

Prioritized Channel Allocation for Cellular Mobile Systems Using Simulated Annealing*

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Abstract

Under the cutoff priority discipline, the prioritized channel allocation problem is formulated, which minimizes the overall blocking probability while ensuring the co-channel interference constraints. To deal with the problem more conveniently, the concept of pattern is used. A simulated annealing approach is applied to the problem, and computational experiments show that a high-quality solution is obtained.

1. Introduction

With the enormous growth of cellular mobile communication, the frequency spectrum allocated to the mobile system becomes a critical resource. Since the spectrum is a limited resource, it is a vital issue to use the spectrum more efficiently. Thus many channel assignment schemes proposed so far have aimed at making efficient utilization of frequency channels [1, 2, 7-9, 16].

However, in many practical situations, the blocking of a handoff call attempt is critical

since it will result in a disconnection of the call in the middle of conversation. Therefore, several priority schemes to reduce the chances of unsuccessful handoffs have been suggested. The simplest way of giving priority to handoff call attempts is to reserve some frequency channels for calls being handed off into the cell. This scheme will be called cutoff priority scheme (CPS) [5].

In the cutoff priority scheme, priority is given to handoff call attempts by exclusively assigning them y_i channels among C_i channels available in cell i . These channels

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are called *guard channels*. The remaining $C_i - y_i (= x_i)$ channels called *ordinary channels* are shared by both calls. When a new call attempt is generated in cell i , it is blocked if the number of free channels is less than or equal to y_i . However, a handoff call attempt is blocked only when all the C_i channels are busy in the cell. Variants of CPS, which allow storing and queuing of handoff calls [5, 14], new calls [4], and both calls [15], are also considered in the literature. Since these buffering strategies cause new delay problems in waiting calls and the pure CPS might give effective priority to handoff calls [12], blocked calls are assumed to be cleared in this study.

Given a certain number of frequency channels and traffic loads, the objective of channel allocation for CPS is to determine the numbers of guard channels as well as ordinary channels for each cell, which is referred to as the *prioritized channel allocation*. In [12], optimal prioritized channel allocation methods are proposed for a single cell system and one cluster of multicell system. This paper considers, in general multicell environments, the prioritized channel allocation problem to minimize the overall blocking probability, defined by the weighted value of the average blocking probabilities of new call attempts and handoff call attempts, while ensuring the co-channel interference constraints.

This prioritized channel assignment problem is mathematically formulated, but it is combinatorial in nature. To deal with the problem more conveniently, the concept of pattern is used. This concept helps to avoid obtaining many equal symmetric solutions which may occur by permuting channels with unaltered value on the objective function. A pattern is a set of cells to which a channel can be allocated without causing co-channel interference. Then a quite satisfactory approach called simulated annealing is applied and the quality of solutions is shown to be very high by computational experiments.

The remainder of the paper is organized as follows. In section 2, an optimal prioritized channel allocation problem is defined and formulated mathematically. Also a reduced problem using the concept of pattern is suggested. In section 3, a solution procedure is described. In section 4, the results of computational experiments are reported. Finally, section 5 summarizes the results.

2. Prioritized Channel Allocation Problem

We consider a cellular mobile system consisting of N cells and M channels. Let λ_i^n and λ_i^h be the traffic demands in erlangs of new and handoff call attempts in

cell i , respectively, and let x_i and y_i be the numbers of ordinary and guard channels available in cell i , respectively. Then the blocking probabilities of new and handoff call attempts in the cell are, respectively, given by

$$BN_i(\lambda_i^n, \lambda_i^h, x_i, y_i) = \frac{\frac{\lambda_i^{x_i}}{x_i!} + \lambda_i^{x_i} \sum_{j=1}^{y_i} \frac{(\lambda_i^h)^j}{(x_i+j)!}}{\sum_{j=0}^{x_i} \frac{\lambda_i^j}{j!} + \lambda_i^{x_i} \sum_{j=1}^{y_i} \frac{(\lambda_i^h)^j}{(x_i+j)!}}$$

and

$$BH_i(\lambda_i^n, \lambda_i^h, x_i, y_i) = \frac{\frac{\lambda_i^{x_i} (\lambda_i^h)^{y_i}}{(x_i+y_i)!}}{\sum_{j=0}^{x_i} \frac{\lambda_i^j}{j!} + \lambda_i^{x_i} \sum_{j=1}^{y_i} \frac{(\lambda_i^h)^j}{(x_i+j)!}}$$

where $\lambda_i = \lambda_i^n + \lambda_i^h$ [5,12]. And the average blocking probabilities of new call attempts and handoff call attempts in the cellular mobile system are, respectively, given by

$$\sum_{i=1}^N w_i^n BN(\lambda_i^n, \lambda_i^h, x_i, y_i)$$

and

$$\sum_{i=1}^N w_i^h BH(\lambda_i^n, \lambda_i^h, x_i, y_i)$$

where $w_i^n = \lambda_i^n / \sum_{i=1}^N \lambda_i^n$ and $w_i^h = \lambda_i^h / \sum_{i=1}^N \lambda_i^h$ are the traffic weighting factors.

Then, the prioritized channel allocation problem (PCAP) which minimizes the over-

all blocking probability, defined by the weighted value of the average blocking probabilities of new call attempts and handoff call attempts, while ensuring the co-channel interference constraints is as follows:

(PCAP)

$$\begin{aligned} \min \alpha & \sum_{i=1}^N w_i^n BN_i(\lambda_i^n, \lambda_i^h, x_i, y_i) + (1-\alpha) \\ & \sum_{i=1}^N w_i^h BH_i(\lambda_i^n, \lambda_i^h, x_i, y_i) \\ \text{s.t. } x_i + y_i &= \sum_{j=1}^M f_{ij}, \text{ for } i=1, \dots, N, \end{aligned} \quad (1)$$

$$f_{sj} + f_{tj} \leq 1, \text{ for } j=1, \dots, M, \text{ and all interfering cell pairs } (s,t), \quad (2)$$

$$x_i, y_i : \text{nonnegative integer, for } i=1, \dots, N, \quad (3)$$

$$f_{ij} = 0 \text{ or } 1, \text{ for } i=1, \dots, N, j=1, \dots, M, \quad (4)$$

where α is a weighting factor. Since the blocking probability of handoff call attempts is considered to be more important than the blocking probability of new call attempts, in general $\alpha \leq 0.5$.

This is a nonlinear integer programming problem, which is NP-hard because it can be easily reduced to the independent set problem known to be NP-complete [3, 10, 11]. The decision variable f_{ij} is a binary integer variable indicating channel allocation where $f_{ij}=1$ represents that channel j is allocated to cell i and $f_{ij}=0$ otherwise.

The decision variables x_i and y_i , representing the number of ordinary and guard channels allocated to cell i respectively, are determined by channel allocation f_{ij} 's as shown in equation (1). The equation (2) means that the same channel j should not be allocated to different cells s and t simultaneously if the cells s and t are within interference zone of each other.

To deal with the proposed problem more conveniently, we introduce the concept of pattern. A channel cannot be allocated to adjacent cells simultaneously because of the co-channel interference. If an identical channel is allocated to a set of cells without causing co-channel interference between pairs of cells, these cells are called co-channel cells. This set of co-channel cells forms a pattern.

Now suppose that we generate P patterns such that every cell belongs to at least one of these patterns. Then our problem reduces to the problem of allocating M channels to P patterns. Let the decision variable t_p denote the number of channels allocated to pattern p . Then using the P patterns, the problem (PCAP) reduces to the following problem:

(RPCAP)

$$\min \alpha \sum_{i=1}^N w_i^n B N_i(\lambda_i^n, \lambda_i^h, x_i, y_i) + (1 - \alpha)$$

$$\sum_{i=1}^N w_i^h B H_i(\lambda_i^n, \lambda_i^h, x_i, y_i)$$

s. t. $\sum_{p=1}^P t_p \leq +M,$ (5)

$$(x_i + y_i) \leq \sum_{p \in S_i} t_p, \text{ for } i=1, \dots, N, \quad (6)$$

$$x_i, y_i : \text{nonnegative integer, for } i=1, \dots, N, \quad (7)$$

$$t_p : \text{nonnegative integer, for } p=1, \dots, P, \quad (8)$$

where S_i is the set of patterns which cover cell i . Here, the right-hand side of (6) is the number of channels available in cell i . If all feasible patterns are considered, the problem (RPCAP) is equivalent to the original problem (PCAP). Due to the property of a pattern, a solution of the problem (RPCAP) always satisfies the co-channel interference constraints. This problem (RPCAP) avoids obtaining many equal symmetric solutions which may occur by permuting channels with unaltered value on the objective function in the problem (PCAP).

The problem (RPCAP) is an approximation to the original problem (PCAP). The choice of patterns considered in the problem (RPCAP) may have an influence on the quality of solutions relative to the original problem (PCAP). Thus it is very important to find good candidate patterns. Kim & Chang [8] suggested three pattern generation procedures considering two facts:

mutual distances between adjacent co-channel cells and the traffic demand distribution. These procedures find patterns such that mutual distances between adjacent co-channel cells are made short as far as possible, and patterns including cells with high traffic demands as much as possible. In [8], the pattern generation procedure A initially selects some noninterfering close cells, then repeatedly selected a cell which is nearest to the already selected cells while ensuring the co-channel interference constraints. The pattern generation procedure B is a simple modification of the procedure A. In this procedure, a cell which is nearest to the initially selected cells is repeatedly selected. The pattern generation procedure C initially selects some cells among cells with high traffic demands, then repeatedly selects a cell with the largest demand while ensuring the co-channel interference constraints.

3. Simulated Annealing

Simulated annealing is a general approach to obtain an approximate solution of combinatorial optimization problems. It has been applied in such diverse areas as computer aided design of integrated circuits, code design, etc. [2]

Generally, a combinatorial optimization problem consists of a solution set Z and a

cost function C which determines the cost $C(z)$ for each $z \in Z$. Simulated annealing can be considered as a generalization of the iterative improvement scheme (local search). For performing a local search one needs to know the neighbors z' of z . Thus one has to define a neighborhood structure $N(z)$ on z . For each solution z , $N(z)$ determines a set of possible transitions which can be proposed by z .

For local search, starting from an arbitrary solution z , in each step of iterative improvement a neighbor z' of z is proposed at random. The solution z is replaced by z' only if cost does not rise, i.e., $C(z') \leq C(z)$.

Obviously, this procedure terminates in a local minimum, i.e., in a solution whose neighbors do not offer any improvement in cost. Unfortunately, such a local minimum may have a substantially higher cost than the global one. To avoid this trapping in poor local optima, simulated annealing occasionally allows solutions of higher cost according to an acceptance rule. Two acceptance rules are used in the literature [6,11], in which the probability of accepting a move to a neighbor producing a variation in the cost function is given by one of the following functions:

$$\begin{cases} e^{-(C(z')-C(z))/T} & \text{if } C(z')-C(z) > 0 \\ 1 & \text{if } C(z')-C(z) \leq 0 \end{cases}$$

and

$$\begin{cases} \max \{0, 1 - (C(z') - C(z))/T\} & \text{if } C(z') - C(z) > 0 \\ 1 & \text{if } C(z') - C(z) \leq 0 \end{cases}$$

The first acceptance rule is the traditional in simulated annealing literature while the second consists of the first items of its Taylor expansion around zero. Parameter T called temperature is initially set to a relatively large value so that the transition from z to z' occurs more frequently, and then it is gradually decreased as the search proceeds. When T becomes sufficiently small and the solution does not change for many iterations, it is concluded to be *frozen*, and the best solution available by then is output as the computed approximate solution.

Simulated Annealing Algorithm

Step 0. Set a feasible solution $z^0 = (x^0, y^0, t^0) \in Z$, an initial temperature T_0 , $0 < \gamma < 1, L, \epsilon > 0$, and $k = l = 0$.

Step 1. Generate a solution $z' \in N(z^l)$. If $C(z') - C(z^l) \leq 0$, then set $z^{l+1} = z'$, otherwise, set $z^{l+1} = z'$ with probability $q (= e^{-(C(z') - C(z^l))/T_k}$ or $\max \{0, 1 - (C(z') - C(z^l))/T_k\}$) and set $z^{l+1} = z^l$ with probability $1 - q$.

Step 2. If $l \geq L$, then go to Step 3, otherwise, go to Step 1 with replacing $l+1$ to l .

Step 3. If $T_k \leq \epsilon$, then terminate, otherwise set $T_{k+1} = \gamma T_k$ and go to Step 1 with replacing $k+1$ to k .

In Step 1, given a solution z^l , the neighborhood structure $N(z^l)$ is defined as follows:

$$\begin{aligned} N(z^l) = \{ & z' = (x', y', t') \mid t' \neq t^l, \\ & |t'_p - t^l_p| = 0 \text{ or } 1, \text{ for all } p; \\ & \sum_{p \in S_i} t'_p = M; \\ & (x'_i, y'_i) = \operatorname{argmin}_{x_i, y_i} \\ & \{B_i(x_i, y_i) \mid x_i + y_i = \sum_{p \in S_i} t'_p\}, \\ & \text{for all } i \} \end{aligned}$$

where $\operatorname{argmin}_{x_i, y_i} \{B_i(x_i, y_i) \mid x_i + y_i = \sum_{p \in S_i} t'_p\}$ denotes an optimal solution of the following problem:

$$\begin{aligned} (P_i) \quad & \min B_i(x_i, y_i) = a w_i^n B N_i(\lambda_i^n, \lambda_i^h, x_i, y_i) + \\ & (1 - a) w_i^h B H_i(\lambda_i^n, \lambda_i^h, x_i, y_i) \\ \text{s.t. } & x_i + y_i = \sum_{p \in S_i} t'_p, \\ & x_i, y_i : \text{nonnegative integer,} \end{aligned}$$

for a given t' . This problem can be easily solved since the solution space is very small. Note that in defining the neighborhood structure, it is not necessary to consider the inequalities of (5) and (6) since the optimal solution of the problem (*RPCAP*) is obtained when the equalities of (5) and (6) are satisfied. Now, given a solution z' a next solution $z' \in \mathcal{N}(z')$ is generated by the following proportional picking strategy:

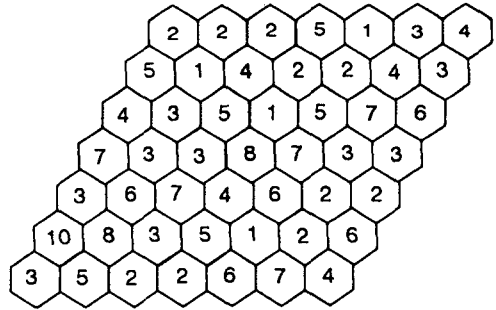
Proportional Picking Strategy

Set $t = t'$. Select two components of t' , t_{p^+} and t_{p^-} , and set $t_{p^+} = t_{p^+} + 1$ and $t_{p^-} = t_{p^-} - 1$. The components t_{p^+} and t_{p^-} ($\neq t_{p^+}$) are selected in proportion to $|S_{p^+}| / \sum_{p=1}^P |S_p|$ and $1 - |S_{p^+}| / \sum_{p=1}^P |S_p|$, respectively, where S_{p^+} is the set of cells covered by pattern p and $|S_{p^+}|$ is the number of elements of the set S_{p^+} .

In Step 2, if it is concluded that a sufficient number of trials have been made with the current T_k (i.e., in *equilibrium*), then the temperature T_k is updated. In Step 3, if the current T_k is concluded to be sufficiently small (i.e., *frozen*), then the algorithm terminates with the current best feasible solution.

4. Computational Experiments

To evaluate the solution quality, the proposed algorithm is compared with uniform channel allocation [7, 9] using cutoff priority scheme (CPS). As a test example, a 49-cell system with 70 available channels like Fig. 1 is considered, which is presented in [8, 16]. The numbers represent the total traffic demands in erlangs. The minimum reuse distance d is $\sqrt{21}r$, where r is the cell radius.



[Fig. 1] Cellular mobile system with nonuniform traffic distribution

In simulated annealing, T_0 is set to 10 and r is set to 0.5. The simple approach of permitting $L=50$ state transition at each T_k level is used. The initial feasible solution z^0 is given by

$$t_p^0 = \lfloor M/P \rfloor + I_p, \text{ for all } p,$$

$$(x_i^0, y_i^0) = \operatorname{argmin}_{x_i, y_i} \{B_i(x_i, y_i) \mid x_i + y_i = \sum_{p \in S_i} I_p^0\}, \text{ for all } i,$$

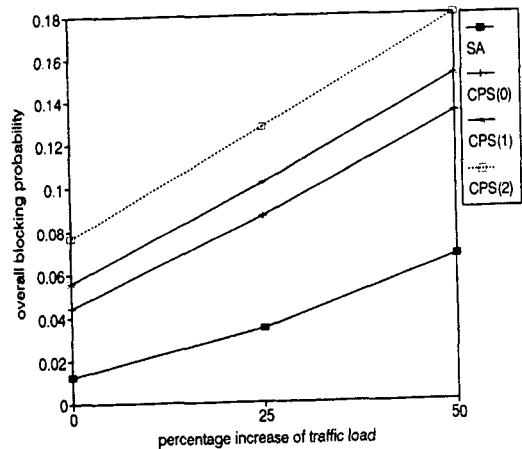
where $\lfloor M/P \rfloor$ denotes the largest integer smaller than or equal to M/P , and $I_p = 1$ if $p \leq M - \lfloor M/P \rfloor * P$, otherwise $I_p = 0$. In Step 1, the probability $q = e^{(C(z') - C(z))/T_k}$ is used. When T_k is less than $\epsilon = 10^{-10}$, it is concluded to be frozen. Patterns are generated by procedures suggested in [8] with parameters $n_a = 1, \delta = d$ and $n_c = 2$. These parameters determine the cells which are initially selected to generate a pattern. In [8], if $n_a = 1$ and $\delta = d$, then the pattern generation procedures A and B initially select each cell of the cellular mobile system to generate a pattern. If $n_c = 2$, then the pattern generation procedure C initially select each subset, whose the number of elements is not greater than $n_c = 2$, of the set of cells such that the total traffic demand λ_i is greater than a predetermined value, for example, $\bar{\lambda} + \sqrt{v}$, where $\bar{\lambda}$ and v are mean and variance of λ_i 's, respectively. The number of patterns generated is 38.

The test results are summarized in [Fig. 2] - [Fig. 6]. In the test, the presented algorithm successfully terminates in several minutes on an IBM PC. In the figures, SA represents the simulated annealing approach,

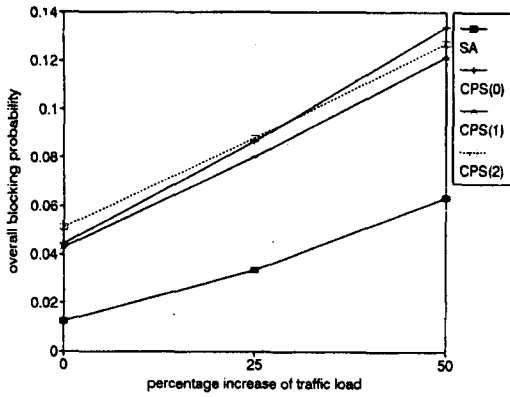
and $\text{CPS}(y_i)$ represents uniform channel allocation using CPS with y_i guard channels. [Fig. 2] - [Fig. 5] show the overall blocking probability of each method as a function of traffic load for the following cases:

- Case 1: $\alpha = 0.5, \lambda_i^n = \lambda_i^h$ for all i
- Case 2: $\alpha = 0.3, \lambda_i^n = \lambda_i^h$ for all i
- Case 3: $\alpha = 0.3, \lambda_i^n = 2\lambda_i^h$ for all i
- Case 4: $\alpha = 0.3, \lambda_i^n = 2\lambda_i^h$ for all i

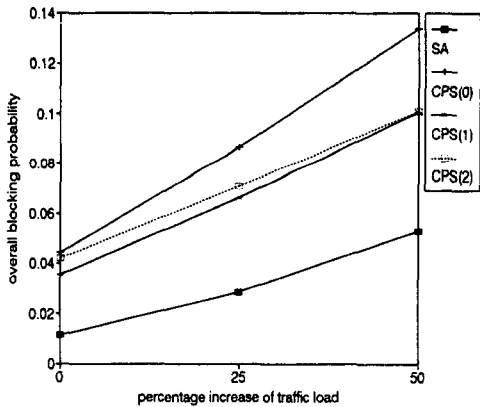
The base traffic load is shown in [Fig. 1]. The traffic load is then increased by 25% and 50% over the base load. The figures show that the solution quality of the proposed approach is better as the traffic load increases.



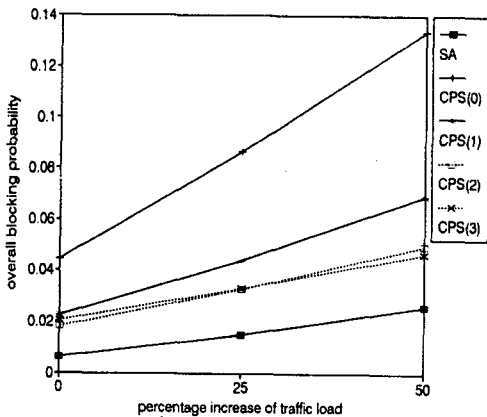
[Fig. 2] Overall blocking probability for case 1



[Fig. 3] Overall blocking probability for case 2

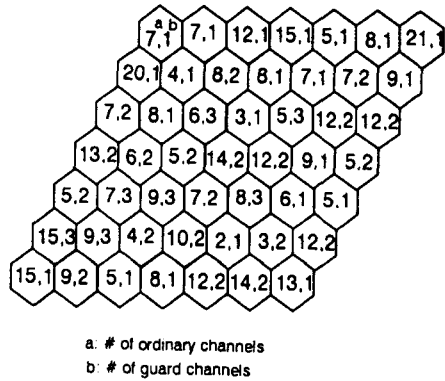


[Fig. 4] Overall blocking probability for case 3



[Fig. 5] Overall blocking probability for case 4

[Fig. 6] shows, for an illustrative purpose, the numbers of ordinary and guard channels assigned to each cell for the case 4 with 50% increased traffic load.



[Fig. 6] Distribution of ordinary and guard channels

5. Conclusions

The prioritized channel allocation problem was suggested to minimize the overall blocking probability while ensuring the co-channel interference constraints in a cellular mobile system with nonuniform traffic distribution. The problem was converted into a simpler form through the concept of pattern. A simulated annealing approach was applied to the simplified problem, and a high-quality solution was obtained.

In this paper, the overall blocking probability, defined by the weighted value of the average blocking probabilities of new call

attempts and handoff call attempts, is considered as the performance metric. However, the proposed approach can be easily applied to the prioritized channel allocation problems with other performance metrics.

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