ instead of τ.

The integration on RHS of equation (A7) can be performed without difficulty for $m=1$, furnishing equation (16), whereas it poses complicated manipulations for $m \geq 2$. In this study, the result only for $m=2$ (denoted by the superscript q) is derived to provide the solution for the example problem considered in the main text as follows:

$$\theta^q(\xi,\ \tau) = \{(u\sqrt{4\tau})\,[-\mathrm{ierfc}(\zeta_1) + e^{u\xi}\,\mathrm{ierfc}(\zeta_2)] + (u\sqrt{4\tau})^2\,[\mathrm{i}^2\mathrm{erfc}(\zeta_1) + e^{u\xi}\,\mathrm{i}^2\mathrm{erfc}(\zeta_2)]\}/u^4 \tag{A8}$$

Appendix B : The effect of end condition

Among three types of the inlet condition for plug-flow region, the case of constant temperature is selected as a representative to estimate the effect of end condition. It is self-evident in equation (22) that the difference between the semi-infinite and adiabatic solutions is

$$\Delta\theta^c(\xi,\ \tau) = e^{uh_p}\,[\mathrm{erfc}(\phi_1) - (u\sqrt{\tau}\,\mathrm{ierfc}(\phi_1)]/2 \tag{B1}$$

According to the properties of the complementary error function, $\Delta\theta^c$ attains its maximum at $\xi = h_p$ (the bottom of the tank) and $u\tau = 1$ (time of a turn-over), i.e. at $\phi_1 = \sqrt{u}(1 + h_p)/2$. With the aid of the formula (Carslaw and Jaeger, 1959)

$$\mathrm{erfc}(\zeta) = \frac{e^{-\zeta^2}}{\pi^{1/2}}\left(\frac{1}{\zeta} - \frac{1}{2\zeta^3} + \frac{1\cdot 3}{2^2\zeta^5} - \frac{1\cdot 3\cdot 5}{2^3\zeta^7} + \cdots\right) \tag{B2}$$

and equation (17), the magnitude of $\Delta\theta^c_{\max}$ can be approximately estimated by expanding it in series and truncating the higher order terms, i.e.

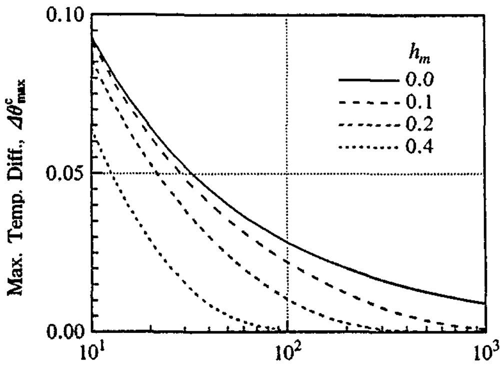

Fig.B1 Maximum temperature differences between the semi-infinite and adiabatic solutions under constant inlet condition as a function of the Peclet number for different mixing depths.

$$\Delta\theta^c_{\max} \approx \frac{2h_p\, e^{-u(1-h_p)^2/4}}{(\pi u)^{1/2}(1+h_p)^2} \tag{B3}$$

Figure B1 shows $\Delta\theta^c_{\max}$ for selected values of the mixing depth $h_m(=1-h_p)$ as a function of the Peclet number. They decrease sharply with increasing both Pe($=u$) and h_m. In view of $\Delta\theta_c \le \Delta\theta^c_{\max}$ and $u \gg 10^2$ in the actual charging process, the difference may be neglected in the analysis.

Since the end condition is concerned only with thermal diffusion, it can be deduced without quantitative evaluation that its effect on the thermal behavior in the tank is insignificant under a convection-dominated (i.e. high Pe) situation. In addition, the difference between the semi-infinite and adiabatic solutions for the other inlet conditions is not likely to exceed the magnitude of equation (B3).