International Journal of Mangement Science
Vol 4, No 2, November 1998

SYNCHRONIZING INDIVIDUALLY OPTIMAL CYCLE TIMES ACROSS MULITI-BUYERS AND MULTI-PRODUCTS

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(Received March 1998; revision received June 1998)

ABSTRACT

A joint problem of order delivery, setup reduction, and cost-sharing in a two-echelon inventory system in which a vendor supplies multiple products to a group of buyers is studied here. The basic premise is that buyers have independently implemented setup reduction programs to acquire benefits from small order sizes. Doing so, however, causes the buyers' individually optimal order cycles to be differ from that of the vendor. In conjunction with this, two models are considered. In the first model, a multi-buyers single product situation is considered in which the vendor implements a joint supply cycle policy. However, buyers, as the dominant party, insist after implementing the individually optimal setup reduction that the vendor accept their individually optimal order schedules. In the second model, a multi-products, single buyer situation is considered in which the buyer implements a joint order policy. Here, the vendor, as the dominant party, refuses to cooperate fully with the buyer's individually reduced joint order schedule, and designs his own individually optimal setup reduction mix for each product under a given budget constraint. This led to a study of an integrated Setup Reduction/Break-even Pricing Policy for each situation to eliminate mismatches in individually optimal cycle times.

1. INTRODUCTION

The benefits of reduced setup are well known in today's manufacturing environment. These include, reduced production lot sizes, lower inventory levels, reduced cycle time, and increased flexibility in the production system. A substantial literature has been developed on how a company should invest in a set-

* The research is supported by the grant from the Daewoo Research Foundation(1997).

up reduction program to reduce its cycle times (see, for example, Porteus [11, 12, 13], Leschke and Weiss [7]). However, earlier researchers have focused mostly on a single-level, single-product setup reduction problem. Relatively limited research has been done from a multi-products (see, for example, Leschke and Weiss [7]) or multi-echelon standpoint. The purpose of this study is to extend the problem of investing in setup reduction from a single-level, single-product setup reduction situation to a two-echelon, multi-products, or multi-buyers situation. Although the idea may be simple, new questions appear, especially related to the joint problem of order delivery, setup reduction coordination, and cost-sharing.

The purpose of this paper, therefore, is to extend the previous research in a way that provides new insights into how managers should coordinate setup reductions in a multi-party situation. Specifically, we model a hypothetical two-echelon, EOQ-like inventory system in which a vendor supplies multiple products to a group of buyers. The nature of the production technology is such ``that the setup reduction program is available to both the vendor and buyers. Assume now that buyers (buyer) have independently initiated setup reduction programs to acquire benefits from small orders. Two models are considered. In the first, a multi-buyers, single product situation is analyzed in which the vendor has been implementing a joint supply policy, and supplying products to each buyer based on a common supply cycle. Buyers' new individually optimal setup reductions cause mismatches between the vendor's joint supply schedule and buyers' individually optimal order schedules. Buyers, as the dominant party, insist that the vendor accepts their separate and individually optimal order schedules. In the second, a multi-products, single buyer situation is considered in which the buyer has been implementing a joint order policy, and placing orders based on a joint order cycle approach. Here, the vendor, as the dominant party, refuses to cooperate fully with the buyers' individually reduced joint order schedule, decides to follow his own individually optimal setup reduction mix for each product under a given budget constraint, and supplies each product separately. An integrated Setup Reduction/Break-even Pricing Policy aimed at synchronizing mismatched cycle times is considered in each model.

The Break-even Pricing approach used in this study is based on studies of quantity discount models to increase vendors' profit. The purpose of these models is to decide on a joint policy of ordering and offering discounts by a vendor to his sole buyer, with the vendor's objective being to entice the buyer to alter his order quantities in order to increase the vendor's profit. Examples of these studies include Monahan [8,9], Lee and Rosenblatt [6], Banerjee [1], Joglekar [3], Kohli and Park [4,5]. Weng and Wong [14], and Weng [15].

2. MULTI-BUYER MODEL

We now consider the first model. Here, a single product is supplied by a vendor to a group of buyers. We will refer to the buyer as k and the vendor as V in our distribution channel. Generally, then, we will use subscripts k and V to designate buyer k's and the vendor's set of parameters. Let us assume a traditional EOQ world in which:

D_k = Yearly demand; P = Current delivered unit price;

Q =Order size; Ψ = Set of buyers (k=1...n);

h_k, H = Holding cost for the buyer k, and the vendor;

r_k, R = Fractional per unit time opportunity cost of capital;

a_k, A, A_k = Setup cost for placing an order by the buyer k; the vendor who practice joint supply cycle approach; and the vendor who supplies each order separately;

a_k^0, A^0, A_k^0 = Setup costs prior to setup reductions;

i_k, I, I_k =Setup reduction investments by the buyer, the vendor who practices a joint supply cycle approach; and the vendor who supplies separately each order;

$\tau_k, \mathrm{T}, \mathrm{T}_k > 0$ = Fraction reduction in setup cost;

$i_k = b_k \ln\left(a_k^0/(1-\tau_k)a_k^0\right)$ = The setup reduction investment of changing the setup cost from a_k^0 to $(1-\tau_k)a_k^0$ (For example, see Porteus [11] for a detailed description of this function.) Rearranging the term shows that $(1-\tau_k)=\exp(-i_k/b_k)$. (See, for example, Leschke and Weiss [7].) Here, $B, B_k, b_k > 0$ represents the cost of making about a 63% reduction in the setup costs for the vendor and buyers. For the sake of convenience, the following notations are used throughout the model: We label j be the base buyer, and let ratios of setup costs be $\theta_k = a_k^0/a_j^0$ and $\theta_V = A^0/a_j^0$. Ratios of setup reduction investment costs be $\beta_k = b_k r_k/b_j r_j$, $\beta_V = BR/b_j r_j$, and $\beta_{Vk} = B_k R/b_k r_k$. Fraction reduction in setup costs, $\overline{\tau}_k = b_k r_k/m_k^R a_k^0 = \left(b_k r_k/m_k^0 a_k^0\right)^2 = 2(b_k r_k)^2/D_k h_k a_k^0$ where $\overline{\tau}_k = 1-\tau_k < 1$. Similarly, $\overline{\mathrm{T}} = BR/m^R A^0$. Here, m_k^R and m^R denote order frequencies for buyer k and the vendor (see (1r) and (2r)). Consider a vendor supplying a product to n buyers. The vendor has been practicing a joint supply cycle approach, and the joint supply cycle per year is determined by $m = D_j/Q_j = D_k/Q_k \quad \forall k \neq j$. Assume the setup reduction option is available to the vendor, and the setup reduction investment function is logarithmic as discussed by Porteus [11]. Subtract production setup, setup reduction investment costs, and holding cost from the gross sales, and let $\Pi(m,I)$ denote the vendor's net annual profit; then (see Leschke and Weiss [7]) the vendor's annual profit is given by the following expression.

$$\Pi(m,I) = \sum_{k=1}^{n}(D_k P_k - HD_k/2m) - mA^0\exp(-I/B) - RI \tag{1}$$

From the vendor's perspective, the individually optimal joint supply frequency (more accurately, "preferred buyer's joint order frequency") and the setup reduction investment cost maximizing profit function (1) are given by $m^* = [\sum_{k=1}^{n} D_k H/2A^0\exp(-I^*/B)]^{1/2}$ and $I^* = B\ln(m^* A^0/BR)$, respectively. Upon solving the two simultaneous equations, the optimal solutions are:

$$I^U = \max[0, B\ln(1/\bar{\mathrm{T}})] \text{ and } m^U = \max\left[\left(\sum_{k=1}^{n} D_k H \Big/ 2A^0\right)^{1/2}, \sum_{k=1}^{n} D_k H \Big/ 2BR\right] \tag{1r}$$

Let $m^0 := \left(\sum_{k=1}^{n} D_k H \;/2A^0\right)^{1/2}$, and $m^R := \sum_{k=1}^{n} D_k H/2BR$. Similarly, let $C(m_k, i_k)$ denote the annual inventory-related cost for buyer k. Then, the cost function and the optimal solutions are given by the following expression:

$$C(m_k, i_k) = P_k D_k + h_k D_k/2m_k + a_k^0 m_k \exp(-i_k/b_k) + r_k i_k \quad \forall k \in \Psi \tag{2}$$

$$i_k^U = \max[0, i_k^R, i_k^R = b_k \ln(1/\bar{\tau}_k), \text{ and } m_k^U = \max[m_k^0, m_k^R], \; m_k^0 = (D_k h_k/2a_k^0)^{1/2},$$
$$m_k^R = D_k h_k / 2b_k r_k \tag{2r}$$

We assume throughout the work that the base buyer's initial optimal investment is $i_j^U = i_j^R > 0$. Given (1r) and (2r), we see that the vendor can no longer now implement the joint supply cycle approach due to mismatches in individually optimal cycle times. For example, between the base buyer j and the vendor, mismatch occurs when $m^U \geq (<) m_j^R$, which leads to $\max[\beta_V, \theta_V/\bar{\tau}_j] \leq (>) \sum_{k=1}^{n} D_k H/D_j h_j$.

2.1 Vendor's Policies

In the following works, a policy aimed at eliminating mismatches in individually optimal cycle times is analyzed from the vendor's perspective.

Break-even Quantity Discount (QD) Policy: The vendor provides Quantity Discount Pricing Schedules to induce buyers to adjust their order frequencies according to a new joint supply(order) cycle. Given that the buyers have accepted this request, they then modify their setup reductions according to the new joint order cycle. The possibility of modifying the setup reduction program is based on Porteus [11], who states that the setup reduction investment can be regarded as a lease that can be broken on occasion, and a new setup cost level selected. All extra costs incurred by buyers from making the cooperative adjustments will be compensated by the vendor through Joint Supply Break-even Quantity Discount Pricing Schedules (JQD). For those buyers who do not agree with the joint order cycle, an individual pair (vendor-single buyer) base order frequency is determined. A Separate Supply Break-even Quantity Discount Schedule (SQD) is then designed as a cost-sharing means to induce the buyer to adjust the schedule. Finally, the vendor invests in the setup reduction program to increase his ability to handle a busier schedule.

2.2 Buyer's Adjustment

Suppose now that the vendor asks buyers to adjust their order frequencies m_k^U by δ_k times. Then, with no other compensating changes, adoption of this request raises the annual cost to

$$C(\delta_k m_k^U, i_k) = P_k D_k + h_k D_k / 2\delta_k m_k^U + a_k^0 \delta_k m_k^U \exp(-i_k/b_k) + \bar{\gamma}_k r_k (i_k^U - i_k) + r_k i_k$$
$$\text{where } \bar{\gamma}_k \in [0,1] \text{ if } i_k^U > i_k, \text{and } \gamma_k = 1 \text{ if } i_k^U \le i_k \tag{3}$$

Here, $\bar{\gamma}_k = 1 - \gamma_k$ represents the non-retrievable fraction of the investment in the setup reduction program. It is included in the formulation to account for the penalties of breaking the "lease," if any, resulting from reducing the investment in the setup reduction program. This function form is chosen to reflect the negative effect incurred by reversing of the process improvement, such as setup reduction. One example of such an effect is the poor quality control resulting from increased production lot size. (See Porteus [12] for the relationship between lot sizing and quality control.) Assume after the order frequency is boosted by δ_k that the buyer has the option of adjusting his current setup reduction investment. Let

$\Delta C(\delta_k, i_k) = C(m_k^U, i_k^U) - C(\delta_k m_k^U, i_k)$ denote the yearly cost difference before and after the order frequency and setup reduction investment adjustment. The buyer's decision problem here is to decide on a new investment level $i_k^* \in [0, i_k^R]$ so as to maximize the cost difference $\Delta C(\delta_k, i_k)$ for a given δ_k.

$$\Delta C(\delta_k, i_k) = m_k^U a_k^0 \left[\exp(-i_k^U/b_k) - \exp(-i_k/b_k)\delta_k\right] + \gamma_k r_k (i_k^U - i_k) + h_k D_k (1 - 1/\delta_k)/2m_k^U \tag{4}$$

For $m_k^U = m_k^R$ ($i_k^U = i_k^R$), let *CY, Y=1,2, 3,* and *4* denote four cases in which case *C1* applies when $1 > \delta_k/\gamma_k > \overline{\tau}_k$, *C2* applies when $\delta_k/\gamma_k \le \overline{\tau}_k$, *C3* applies when $1/\gamma_k \ge \delta_k/\gamma_k \ge 1$, and *C4* applies when $\delta_k > 1$. For $m_k^U = m_k^0$ ($i_k^U = 0$), let *CY, Y=5* and *6* denote two cases in which case *C5* applies when $\delta_k > \overline{\tau}_k^{1/2}$ and *C6* applies when $\delta_k \le \overline{\tau}_k^{1/2}$. The main results are summarized in Proposition 1.

Proposition 1. (**Proof**. See Appendix 1)
(a) $\Delta C(\delta_k, i_k)$ has a unique maximum $i_k^*(\delta_k)$.
(b) The necessary condition for optimizing $\Delta C(\delta_k, i_k)$ is $i_k^*(\delta_k) = i_k^R + b_k \ln(\delta_k/\gamma_k)$ when $m_k^U = m_k^R$; it is partitioned into four cases based on the specific values of δ_k. Similarly, when $m_k^U = m_k^0$, the necessary condition for optimizing $\Delta C(\delta_k, i_k)$ is $i_k^*(\delta_k) = i_k^R/2 + b_k \ln(\delta_k)$; it is partitioned into two cases. The optimal solution for buyer k is given by the following expression:

$$i_k^*(\delta_k) = \begin{bmatrix} when \;\; m_k^U = m_k^R;\; C1{:}\; i_k^R + b_k \ln(\delta_k/\gamma_k);\; C2: 0;\; C3{:}\; i_k^R;\; C4{:}\; i_k^R + b_k \ln(\delta_k) \\ \\ when \;\; m_k^U = m_k^0;\;\; C5{:}\; i_k^R/2 + b_k \ln(\delta_k);\; C6{:}\; 0 \end{bmatrix} \tag{5}$$

The corresponding fraction reduction is adjusted to $(\overline{\tau}_k \gamma_k/\delta_k \;|\, 0 \,|\, \overline{\tau}_k \,|\, \overline{\tau}_k/\delta_k)$ when cases (*C1* | *C2* | *C3* | *C4*) apply, respectively. □

When $m_k^U = m_k^R$, Proposition 1 reveals that buyer *k*'s optimum policies are either (*C1*) rollback of the initial setup reduction investment (here, the condition

$1 > \delta_k/\gamma_k > \overline{\tau}_k$ implies the adjustment amount $b_k \ln(\delta_k/\gamma_k)$ of the initial investment i_k^R to be negative, and the adjusted investment $i_k^R + b_k \ln(\delta_k/\gamma_k)$ to be positive), or (*C2*) not investing in setup reduction ($i_k^R + b_k \ln(\delta_k/\gamma_k) \le 0$), or (*C3*) maintaining the initial setup reduction investment, or (*C4*) further reducing the setup cost from the initially reduced level. Note that the rollback adjustment of the initial setup reduction investment i_k^R does not occur when $m_k^U = m_k^0$, since no investments are made initially. Therefore, cases *C1* and *C3* discussed in $m_k^U = m_k^R$ are not considered here. For case *C1* (the adjusted fraction reduction being $\overline{\tau}_k \gamma_k/\delta_k < 1$), the optimal policy (5) reveals that (i) the higher the retrievable γ_k or (ii) the smaller the multiplier δ_k, the greater the rollback (the more negative the rollback adjustment $b_k \ln(\delta_k/\gamma_k)$) in the initial setup reduction investment that takes place.

Note that by choosing the optimal (δ_k^*, i_k^*) the vendor will offer a discriminated total quantity discount of $\Delta C(\delta_k^*, i_k^*)$ to each buyer , and the unit quantity discount price leads to $P_k^{QD} = P + \Delta C(\delta_k^*, i_k^*)/D$ so that the buyer is indifferent to the adjustment.

2.3 Vendor's Adjustment

We now turn our attention to the vendor. Here, the vendor offers a quantity discount schedule to synchronize each buyer's individually optimal order frequency. The vendor's profit function is given by the following expression:

$$\Pi(\delta_k m_k^U, I) = \sum_{k=1}^{n} \left[D_k P_k - HD_k/2\delta_k m_k^U + \Delta C(\delta_k, i_k) \right] - A^0 \delta_V m^U \exp(-I/B) - IR$$

$$\text{S.T.} \quad \delta_V m^U = \delta_j m_j^R = \delta_k m_k^U = \phi \ \ \forall k \neq j \in \Psi \qquad (6)$$

We seek to maximize the annual profit $\Pi(\delta_k m_k^U, I)$ by optimizing over I and δ_k, $\forall k \in \Psi$. Since $\delta_k = \delta_j m_j^R / m_k^U \ \ \forall k \in \Psi$ and $\delta_V = \delta_j m_j^R / m^U$, our approach in this paper is to optimize over δ_j for a given I first, and then optimize over $I(\delta_j^*)$. Let *CX*, *X=A,B* denote two cases, where case *CA* applies when $\delta_j > \overline{\tau}_j(\beta_V/\theta_V)$, and *CB* applies for the opposite case.

Proposition 2. **(Proof** : See Appendix 2)

(a) $\Pi(\delta_k m_k^U, I)$ has a unique maximum $\delta_k^*(I, i_k)$, and $\Pi(\delta_k^* m_k^U, I)$ has a unique maximum $I^*(\delta_k^*)$.

(b) The necessary conditions maximizing (6) are given by the following expressions:

$$\delta_j^*(I, i_j^*) = \left(\frac{\overline{\tau}_j \sum_{k=1}^{n} D_k (H + h_k) / D_j h_j}{\theta_V \exp(-I/B) + \sum_{k=1}^{n} \theta_k \exp(-i_k^*/b_k)} \right)^{1/2} ; \ \delta_k^* = \delta_j^* m_j^R / m_k^U \text{ , and}$$

$$(I^*, \overline{T}^*) = \left[CB:\ (0,0);\ CA:\ \left(i_j^R B/b_j + B\ln(\delta_j^* \theta_V / \beta_V),\ \overline{T}/\delta_V^*\right) \right]. \tag{7}$$

□

The expression of I^* reveals that the vendor's setup reduction investment policy is closely related to the buyer's setup reduction investment i_j^R, β_V, θ_V and δ_j. Therefore, for example (i) the less the buyer invests in setup reduction, or (ii) the more costly (less costly) the vendor's setup reduction cost (setup cost) compare to the buyer, or (iii) the greater the buyer's order frequency adjustment, the fewer vendor's efforts are expended on setup reduction. The intuition behind Proposition 2(b) becomes clear when we interpret the problem from the perspective of the vendor-buyer joint inventory cost. Let $TSC(\bullet)$ denote vendor-buyer joint inventory cost, then the cost is given by the following expression:

$$TSC(\phi, I, i_k) = \sum_{k=1}^{n} \left[P_k D_k + h_k D_k / 2\phi + a_k^0 \phi \exp(-i_k/b_k) + r_k i_k \right]$$
$$+ \sum_{k=1}^{n} HD_k / 2\phi + \phi A^0 \exp(-I/B) + RI$$

Minimizing with respect to ϕ, and comparison with Proposition 2(b) reveal that $\phi^* = \left[\sum_{k=1}^{n} D_k (H + h_k) / 2\left(A_0 \exp(-I/B) + \sum_{k=1}^{n} a_k^0 \exp(-i_k/b_k)\right)\right]^{1/2} = m_j^R \delta_j^*$. Hence, we see that the optimal boost factor δ_j^* provided in Proposition 2(b) is designed to adjust current order frequency m_j^R to meet the joint optimal order frequency

Assuming now that $m_k^U = m_k^R \quad \forall k \in \Psi$, and $m^U = m^R$. Substituting the optimal policies of joint supply (JQD) and separate supply (SQD) into supply chain cost reveals:

$$\underset{JQD}{TSC}\left(\phi^*, I_k^*, i_k^*\right) - \sum_{k \in \Psi} \underset{SQD}{TSC}\left(\phi_k^*, I_k^*, i_k^*\right) \geq 0 \Rightarrow$$

$$\left\{\left[\sum_{k \in \Psi} a_k^0 \exp\left(-i_k^*/b_k\right) + A^0 \exp\left(-I^*/B\right)\right] \sum_{k \in \Psi} D_k\left(h_k + H\right)\right\}^{1/2}$$

$$- \sum_{k \in \Psi} \left\{\left[a_k^0 \exp\left(-i_k^*/b_k\right) + A_k^0 \exp\left(-I_k^*/B_k\right)\right] D_k\left(h_k + H\right)\right\}^{1/2} \geq \frac{R\left(\sum_{k \in \Psi} I_k^* - I^*\right)}{\sqrt{2}} \qquad (8)$$

If this condition applies, the vendor will abandon the joint supply approach and adopt the separate supply. Assuming now that case *CA1* applies to all buyers and the vendor both for joint supply and separate supply policies, then (8) can be rearranged to:

$$\frac{\left\{\sum_{k \in \Psi} \left[\left(\frac{a_k^0 \bar{\tau}_k \gamma_k}{\delta_k^{SQD}} + \frac{A_k^0 \bar{T}_k}{\delta_{Vk}}\right) D_k\left(h_k + H\right)\right]^{1/2}\right\}^2}{\left\{\left(\sum_{k \in \Psi} \frac{a_k^0 \bar{\tau}_k \gamma_k}{\delta_k^{JQD}} + \frac{A^0 \bar{T}}{\delta_V}\right) \sum_{k \in \Psi} D_k\left(h_k + H\right)\right\}} \leq \left\{1 - \frac{R\left[\sum_{k \in \Psi} B_k \ln\left(\delta_{Vk}/\bar{T}_k\right) - B \ln\left(\delta_V/\bar{T}\right)\right]}{\left\{2\left(\sum_{k \in \Psi} \frac{a_k^0 \bar{\tau}_k \gamma_k}{\delta_k^{JQD}} + \frac{A^0 \bar{T}}{\delta_V}\right) \sum_{k \in \Psi} D_k\left(h_k + H\right)\right\}^{1/2}}\right\}^2$$

where δ_k^{JQD} and δ_k^{SQD} denote adjustment factors for buyer k when the buyer accepts (JQD) and refuses (SQD) the joint supply policy. Similarly, δ_V and δ_{Vk} denote adjustment factors for the vendor implementing joint supply and separate supply policies. Here, $I_k^* = \max\left(0, B_k \ln\left(\delta_{Vk}/\bar{T}_k\right)\right)$ and $\bar{T}_k = B_k R / m_{Vk}^R A_k^0$, where $m_{Vk}^R = D_k H / 2 B_k R$ denotes individually optimal separate supply frequency analogous to (1r). The above condition holds when (i) the variability in $D_k\left(h_k + H\right)$ or (ii) the variability in $a_k^0 \bar{\tau}_k \gamma_k + A_k^0 \bar{T}_k = \left[\left(b_k r_k\right)^2 / D_k h_k + \left(B_k R\right)^2 / D_k H\right]$ across buyers is large. The variability increases with increased variability in individual setup reduction investment costs ($b_k r_k$ or $B_k R$). We also see that the above condition applies when (iii) the sum of individual investment $\sum_{k \in \Psi} B_k \ln\left(\delta_{Vk}/\bar{T}_k\right)$ is relatively less costly compared to joint investment $B \ln\left(\delta_V/\bar{T}\right)$. This condition applies when the joint setup reduction investment cost B is significantly larger than the sum of individual setup reduction program $\sum_k B_k$. These observations are all based on the assumption that case *CA1* applies.

Combining the two partitioning systems, we obtain cases *CXY, X=A,B* and *Y=1,2,3,* and *4.* For example, $CA1 := \{\delta_j^* > \overline{\tau}_j\,(\beta_V/\theta_V)(CA),\ \gamma_j > \delta_j^* > \gamma_j\overline{\tau}_j\ (C1)\}$. Other cases can be defined similarly.

Figure 1 illustrates the optimal setup reduction policies for the vendor and the base buyer. The horizontal coordinate is divided into four sub-areas based on the index set $(\gamma_j\overline{\tau}_j, \gamma_j, 1)$ provided in Proposition 1. This index set describes the buyer's setup reduction investment policy. For example, when $\delta_j^* \leq \gamma_j\overline{\tau}_j$, Case 2 applies, and the buyer's optimal setup reduction policy results in no investment in setup reduction. Figure 1 also provides the second index $\overline{\tau}_j\,(\beta_V/\theta_V)$ that splits the vendor's setup reduction policies provided in Proposition 2. Figure 1 reveals

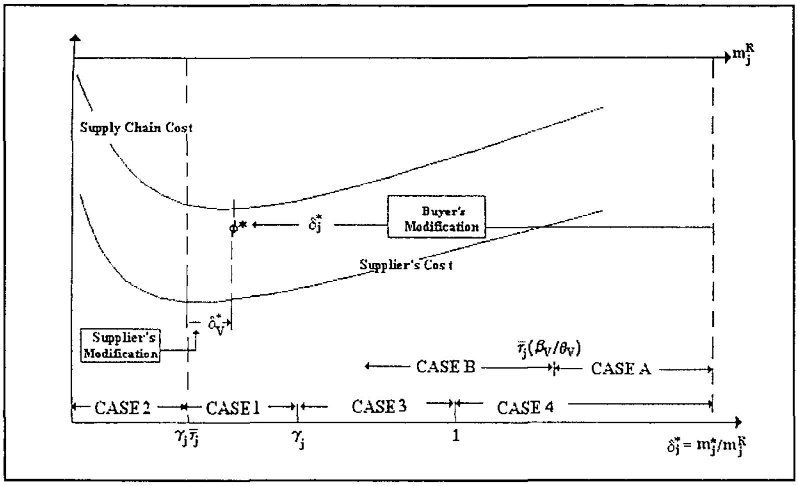

Figure 1 reveals Figure 1. Feasible Areas for Different Cases

that the possibility of both the buyer and the vendor making the investment in setup reduction increases as *Case 1+Case 3+Case 4* and *Case A* increase. We see that these areas increase as $\overline{\tau}_j$, β_V and γ_j decrease or θ_V increases. We will explain this result, but before doing so we need to derive a simultaneous solution $\delta_j^*(I^*, i_j^*)$.

2.4 Simultaneous Solutions

In this section we provide simultaneous solutions based on the results provided in Propositions 1 and 2. Let ΨY, *Y=1,2,3,4,5,* and *6* denote sets of buyers satisfying cases *CY, Y=1,2,3,4,5,* and *6,* and $\Psi = \bigcup_{Y=1}^{6} \Psi Y$.

Proposition 3. (**Proof** : See Appendix 3)

(a) Let $\varphi_X = 1$ or 0, when *CX=A* or *B* apply, and $\varphi_A + \varphi_B = 1$. Also for the purpose of the following proposition, let $\Gamma_1 = \left(\overline{\tau}_j \beta_V \varphi_A + \sum_{k \in \Psi 1} \overline{\tau}_j \beta_k \gamma_k + \sum_{k \in \Psi 3,4\&5} \overline{\tau}_j \beta_k\right)$, $\Gamma_2 = \left(\theta_V \varphi_B + \sum_{k \in \Psi 2\&\Psi 6} \theta_k\right)$ respectively. The simultaneous optimal adjustment factors are given by the following expressions:

Optimal Policy δ_j^*

CASE_ SIMPLE	*CASE_ NOT_ SIMPLE*
if ψ*1,3,4,5=*ψ *and CA applies*	*Otherwise*
$\dfrac{\sum_{k \in \Psi} D_k (H_k + h_k) / D_j h_j}{\beta_V + \sum_{k \in \Psi 1} \gamma_k \beta_k + \sum_{k \in \Psi 3,4\&5} \beta_k}$	$\dfrac{-\Gamma_1 + \sqrt{\Gamma_1^2 + 4\Gamma_2 \overline{\tau}_j \sum_{k \in \Psi} D_k (H_k + h_k) / D_j h_j}}{2\Gamma_2}$

(b) Let $\Omega = (H + h)/h$. For SQD schedule, since only one buyer is considered, we drop subscripts *k* and *V.* Let δ_{XY}, *X=A,B* and *Y=1,2,3* denote optimal adjustment factors $\delta^*(I^*, i^*)$ obtained from substituting $I^*(\delta^*)$ and $i^*(\delta^*)$as provided in Propositions 1 and 2 (see Proposition 3(c).)

For case *X=A* and *B*:

(i) $\delta_{x1}/\gamma < (\ge) 1$ *iff* $\delta_{x3}/\gamma < (\ge) 1$, and it implies $\delta_{X1} < (\ge) \delta_{X3}$.

(ii) $\delta_{x1}/\gamma > (\le) \overline{\tau}$ *iff* $\delta_{x2}/\gamma > (\le) \overline{\tau}$, and it implies $\delta_{X2} < (\ge) \delta_{X1}$.

(iii) $\delta_{x2}/\gamma \le \overline{\tau}$ $(\delta_{x2}/\gamma > \overline{\tau})$ *iff* $\delta_{x4} \le 1$ $(\delta_{x4} > 1)$, and it implies $\delta_{X2} \ge (<) \delta_{X4}$.

(iv) $\delta_{x3} \ge (<) 1$ *iff* $\delta_{x4} \ge (<) 1$, and it implies $\delta_{X4} < (\ge) \delta_{X3}$.

(v) For case Y=*1,2 3,* and *4*: $\delta_{BY} \le (>) \overline{\tau}(\beta/\theta)$ *iff* $\delta_{AY} \le (>) \overline{\tau}(\beta/\theta)$, and implies $\delta_{BY} \ge (<) \delta_{AY}$.

(c) For the purpose of the following proposition, let $J(X,Y) := (X)^2 / \theta + (Y)^2$. Cases *CXY,* after substituting simultaneous optimal solutions $\delta^*(I^*, i^*)$, result in the following eight mutually exclusive and collectively exhaustive cases.

Cases CA1- CB4

$$\begin{aligned}
CA1 &:= \{(\gamma+\beta)\beta\bar{\tau} < \Omega\theta, (\gamma+\beta)\gamma > \Omega > \bar{\tau}(\gamma+\beta)\gamma\} \\
CB1 &:= \{(\gamma+\beta)\beta\bar{\tau} \ge \Omega\theta, J(\gamma\bar{\tau}\theta, \gamma\bar{\tau}) < \Omega\bar{\tau} < J(\gamma\theta, \gamma\bar{\tau})\} \\
CA2 &:= \{J(\beta,\beta) < \Omega\theta/\bar{\tau}, \Omega \le \bar{\tau}(\gamma+\beta)\gamma\} \\
CB2 &:= \{J(\beta,\beta) \ge \Omega\theta/\bar{\tau}, \Omega\bar{\tau} \le J(\gamma\bar{\tau}\theta, \gamma\bar{\tau})\} \\
CA3 &:= \{J(\beta\bar{\tau}, \beta\sqrt{\bar{\tau}}) < \Omega\theta, 1+\beta \ge \Omega \ge (\gamma+\beta)\gamma\} \\
CB3 &:= \{J(\beta\bar{\tau}, \beta\sqrt{\bar{\tau}}) \ge \Omega\theta, \bar{\tau}+\theta \ge \Omega\bar{\tau} \ge J(\gamma\bar{\tau}\theta, \gamma\bar{\tau})\} \\
CA4 &:= \{(1+\beta)\beta\bar{\tau} < \Omega\theta, \Omega > 1+\beta\} \\
CB4 &:= \{(1+\beta)\beta\bar{\tau} \ge \Omega\theta, \Omega\bar{\tau} < \bar{\tau}+\theta\}
\end{aligned}$$

Cases *CA1-CB4* apply if and only if the simultaneous solutions $\delta^*(I^*, i^*)$ are:

Optimal Order Frequency Adjustment $\delta^*(I^*, i^*)$

CA1,4:	$\Omega/(\beta+\gamma_1)$	*CB1,4:*	$\left(-\gamma_1\bar{\tau}+\sqrt{(\gamma_1\bar{\tau})^2+4\Omega\theta\bar{\tau}}\right)/2\theta$
CA2,3:	$\varphi\left(-\beta+\sqrt{\beta^2+4\Omega/\varphi}\right)/2$	*CB2,3:*	$\sqrt{\Omega\bar{\tau}/(\theta+\varphi)}$

where $\varphi = \bar{\tau}$ *when CA2 or CB3 applies and* $\varphi = 1$ *otherwise; and* $\gamma_1 = \gamma$ *when C1 applies and* $\gamma_1 = 1$ *otherwise. CA3* ($\Omega \le 1+\beta \Rightarrow \beta \ge H/h$) *and CB3* ($\bar{\tau}+\theta \ge \Omega\bar{\tau} \Rightarrow \theta \ge \bar{\tau}H/h$) *ensure that* δ_{A3} *(CA3) and* δ_{B3} *(CB3) are greater or equal to 1. These conditions were discussed in section 2. Similarly, we obtain optimal policies for CXY , X=A,B and Y=5,6: A5:* $\sqrt{\bar{\tau}}\Omega/(\sqrt{\bar{\tau}}\beta+1)$; *A6:* $\bar{\tau}\left(-\beta+\sqrt{\beta^2+4\Omega/\bar{\tau}}\right)/2$; *B5:* $\sqrt{\bar{\tau}}\left(-1+\sqrt{1+4\Omega\theta}\right)/2\theta$; and *B6:* $\sqrt{\Omega\bar{\tau}/(\theta+1)}$. □

Consider the single buyer-vendor case. We see that the vendor can invest in setup reduction in cases *CA1,3,* and *4,* given that the buyer can invest in setup reduction (cases *1, 3,* and *4*). Proposition 3(b) reveals that the vendor can invest in setup reduction if the first conditions of *CA3:* $\beta^2\bar{\tau}(\bar{\tau}/\theta+1) < \Omega\theta$, and *CA1:* $(\gamma+\beta)\beta\bar{\tau} < \Omega\theta$ apply. Given that the vendor can invest in setup reduction (case *A*), the buyer can invest in setup reduction in all cases except *CA2,* which leads to $\Omega > \bar{r}\gamma(\gamma + \beta)$. Therefore, if these three conditions are satisfied, both vendor and buyer can invest in setup reduction. We see that the first two conditions hold when (i) BR is relatively small (the vendor can invest in the setup reduction less costly), or (ii) θ is relatively large (causing the buyer's individually optimal order frequency to be higher than the vendor's, hence increasing the vendor's need to reduce the setup cost), or (iii) H is relatively large (leading to a higher vendor's individually optimal order frequency which requires a lower setup cost to offset the increased setup frequency). $\Omega > \bar{r}\gamma(\gamma + \beta)$holds when (iv) br is relatively small (leads to a relatively lower initially reduced buyer's setup cost; therefore, the possibility that the buyer's new setup cost after the roll-back adjustment is still lower than the originally unreduced setup cost increases), or (v) γ is relatively small (it is not rational for the buyer to significantly adjust his originally reduced setup cost). Note that once *CA1* or *CA3* apply, and it is optimal for both the vendor and buyer investing in setup reductions, if D is increased, they will continue to invest in setup reduction. The reason behind it can be described keeping in mind that the buyer's individually optimal order frequency m^R_{Buyer} is increasing in the demand rate, whereas the optimal adjustment factors δ_{A1} and δ_{A3} do not depend on that. Hence, if the demand rate increases, the joint optimal order frequency $m^R_{Buyer}\delta^*$ will increase. Since both the vendor's and the buyer's optimal setup reduction investments and m^R_{Buyer} are proportional, each will continue to invest in the setup reduction. When both vendor and buyer can invest in setup reductions, the buyer will adjust more (smaller δ_{A1}) when (i) a large fraction (γ) of the investment can be retrieved, or (ii) the vendor's setup reduction cost is relatively expensive (leading to a relatively larger optimal production lot size for the vendor), or (iii) H/h is relatively small (leading to a smaller m^U and a smaller m^R_{Buyer}). These observations are all based on the assumption that *CA1* or *CA3* apply. Analysis of the other cases is similar and therefore omitted.

Here the numerical example furnished in Monahan (1988) is provided to better understand the model. We assume the distribution channel consists of a supplier and sore buyer. Let D=10,000, $h = 2$, $r, R = 0.2$, and $P = 10$. All numerical parameters have been kept with a few exceptions to the new parameters. We let $a^R = 100$ (buyer's individually reduce setup cost), $b = 5000$, and $a^0 = 500$. The model experiments with different levels $H = 1.4$, 2, $A^0 = 800$, 500, and β=1, 2.5, and 10. For the purpose of comparison, we will restrict our analysis on three relevant cost components: setup cost, holding cost, and setup reduction costs. Table 1 summarizes the numerical experiment. For example, "DN cost" (order frequency being individually determined by the buyer) gives the cost of "Do Nothing" policy "Cost" exhibits the cost resulted from following the optimal policy provided in our model. For example, R1 and R2 give the vendor's and buyer's cost before Quantity Discount. The Break-even Quantity Discount (QD) can be obtained from R2–R4. The vendor's (buyer's) costs including (excluding) QD can be obtained from R1+QD (R2-QD). The vendor's cost savings can be obtained by R5= ma$[0, R3 - R1]$, which after subtracting the "Break-even Quantity Discount" results in the "Net cost savings".

Table 1. Results of the Numerical Example

	$a^0/a^R = 8$				$a^0/a^R = 5$			
	β=1	β=2.5	β^*=2.5	β=10	β=1	β^*=1	β=2.5	β=10
CASE	CA1	CA1	CA1	CB1	CA1	CA1	CB1	CB1
δ	0.68	0.38	0.43	0.35	0.68	0.75	0.43	0.43
S_1	148	259	233	283	148	133	233	233
S_2	148	647	583	800	148	133	500	500
COST								
***R1*:Vendor**	3205	3935	4458	3817	2735	2989	2961	2961
• QD	89	639	484	790	89	45	484	484
Includes QD	3294	4257	4942	4607	2824	3034	3445	3445
***R2*:Buyer**	3698	4248	4093	4399	3698	3654	4093	4093
Excludes QD	3609	3609	3609	3609	3609	3609	3609	3609
(R1+R2)	6903	8183	8551	8216	6433	6643	7054	7054
				DN Cost				
S_2	100	250	250	800	100	100	250	500
***R3*:Vendor**	3429	5758	5907	8350	2959	3109	4582	5350
***R4*:Buyer**	3609	3609	3609	3609	3609	3609	3609	3609
(R3+R4)	7038	9367	9516	11959	6568	6718	8189	8959
				Vendor's Cost Savings From CS/SC Policy				
♦*R5:* Savings	224	1823		4533	224		1621	2389
R5-QD	135	1184		743	135		1137	1905
% to ***R3***	4.0	20.1		44.8	4.6		24.8	35.6

•The vendor's break-even quantity discount =max[0, R2-R4]

*The β^* corresponds to the case $H = 2$.

♦The vendor's Savings =max [0,R3-R1]

Table 1 verifies (a^*, A^*, δ^*) (i) increasing in β, and (ii) decreasing in H. It shows us that the vendor's "Net cost savings" increases as vendor's setup reduction cost (BR) increases. In the optimal policy, an expensive vendor's setup reduction cost leads to a smaller reduction in the vendor's setup cost. Hence, comparing to "reactively" maintaining the buyer's individually optimal order frequency (as does in the DN policy) and investing in a rather expensive setup reduction program, more savings are generated by "actively" inducing the buyer to adjust his order frequency by the δ times. The numerical example shows that our model provides a cooperative policy that minimizes the total cost of the supply chain.

We now give the algorithm for the muti-buyers case. This algorithm uses optimal policies provided in Proposition 3 to search for the optimal $\delta_k^* \ \forall k \in \Psi$.

The ALGORITHM MULTI_BUYER

Step 1: Let $\Psi2^{OLD}, \Psi3^{OLD}, \Psi6^{OLD} = \phi$, $\Psi1^{OLD}, \Psi4^{OLD}, \Psi5^{OLD} = \Psi^{OLD}$, $CX^{OLD} = CA$, and δ_j =CASE_SIMPLE.

Step 2: Check conditions $CY \ \forall k \in \Psi Y^{OLD}$. Let $k \in \Psi Y^{NEW}$ if conditions CY holds. Check the condition CX for vendor. Let it be CX^{NEW}. If $\Psi Y^{OLD} = \Psi Y^{NEW}$ and $CX^{OLD} = CX^{NEW}$
then Stop. Let $\delta_j^* := \delta_j$, $\delta_k^* := \delta_j^* m_j^R / m_k^U$. Else let $\Psi Y^{OLD} := \Psi Y^{NEW}$ and $CX^{OLD} := CX^{NEW}$.
Go to Step 3

Step 3: Compute δ_j (by CASE_NOT_SIMPLE), and let $\delta_k := \delta_j m_j^R / m_k^U$. Go to Step 2.

To illustrate the preceding analysis, consider the following example involving two buyers. Let D_1=1000 denote buyer 1 and D_2=2000, buyer 2. Let a_1^0=\$15/setup for buyer 1 and a_2^0=\$60/setup for buyer 2, and cite an inventory holding cost of h_1=\$3/unit/year and h_2=\$6/unit/year for buyers 1 and 2, respectively. We also let the opportunity cost rate for buyer 1 and 2 be $r_1 = r_2$=0.2, and setup reduction constant b_1=\$500 and b_2=\$1,000 for buyers 1 and 2, respectively. Similarly, let the vendor's setup cost be A_1^0=\$75/setup and A_2^0=\$100/setup, and an inventory holding cost of H=\$2/unit/year and opportunity cost rate R=0.2. We let the setup reduction constant be B_1=\$2,000 and B_2=\$1,500. If the vendor passively gives in to the buyer's individually optimal order schedule, and does nothing to alter the order frequencies, then the optimal setup reduction investment level will be decided independently for each buyer with order frequencies m_k^U. The corresponding total transactions cost incurred by the vendor and the buyers is \$3633, of which buyer 1's (2's) total cost is \$281 (\$839). The vendor's cost corresponding to buyer 1(2) is \$880 (\$1,632). Buyers order frequencies are m_1^R=15 (m_2^R=30) for buyer 1(2). The setup cost is reduced to $\overline{\tau}_1$=44% ($\overline{\tau}_2$=11%) of the original setup cost for buyer 1(2). If the vendor offers a SQD for each buyer, the optimal order frequency is $\delta_1 m_1^R$=0.94(15)=14.1 for buyer 1 and $\delta_2 m_2^R$= 0.53(30)=15.9 for buyer 2, where the corresponding total transactions cost incurred by the vendor and buyers is \$2,866, in which buyer 1's (2's) total cost is \$281 (\$889); the vendor's cost corresponding to buyer 1(2) is \$859 (\$835). The quantity discount given to buyers is \$50. Now consider joint orders for the two products. Let the vendor's joint setup cost be A^0=\$110, and the setup reduction constant be B=\$2,500. The optimal joint order frequency is synchronized to $\delta_1 m_1^R = \delta_2 m_2^R = \delta_V m^R$=13.12. The total transaction cost incurred by the vendor and the buyers is \$2,471, in which buyer 1's(2's) total cost is \$282 (\$929); and the vendor's cost is \$1,350. The quantity discount given to buyers is \$91. Therefore, separate setup reduction and quantity discount policy reduces the total transaction cost by 26.76%, from \$3,633 to \$2,866. Joint order policy further reduces the total transaction cost by 16%, from \$2,866 to \$2,471.

3. MULTI-PRODUCTS MODEL

We now consider the second model. Here, a vendor supplies a set of k products to a buyer. We will use subscripts k and B to designate product k's and buyer's parameters. The notations used in section 2 are mostly preserved, except that the meanings have been changed here accord with the new problem set. For example, D_k denotes the demand rate for product k. Similarly, A_k^0, B_k, A^0, H_k, and B denote the vendor's set of parameters, and a^0, h_k, and b denote the buyer's set of parameters. Let $\theta_k = A_k^0/a^0$; $\beta_k = B_k R/br$; $\overline{\tau} = \left(br/m^0 a^0\right)^2$; and $\overline{\mathrm{T}}_k = B_k R/m_k^{eR} A_k^0 = \left(B_k R/m_k^0 A_k^0\right)^2$ (see (9r.3), for m_k^{eR}). Here, the buyer's joint order cycle is determined by $m = D_j/Q_j = D_k/Q_k \quad \forall k \neq j$. Assume now that the buyer has independently implemented a setup reduction program to reduce the joint order (setup) cost, that is, his order frequency is being increased to $m^R = \sum_{k=1}^{n} D_k h_k / 2br > m^0 = \left(\sum_{k=1}^{n} D_k h_k / 2a^0\right)^{1/2}$, with setup reduction investment cost $i^R = b\ln(1/\tau)$. The vendor, as the dominant party, refuses to cooperate fully with the buyer's initiation, and decides to design his own individually optimal setup reduction mix for each product under a budget constraint. In conjunction with this, an adjustment process is proposed from the buyer's perspectives.

(1) *Vendor's supply schedule adjustment and setup reduction investment.* The buyer provides a Premium Pricing Schedule to induce the vendor to adjust his supply frequencies for each product. Given that the vendor has accepted this request, he then designs his setup reduction mix under a given budget constraint. All extra costs incurred by the vendor from making the cooperative adjustments will be compensated by the buyer through a Break-even Premium Pricing Schedule. Furthermore, when no further setup reduction investment is possible due to the budget constraint, the buyer will substitute for the vendor, and invest in the vendor's setup reduction if he so decides. A similar concept has been reported in Cheng and Podolsky [2] p. 109:

> ...The completion of supplier audit should set the stage for education of supplier.....A 'supplier quality training plan' was developed to assist the supplier in improving the quality and ensuring on-time delivery.....

In terms of Total Quality Cost, such training cost is often referred to as prevention costs, and sometimes can occupy as much as 4% of total sales revenue. (See, for example, Oakland and Porter [10], p. 132.)

(2) *Buyer's joint order schedule and setup reduction adjustment.* The buyer adjusts his joint order frequency and modifies investment in setup reduction.

Suppose now that the buyer asks the vendor to adjust his individually optimal supply frequency m_k^U (see optimal policy (9r.3)) by δ_k. Then, with no other compensating changes, adoption of this request changes the annual cost to:

$$\Pi\left(\delta_k m_k^U, I_k, \varpi\varsigma_k\right) = \sum_{k=1}^{n}\left[D_k P_k - H_k D_k / 2\delta_k m_k^U - \delta_k m_k^U A_k^0 \exp\left(-I_k/B_k\right)\exp\left(-\varpi\varsigma_k/B_k\right) - RI_k\right]$$

$$\text{S.T. } \sum_{k=1}^{n} I_k \le BGT; \quad \varpi = 0(=1) \text{ if } \sum_{k=1}^{n} I_k < (=) BGT \tag{9}$$

Here, *BGT* denotes the available setup reduction investment. The setup reduction investments $\sum_{k=1}^{n}\varsigma_k$ in excess of *BGT* will be invested by the buyer as a supplement. Let $\Delta\Pi(\delta_k, I_k, \varpi\varsigma_k) = \Pi\left(m_k^U, I_k^U\right) - \Pi\left(\delta_k m_k^U, I_k, \varpi\varsigma_k\right)$ denote the supplier's profit loss resulting from making the adjustment. When $m_k^U = m_k^{eR}$ (see (9r.3)), let *CA* denote the case in which $\delta_k > \overline{T}_k$, and *CB*, for the opposite case. Similarly, when $m_k^U = m_k^0$ (see (9r.3)), let *CA* denote the case in which $\delta_k > \overline{T}_k^{1/2}$, and *CB*, for the opposite case. Without the budget constraint, the optimal solutions are given by the following expression:

$$I_k^{\max} = \begin{bmatrix} \text{when } m_k^U = m_k^{eR} & I_k^{eR} + B_k \ln(\delta_k) & \text{if } CA;\ 0 \ \text{if } CB \\ \\ \text{when } m_k^U = m_k^0 & I_k^{eR}/2 + B_k \ln(\delta_k) & \text{if } CA;\ 0 \ \text{if } CB \end{bmatrix}. \tag{9r.1}$$

where $I_k^{eR} = B_k \ln(1/\overline{T}_k)$ gives the unconstrained optimum before the δ_k − adjustment. With the given budget constraint, a marginal analysis similar to that of Leshke and Weiss [7] can be implemented to decide the optimal investment level for each product. The marginal function for the product k is given by the following expression:

$$\partial \Delta \Pi(\delta_k, I_k, \varpi\varsigma_k)/\partial I_k = -\exp(-(I_k + \varpi\varsigma_k)/B_k) m_k^U A_k^0 \delta_k / B_k + R. \tag{9r.2}$$

The optimal investment level is seen to be $I_k^* \in [0, I_k^{\max}]$. Let I_k^{Pre} denote the optimum investment level decided from the marginal analysis, before being adjusted δ_k times such that $\sum_{k=1}^{n} I_k^{\mathrm{Pre}} \le BGT$. Then the individually optimal order frequencies for product k are given by the following expression, analogous to (1r).

$$m_k^U = \max[m_k^0, m_k^R \ ; \ m_k^0 = [D_k H_k / 2A_k^0]^{1/2}, \text{ and } m_k^R = [D_k H_k / 2A_k^0 \exp(-I_k^{\mathrm{Pre}}/B_k)]^{1/2} \tag{9r.3}$$

If $I_k^{\mathrm{Pre}} = I_k^{eR}$, then the second term is reduced to $m_k^{eR} = D_k H_k / 2B_k R$. Now, let $C(\delta_k m_k^U, i, \varsigma_k)$ denote the buyer's yearly cost when he offers an incentive to the vendor so that the latter will accept the joint order (supply) approach. The cost function is given by the following expression:

$$C(\delta_k m_k^u, i, \varsigma_k) = \sum_{k=1}^{n} [P_k D_k + h_k D_k / 2\delta_k m_k^u + \Delta\Pi(\delta_k, I_k^*, \varpi\varsigma_k)]$$

$$+a^0\delta_B m^R \exp(-i/b)+ri+\bar{\gamma}r\left(i^R-i\right)+r\sum_{k=1}^{n}\varsigma_k$$

$$\text{S.T. } \delta_B m^R=\delta_j m_j^U=\delta_k m_k^U \quad \forall k\neq j\in\Psi \tag{10}$$

As mentioned, the extra investments ς_k will be paid by the buyer when $\sum_{k=1}^{n} I_k^* =$BGT. Here, we seek to minimize the annual cost $C\left(\delta_k m_k^U, i, \varsigma_k\right)$ by optimizing over i, ς_k and δ_k. We optimize over δ_B for a given i and ς_k first, and then optimize over $i\left(\delta_B^*\right)$ and $\varsigma_k\left(\delta_B^*\right)$. Let CY, *Y=1,2, 3,* and *4* denote four cases in which case *C1*: $1>\delta_B/\gamma>\bar{\tau}$, *C2*: $\delta_B/\gamma\le\bar{\tau}$, *C3*: $1/\gamma\ge\delta_B/\gamma\ge 1$, and *C4*: $\delta_B>1$. Let *Ca*: $R/r>m_k^{eR}\exp\left[\left(I_k^*-I_k^{\max}\right)/B_k\right]/m_k^U$, and *Cb* for the complementary case. In particular, if $m_k^U=m_k^R$, then *Ca*: $R/r>\exp\left[\left(I_k^*-I_k^{\max}+\left(I_k^{\mathrm{Pre}}-I_k^{eR}\right)/2\right)/B_k\right]$

Proposition 4. (**Proof** : See Appendix 4)

(a) The conditions necessary to minimize (10) are given by the following expressions, respectively:

$$\delta_B^*\left(i,\varsigma_k,I_j^*\right)=\left(\frac{\bar{\tau}\sum_{k=1}^{n}D_k\left(H_k+h_k\right)\Big/\sum_{k=1}^{n}D_k h_k}{\exp(-i/b)+\sum_{k=1}^{n}\theta_k\exp\left(-\left(I_k^*+\varpi\varsigma_k\right)/B_k\right)}\right)^{1/2} \tag{11.1}$$

$$i^*=\left[C1:\ i^R+b\ln\left(\delta_B^*/\gamma\right);C2:\ 0;\ C3:\ i^R;\ C4:\ i^R+b\ln\left(\delta_B^*\right)\right] \tag{11.2}$$

$$\varsigma_k^*+I_k^*=\left[Ca:I_k^{\max}+B_k\ln\left(Rm_k^U/r\,m_k^{eR}\right);\ \ Cb:I_k^*\right] \tag{11.3}$$

(b) When $\varsigma_k^*=0 \quad \forall k\in\Psi$, the optimal adjustment factor takes one of the following solutions:

Optimal Policy δ_B (12)

C1 or C4 applies	*C2 or C3 applies*
$\dfrac{-\bar{\tau}\gamma_{14}+\sqrt{\left(\bar{\tau}\gamma_{14}\right)^2+4\Theta\left(I_k\right)\bar{\tau}\sum_{k=1}^{n}D_k\left(H_k+h_k\right)\Big/\sum_{k=1}^{n}D_k h_k}}{2\Theta\left(I_k\right)}$	$\sqrt{\dfrac{\bar{\tau}\sum_{k=1}^{n}D_k\left(H_k+h_k\right)\Big/\sum_{k=1}^{n}D_k h_k}{\bar{\tau}_{23}+\Theta\left(I_k\right)}}$

where $\Theta(I_k)=\sum_{k=1}^{n}\theta_k \exp(-I_k/B_k)$, $\overline{\tau}_{23}=\overline{\tau}$ (=1) when *C3*(*C2*) holds, and $\overline{\tau}_{23}=0$ otherwise. $\gamma_{14}=\gamma$ (=1) when *C1* (*C4*) applies, and γ_{14}=0 otherwise.

Let ΨX, *Y*=*A* and *B* denote sets of products satisfying cases *CX*, *X*=*A* and *B* and $\Psi = \Upsilon_{X=A}^{B}\Psi X$. Let $\Gamma_2=\left(\overline{\tau}_{23}+\sum_{K\in\Psi B}\theta_K\right)$ $\Gamma_1=\overline{\tau}\left(\gamma_{14}+\sum_{K\in\Psi A}\beta_k\right)$. When $I_k^*=I_k^{\max}$ $\forall k\in\Psi$, the optimal solution takes one of the following forms.

Optimal Policy δ_B when $I_k=I_k^{\max}$ $\forall k\in\Psi$ (13)

CASE_ SIMPLE	*CASE_NOT_SIMPLE*
ΨA=Ψ and C1 or C4 applies	*Otherwise*
$\dfrac{\sum_{k=1}^{n}D_k(H_k+h_k)\Big/\sum_{k=1}^{n}D_k h_k}{\gamma_{14}+\sum_{k\in\Psi A}\beta_k}$	$\dfrac{-\Gamma_1+\sqrt{\Gamma_1^2+4\overline{\tau}\Gamma_2\sum_{k=1}^{n}D_k(H_k+h_k)\Big/\sum_{k=1}^{n}D_k h_k}}{2\Gamma_2}$

□

Note that by choosing $(\delta_k^*, I_k^*, \varsigma_k^*)$ the buyer will offer a total premium price of $\Delta\Pi(\delta_k, I_k^*, \varpi\varsigma_k)$ to the supplier for each product, and the unit premium price is $P_k^{PR}=P-\Delta\Pi(\delta_k, I_k^*, \varpi\varsigma_k)/D$ so that the vendor is indifferent to the adjustment. Proposition 4(a), optimum policy (11.3), reveals that, when $m_k^U=m_k^{eR}$, $\varsigma_k^*+I_k^*$ amounts to the unconstrained optimum $I_k^{\max}$ when $R=r$, and more (less) than $I_k^{\max}$ by $B_k\ln(R/r)$ when $R>r$ ($R<r$). That is, the buyer will more likely substitute for the vendor (*ca* applies) if (i) his opportunity cost rate is less costly compared to the vendor, or (ii) the vendor has a tighter budget constraint (larger $I_k^{\max}-I_k^*$ or $I_k^{\mathrm{Pre}}-I_k^{eR}$). We now provide an algorithm to solve the problem. First, we design a subroutine for marginal analysis similar to that in Leschke and Weiss [7]. The subroutine MARGINAL_ana uses the marginal function (9r.2), and trial optimal policy (12) and (11.2) to search for trial optimal investment level I_k^{Trial}. We see that, the marginal function (9r.2) is a strictly increasing function of I_k^{Trial}; hence, a unique optimal investment level can be obtained. Assume I_k^{Pre} and m_k^U are found from the previous decision making process.

SUBROUTINE MARGINAL_ana

Step 0 Let $\breve{\chi} = \varnothing$ and $\chi = \Psi$. Let $BGT^{Left} = BGT$. Search $k(1) := \max_{k \in \chi}\left[m_k^U A_k^0 / B_k - R\right]$.

Let $k(1) \in \breve{\chi}$, $\chi - k(1)$, and *W*=2. Compute I_1^{NEW} by (*SM*), and search $\delta_{B(1)}^{Trial}$ by (11.2), (12).

Step 1 Search $k(W) := \max_{k \in \chi}\left[m_k^U A_k^0 / B_k - R\right]$.

If $\delta_{B(W-1)}^{Trail}\, m^R A_W^0 / B_W - R \le 0$, go to Step 2.

Else Y=1 to W-1 do :

Computes $I_Y^{NEW} = B_Y \ln\left(A_Y^0 B_W / A_W^0 B_Y\right)\ \ I_Y^{OLD}$ .... (SM)[1]

Check $\sum_{Y=1}^{W-} I_Y^{NEW} > BGT^{Left}$

If *yes* for *Y*=1 to *W-1*, let $I_Y^{Trail} := \min\left[I_Y^{\max}, I_Y^{OLD} + \left(BGT^{Left}/(W-1)\right)\right]$.

If $\sum_{Y=1}^{W-1} I_Y^{Trial} < BGT$, assign the remaining budget to products whose investment amount is less than $I_Y^{\max}$. Search $\delta_{B(W)}^{Trial}$ and i^{Trial} by (11.2) and (12), and let $\delta_B^{Trial} := \delta_{B(W)}^{Trial}$. Go to **MULTI_PRODUCT** .

Else, let $BGT^{Left} := BGT^{Left}\ \ \sum_{Y=1}^{W-1} I_Y^{NEW}$, for *Y*=1 to *W*-1, let $I_Y^{OLD} := I_Y^{OLD} + I_Y^{NEW}$, $k(W) \in \breve{\chi}$, $\chi - k(W)$, and W:=W+1.

If W $\le$ *n*-1 go to Step 1 . Else, if W>*n*-1, go to Step 2.

Step 2 For *Y*=1 to *W* do: $I_Y^{Trail} = \min\left[I_Y^{\max}, I_Y^{OLD} + \left(BGT^{Left}/W\right)\right]$.

If $\sum_{Y=1}^{W} I_Y^{Trial} < BGT$, assign the remaining budget to products whose investment amount is less than $I_Y^{\max}$.

Search $\delta_{B(W)}^{Trial}$ and i^{Trial} by (11.2) and (12). Stop; go to **ULTI_PRODUCT** .

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1) We equating two marginal functions $\exp(-I_Y/B_Y) m^R \delta_{B(W-1)}^{Trial} A_Y^0 / B_Y - R$, and $\exp(-0/B_W) m^R \delta_{B(W-1)}^{Trial} A_W^0 / B_W - R$, and obtain $I_Y^{NEW} = B_Y \ln\left(\dfrac{m^R \delta_{B(W-1)}^{Trial} A_Y^0 B_W}{m^R \delta_{B(W-1)}^{Trial} A_W^0 B_Y}\right) - I_Y^{OLD}$

We now give the main algorithm for the optimal solution. Algorithm ULTI_PRODUCT uses optimal policies provided in Proposition 5 to search for the optimal δ_k^* .

The ALGORITM MULTI_PRODUCT

Step 1 Let $I_k^* = I_k^{\max}$ $\forall k \in \Psi$. Call Subroutine search for δ_k^{Guess} $\forall k \in \Psi$ and δ_B^{Guess}. Compute ($I_k^{\max}, i^{Guess}$) by (9r.1), (11.2) and (13). (The subroutine follows an approach similar to that in Algorithm **MULTI_BUYER**)
If $\sum_{k=1}^{n} I_k^{\max} \leq BGT$ Stop. Let $I_k^* = I_k^{\max}$ $\forall k \in \Psi$ and $i^* = i^{Guess}$; $\delta_k^* = \delta_k^{Guess}$ $\forall k \in \Psi$ and $\delta_B^* = \delta_B^{Guess}$. Else, if $\sum_{k=1}^{n} I_k^{\max} > BGT$, call **MARGINAL_ana.**

Step 2 Let $I_k^* := I_k^{Trial}$ $\forall k \in \chi$ and $I_k^* = 0$ $\forall k \in \chi$. Compute ς_k^* by (11.3). Let $\delta_B^* := \delta_B^{Trial}$, and let $\delta_k^* := \delta_B^* m^R / m_k^U$. Stop.

To illustrate the preceding analysis, consider the following example involving two products. Let D_1=1,000 and D_2=2,000. Let the vendor's setup cost be A_1^0=\$75/setup and A_2^0=\$100/setup, and the inventory holding cost be H_1=\$5/unit/year and H_2=\$6/unit/year. We let R=0.2, and the vendor's setup reduction constants be B_1=\$1,500 and B_2=\$2,000. Similarly, let the buyer's set-up cost be a_1^0=\$15/setup and a_2^0=\$60/setup, and set an inventory holding cost of h_1=\$10/unit/year h_2=\$12/unit/year and an opportunity cost rate r=0.2. We let setup reduction constants be b_1=\$500 and b_2=\$1,000. Finally, we assume the budget limit to be \$4,500. If the buyer follows the vendor's individually optimum policy, then the corresponding total transactions cost incurred by the buyer and the vendor is \$4,166, in which the vendor's cost corresponding to product 1(2) is \$820 (\$1,322); the buyer's cost corresponding to product 1(2) is \$722 (\$1,300). The sum of the two amounts to \$2,022. The vendor's supply frequencies are m_1^{eR}=8.33 (m_2^{eR}=15) for product 1(2). The corresponding setup reduction investments are I_1^{eR}=\$1,100 and I_2^{eR}=\$2,643. The sum of the two investments does not go over the budget limit. The vendor's (buyer's) setup cost is reduced to $\overline{\mathrm{T}}_1$=48% ($\overline{\tau}_1$=80%), and $\overline{\mathrm{T}}_2$=27% ($\overline{\tau}_2$=22%). Let the buyer's joint setup cost be a^0=\$65, and the setup reduction constant be b=\$1,250. The unconstrained optimum result in $I_1^{\max}$=\$2,147 and $I_2^{\max}$=\$2,869, where the sum of two exceeds the budget limit. The corresponding order frequency via (13) can be shown to be $\delta_B m^R$=0.395(42.5)=16.78. By subroutine MARGINAL_ana, we obtain the optimal order frequency $\delta_B m^R$=0.367(42.5)=15.58, where the corresponding investment cost is \$2,837 (\$1,662) for product 2(1). The corresponding total transactions cost incurred by the vendor and the buyers is \$3,898 (6.4% reduction from \$4,166), in which the vendor's cost corresponding to product 1(2) is \$879 (\$1,329); the buyer's cost is \$1,755 (13.2% reduction from \$2,022). The premium price given to the vendor is \$76. Now consider the buyer's option of setup reduction investment substitution. Since product 1's (2's) investment is \$485 (\$32) less than the unconstrained optimum $I_1^{\max}$=\$2,147 ($I_2^{\max}$=\$2,869), the buyer substitutes for this amount. Now the optimal joint order frequency is synchronized to $\delta_B m^R$=16.78. The total transaction cost incurred by the vendor and the buyers is \$3,840 (7.8% reduction from \$4,166), in which the vendor's cost corresponding to product 1(2) is \$879 (\$1,331); the buyer's cost is \$1,698 (16% reduction from \$2,022). The premium price given to the vendor is \$67.

4. CONCLUSION

In this research, an EOQ-like inventory system is presented in which a vendor supplies multiple products to a group of buyers. Mismatches occur in individually optimal cycle times due to the buyers' and vendor's setup reduction programs. Two models aimed at rectifying this problem are analyzed. In the multi-buyers model, a single product is supplied by a vendor to a group of buyers. The vendor supplies products to each buyer based on a joint supply cycle approach. Here, buyers, as the dominant party, insist that the vendor accept buyers' separate and individually optimal order schedules. In the multi-products model, a group of products are supplied by a vendor to a buyer. The buyer's order is based on a joint order cycle approach. The vendor, as the dominant party, refuses to cooperate fully with the buyers' individually reduced joint order cycle schedule and decides to follow his own individually optimal setup reduction program for each product, and supplies each product separately. An integrated Setup Reduction/Break-even Pricing Policy is considered in each model. In the first model, a quantity discount pricing schedule is presented by the vendor to induce buyers to adjust his order cycle. In the second model, a premium pricing schedule is presented by the buyer to induce the vendor to adjust his supply cycles for each product.

Appendix 1. Proof of Proposition 1

(a) We see that $\partial\Delta C(\delta_k,i_k)/\partial i_k = \exp(-i_k/b_k)\delta_k m_k^U a_k^0/b_k - \gamma_k r_k$, and the second derivative is seen to be strictly negative. Proposition 1(a) follows.

(b) Straightforward computation shows that $\partial\Delta C(\delta_k,i_k)/\partial i_k=0$ leads to $i_k^*(\delta_k)=b_k\ln\left(m_k^R a_k^0/r_k b_k\right)+b_k\ln(\delta_k/\gamma_k)$ when $m_k^U=m_k^R$, and $i_k^*(\delta_k)=b_k\ln\left(m_k^R a_k^0/r_k b_k\right)+b_k\ln(\delta_k)+b_k\ln\left(m_k^0/m_k^R\right)$when $m_k^U=m_k^0$, which after substituting $m_k^0/m_k^R=\overline{\tau}_k^{1/2}$, $m_k^R a_k^0/r_k b_k=1/\overline{\tau}_k$, and reorganizing the terms result in $i_k^*(\delta_k)=i_k^R+b_k\ln(\delta_k/\gamma_k)$ and $i_k^*(\delta_k)=i_k^R/2+b_k\ln(\delta_k)$. The case $m_k^U=m_k^R$ is further partitioned into four cases given in (5) based on the specific values of δ_k and the boundary condition $[0,i_k^R]$. For example, if δ_k is such that that $b_k\ln(\delta_k/\gamma_k)<0$ and $i_k^R+b_k\ln(\delta_k/\gamma_k)>0$ (leading to *C1*: $1>\delta_k/\gamma_k>\overline{\tau}_k$), then the rollback adjustment results in a new setup cost $i_k^*(\delta_k)=i_k^R+b_k\ln(\delta_k/\gamma_k)$. On the contrary, if $b_k\ln(\delta_k/\gamma_k)<0$ (that is, $i_k^*\geq i_k^R$) then, by definition $\gamma_k=1$ (see (3)). Assume now $b_k\ln(\delta_k/\gamma_k)\geq 0$ and $\delta_k\leq 1$ (leading to *C3*: $1/\gamma_k\geq\delta_k/\gamma_k\geq 1$); then, case *C3* applies. □

Appendix 2. Proof of Proposition 2

(a) The first differentiation of the profit function reveals that $\partial\Pi\left(\delta_j m_j^R,I\right)/\partial\delta_j=\sum_{k=1}^n D_k(H+h_k)/2m_j^R\delta_j^2-m_j^R\left[A^0\exp(-I/B)+\sum_{k=1}^n a_k^0\exp(-i_k/b_k)\right]$....(A2.1). The second derivative shows that the function is strictly concave. Similarly, $\partial\Pi\left(\delta_j m_j^R,I\right)/\partial I=\exp(-I/B)\delta_j m_j^R A^0/B-R$.....(A2.2), and the second derivative reveals that the function is strictly concave.

(b) The optimal δ_j^* follows from equating (A2.1)=0. Similarly, by equating (A2.2) to 0, we obtain $I^*=B\ln\left(m_j^R a_j^0/b_j r_j\right)+B\ln\left(\delta_j^*\theta_V/\beta_V\right)$, , which leads to the desired result. This condition is further partitioned into two cases based on the specific values of δ_j^* and the boundary condition $I^*\geq 0$. For example, if δ_j^* is such that $i_j^R B/b_j+B\ln\left(\delta_j^*\theta_V/\beta_V\right)>0$ (leading to *CA*: $\delta_j^*>\overline{\tau}_j(\beta_V/\theta_V)$), then the vendor's optimal investment is $\delta_j^*>\overline{\tau}_j(\beta_V/\theta_V)$ □

Appendix 3. Proof of Proposition 3

(a) We see that $i_k^* = b_k \ln(\delta_j m_j^R a_k^0 / r_k b_k \gamma_k) = i_j^R b_k / b_j + b_k \ln(\delta_j \theta_k / \beta_k \gamma_k)$, $\forall k \neq j$, substituting i_k^* in (5) and I^* in (7) into the optimal $\delta_j^*(I, i_j)$ reveals the desired result.

(b) (i) We now prove Proposition 3(b).i. Assume CA1 ($\delta_{A1}/\gamma < 1$ holds) applies. Suppose, for the purpose of contradiction, that $\delta_{A3}/\gamma \geq 1$ holds. Via Proposition 2, δ_{AY}, Y=1,3 can be rearranged to $\delta_{A1}(\beta+\gamma) - \Omega = 0$ ....(A3.1), and $\delta_{A3}^2 + \delta_{A3}\beta - \Omega = 0$ ....(A3.2). Substituting $\delta_{A3} \geq \gamma$ into (A3.2) yields $\delta_{A3}(\gamma+\beta) - \Omega \leq 0$ ....(R.3.1). Result (R.3.1) and equation (A3.1) imply $\delta_{A1} \geq \delta_{A3}$. This then contradicts $\delta_{A1}/\gamma < 1$ and $\delta_{A3}/\gamma \geq 1$. Therefore, it follows that, for case *CA1*, $\delta_{A3}/\gamma < 1$; this leads to $\delta_{A3} > \delta_{A1}$ through a similar analysis. For case *CA3* ($\delta_{A1}/\gamma \geq 1$ holds), we assume $\delta_{A3}/\gamma < 1$ holds and derives contradictions. We then conclude $\delta_{A1}/\gamma \geq 1$ and $\delta_{A3}/\gamma \geq 1$ hold; this leads to $\delta_{A3} \leq \delta_{A1}$. Case *B* can be demonstrated through a similar argument.

(iii) We now prove Proposition 3(b).iii. Assume CA2 ($\delta_{A2}/\gamma \leq \bar{\tau} < 1$ holds) applies. Suppose, for the purpose of contradiction, that $\delta_{A4} > 1$ holds. We see that δ_{AY}, Y=2,4 can be rearranged to $(\delta_{A2})^2/\bar{\tau} + \delta_{A2}\beta - \Omega = 0$ ....(A3.3), and $\delta_{A4}\beta + \delta_{A4} - \Omega = 0$ ....(A3.4). Substituting $\delta_{A2} \leq \bar{\tau}\gamma$ into (A3.2) yields $\delta_{A2}\gamma + \delta_{A2}\beta - \Omega \geq 0$ ....(R.3.2). Result (R.3.2) and equation (A3.4) imply $\delta_{A2} \geq \delta_{A4}$. This then contradicts $\delta_{A2}/\gamma < 1$ and $\delta_{A4} > 1$. Therefore, it follows that, for case *CA2*, $\delta_{A4} \leq 1$ holds; this leads to $\delta_{A2} \geq \delta_{A4}$ through a similar analysis. For case *CA4* ($\delta_{A4} > 1$ holds), we assume $\delta_{A2}/\gamma \leq \bar{\tau}$ holds and derives contradictions. We then conclude $\delta_{A4} > 1$ and $\delta_{A2}/\gamma > \bar{\tau}$ hold; this leads to $\delta_{A2} < \delta_{A4}$. Case *B* can be demonstrated through a similar argument. Other cases can be verified similarly.

(c) We see that *CXY* can be represented by the following eight exhaustive cases:

$CA1 := \{\delta_{A1} > \bar{\tau}(\beta/\theta), \gamma > \delta_{A1} > \gamma\bar{\tau}\}$,	$CB1 := \{\delta_{B1} \leq \bar{\tau}(\beta/\theta), \gamma > \delta_{B1} > \gamma\bar{\tau}\}$
$CA2 := \{\delta_{A2} > \bar{\tau}(\beta/\theta), \delta_{A1} \leq \gamma\bar{\tau}\}$,	$CB2 := \{\delta_{B2} \leq \bar{\tau}(\beta/\theta), \delta_{B1} \leq \gamma\bar{\tau}\}$
$CA3 := \{\delta_{A3} > \bar{\tau}(\beta/\theta), \gamma \leq \delta_{A1}, 1 \geq \delta_{A3}\}$,	$CB3 := \{\delta_{B3} \leq \bar{\tau}(\beta/\theta), \gamma \leq \delta_{B1}, 1 \geq \delta_{B3}\}$
$CA4 := \{\delta_{A4} > \bar{\tau}(\beta/\theta), 1 < \delta_{A3}\}$,	$CB4 := \{\delta_{B4} \leq \bar{\tau}(\beta/\theta), 1 < \delta_{B3}\}$

The conditions listed in Propoition 3(c) are obtained by substituting $\delta^* = \delta_{XY}$ into eight cases listed above. For example, *CA3* satisfies $\delta_{A3} > \bar{\tau}(\beta/\theta)$ (case *A*), $\gamma \le \delta_{A1}$ (case 3), and $1 \ge \delta_{A3}$ (situation 1). Given that *CA3* applies, substitution of $\delta^* = \delta_{A3} = -\beta + \sqrt{\beta^2 + 4\Omega}/2$ into $\delta_{A3} > \bar{\tau}(\beta/\theta)$, $\gamma \le \delta_{A3}$ and $1 \ge \delta_{A3}$ lead to the conditions listed in Proposition 3(c). We now show that *CA1* is disjointed from the other cases. Using a similar argument, it is not difficult to show the remaining cases. *CA1, CA2, CA3,* and *CA4* are easily seen to be disjointed. We now show the exclusivity of the rest of the cases:

CA1 and *CB3,CB4*: *CA1* implies $\delta_{A1} > \delta_{B1}$ (Proposition 3(b).v). This and the condition $\delta_{A1} < \gamma$ in *CA1* imply $\delta_{B1} < \gamma$, which contradicts *CB3* and *CB4.*

CA1 and *CB2*: *CB2* implies $\delta_{A2} \le \delta_{B2}$ (3(b).v) and $\delta_{B2} \le \bar{\tau}\gamma$ (3(b).ii). Together, they imply $\delta_{A2} \le \bar{\tau}\gamma$. Similarly, *CA1* implies $\delta_{A2} > \bar{\tau}\gamma$ (3(b).ii), which contradicts *CB2.*

CA2 and *CB1* are disjointed. *CA2* implies $\delta_{A2} > \delta_{B2}$ (3(b).v) and $\delta_{A2} \le \bar{\tau}\gamma$ (3(b).ii). Together, they imply $\delta_{B2} \le \bar{\tau}\gamma$, which contradicts *CB1* since *CB1* implies $\delta_{B2} > \bar{\tau}\gamma$ (3(b).ii)

CA2 and *CB3*: *CA2* implies $\delta_{A2} > \delta_{B2}$ (3(b).v) and $\delta_{A2} \le \bar{\tau}\gamma$. Together, they imply $\delta_{B2} \le \bar{\tau}\gamma$, which then implies $\delta_{B1} \le \bar{\tau}\gamma$ (3(b).ii), which contradicts *CB3.*

CA2 and *CB4* are disjointed. *CA2* implies $\delta_{A2} > \delta_{B2}$ (3(b).v) and $\delta_{A2} \le \bar{\tau}\gamma$. Together, they imply $\delta_{B2} \le \bar{\tau}\gamma$. *CB4* reveals $\delta_{B4} > 1$ $\delta_{B2} > \gamma\bar{\tau}$ (3(b).iii), which contradicts *CA2.* Other cases can be verified similarly. The eight necessary conditions given in Proposition 3(c) are obtained from substituting $I^*(\delta^*)$ as given in Proposition 1, and $i^*(\delta^*)$ as provided in Proposition 2 into $\delta^*(I,i)$.

Appendix 4. Proof of Proposition 4

(a) Since $\partial C(\delta_B m^R, i, \varsigma_k)/\partial\delta_B = -\sum_{k=1}^{n} D_k (H_k + h_k)/2m^R\delta_B^2 + m^R\left[a^0\exp(-i/b) + \sum_{k=1}^{n} A_k^0 \exp\left(-\left(I_k^* + \varpi\varsigma_k\right)/B_k\right)\right]$ ....(A4.1), the optimal δ_B^* follows from equating (A4.1)=0. Similarly, $\partial C(\delta_B m^R, i, \varsigma_k)/\partial i = -\exp(-i/b)\delta_B m^R a^0/b + \gamma r$ ....(A4.2). By equating (A4.2) to 0, we obtain the desired result. Lastly, upon assuming $\varpi = 1$, we obtain

$\partial C(\delta_B m^R, i, \varsigma_k)/\partial \varsigma_k = -\exp\left(-\left(I_k^* + \varsigma_k\right)/B_k\right)\delta_B m^R A_k^0 / B_k + r$ ....(A4.3). (11.3) can obtained by equating (A4.3) to 0.

(b) (12) obtained from substituting i^* into (11.1), and (13) obtained from substituting I_k^{max} and i^* into (11.1) □

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