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Multi-Criteria Decision Making Procedure under Incompletely Identified Preference Information

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Abstract

The paper deals with interactive multiple criteria decision making procedure when decision maker (DM) specifies her or his preference in incomplete ways. Usually DM is willing or able to provide only incomplete information, because of time pressure and lack of knowledge or data. Under incomplete information on utility and attribute weight, the pairwise dominance checks result in strict or weak dominance values. Considering only strict dominance values sometimes fails to prioritize alternatives because of fuzziness of preference information. Further there exists some information loss useful if used, otherwise. In this paper, we consider the outranking concept which implies the willingness of DM's taking some risk under the least favorable situation because s/he has enough reasons to admit the results. By comparing the magnitude of net preference degree of alternatives which is defined by difference between outranking and outranked degree of each alternative, we can prioritize alternatives.

1. Introduction and Background

There are many methods in the field of decision analysis that help decision makers come up with an optimal or satisfying solution. The gap between theoretical work and practical needs is due to the fact that the decision problem or the preferences of the decision maker (DM) are not (yet) structured enough to allow the successful application of most decision analysis methods. For example,

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the DM can not provide exact estimation of probability distributions or s/he is incapable of specifying the preference information required by the method of interest (Weber, 1987). The primary reasons that DM provides only incomplete information are 1) decision should be made under time pressure and lack of data 2) many of attributes are intangible or non-monetary, 3) DM has limited attention and information processing capabilities (Kahneman, et. al., 1982).

General single stage multi-attribute decision problem consists of a multitude of subjects such as decision alternatives, criteria, and preference information. According to the type or degree of preference information of the DM, various solution techniques can be applied in estimating unknown parameters as shown in Table 1.

Most of previous studies have assumed that the information about at least two of three parameters are (partially) identified. Recently, Park et al.(1997) shows that in the case of attribute weight and utilities being known incompletely, there exists a necessary and sufficient condition for establishing dominance relations between the alternatives under the assumption of functional independence. Under the assumption (the functional independence assumptions are appropriate in many realistic problems, and they are operationally verifiable in practice), their method imposes less preference articulation burden to the DM.

When information about attribute weights and utilities is incomplete, we can construct a preference order of alternatives through pairwise dominance relationship because common attribute weights should be applied. The

〈Table 1〉 Review of decision making approaches

Alternative (Rank)	Criteria (Weight)	Utility (Outcome)	Related works
ordinal	cardinal	unknown	Jacquet-Lagreze and Siskos (1982)
unknown	ordinal	ordinal	Cook and Kress (1991)
ordinal	unknown	cardinal	Srinivasan and Shocker(1973), Pekelman and Sen(1974)
ordinal	unknown	LPI*	White et. al. (1984)
unknown	ordinal	cardinal	Sarin (1977), Hannan (1981), Kirkwood and Sarin(1985)
unknown	LPI	cardinal	Kmietowicz and Pearman (1984), Kofler, et.al. (1984)
unknown	LPI	LPI	Sage and White (1984), Weber (1987), Park, et. al.(1996), Park and Kim (1997)

* LPI : Linear Partial Information

pairwise dominance checks result in strict or weak dominance values. Considering only strict dominance values sometimes fails to prioritize alternatives because of fuzziness of preference information. Further there may exist some information loss useful if used, otherwise.

In this paper, we consider the outranking concept which implies the willingness of DM's taking the results although the contrary results occur under the least favorable situation, because s/he has enough reasons to admit the results. By comparing the magnitude of net preference degree of alternatives which is defined by difference between outranking and outranked degree of each alternative, it is possible to prioritize alternatives.

Although the proposed method gives less burden in the process of eliciting preference information, it requires resolving possible inconsistency cases due to the infeasible preference space for the special case of linear inequalities.

2. Multi-Criteria Decision Making with Incomplete Information

In multi-criteria decision making, one usually considers a set of alternatives, which are valued by a family of criteria. A classical evaluation of alternatives leads to the aggregation of all criteria into a unique criterion, called a utility function. We assume that the attributes are additive independent which leads to an additive multi-attribute utility function (Keeney and Raiffa, 1976). The aggregated value or expected utility of alternative under certainty is given as follows :

$$EU(a_i) = \sum_{k=1}^{N} w_k u_k(a_i)$$

2.1 Notation

$A = \{a_i\}_{i=1,M}$: a discrete finite set of M possible alternatives.

$W = \Phi_w \cap \{\sum_k w_k = 1,\ w_k \geq 0\}$: the set of constraints or all possible values on the attribute weights, $w \in W$ where Φ_w is a set derived from DM's incomplete information regarding the relative importance of attributes.

$U = \Phi u_k \cap \{1 \geq u_k(a_i) \geq 0, 1 \geq u_k(a_j) \geq 0\}$: the set of constraints on the utilities where Φu_k is a set of constraints in the utility information obtained by the DM, for the consequences when attribute k is given.

$\Psi(a_i, a_j) = \{W, U\}$, where. $U = \{U_k\}_{k=1,N}$.

Ω: the set of collecting dominance relations between the alternatives, $\Omega \subseteq A \times A$; for example, $(a_i, a_j) \in \Omega$ means that alternative a_i is at least as preferred as alternative a_j.

2.2 Incomplete information about attribute weight/utility

Though no definite terminology has been

developed for the general case, some authors describe 'partial information', 'imprecise information', 'incomplete knowledge' or 'linear partial information', interchangeably (Kmietowicz and Pearman, 1984; Kirkwood and Sarin, 1985; Hazen, 1986). The decision situation is incomplete if and only if at least one of the functions (attribute weight, utility, and probability) is not exactly specified. The DM may not be sure about (1) the utility function either because s/he allows a range for the weights of the conditional utility functions or s/he is not sure about the risk and value judgments on one or several attributes, (2) in probability distribution there may only be ordinal information or interval statements on the probability of states available, or (3) the DM may not be sure about how the consequence of an alternative should be evaluated in the attribute space.

The incomplete information types we are considering here are linear inequalities. Examples of linear inequalities on attribute weights are given by the following forms:

Form 1. A weak ranking: $\{w_i \geq w_j\}$

Form 2. A strict ranking:

$\{w_i - w_j \geq \alpha_i\}$,

Form 3. A ranking with multiples:

$\{w_i \geq \alpha_i w_j\}$,

Form 4. An interval form:

$\{\alpha_i \leq w_i \leq \alpha_j + \varepsilon_i\}$,

Form 5. A ranking of differences:

$\{w_i - w_j \geq w_l - w_m\}$,

for $j \neq l \neq m$

Forms 1-4 are well known types of imprecise information. A difficulty in taking the information of Forms 2-4 is to precisely justify their constants, since these forms contain numerical values such as α_i and ε_i. Form 5 is a ranking of differences of adjacent parameters obtained by ranking between two parameters, which can be subsequently constructed based on Form 1. Incompletely specified information about attribute weight and utility can be regarded as set of constraints, W and U, respectively.

2.3 Basic solution technique

For the case of decision making under outcome certainty, we say that alternative a_i has a higher value score than alternative a_j when

$$\min_{\Psi(a_i, a_j)} z(a_i, a_j) = \sum_{k=1}^{N} w_k[u_k(a_i) - u_k(a_j)] \geq 0 \tag{1}$$

Assuming that the incomplete information of utility values for each attribute are functionally independent, which is formally denoted by a notation, $U_i \perp U_k \ \forall i \neq k$, (1) becomes separable for each attribute thus yielding a set of the following linear programs:

$$z_{Min}(a_i, a_j) = \min_{W} z(a_i, a_j) = \sum_{k=1}^{N} w_k v_k(a_i, a_j)$$

with

$$v_k(a_i, a_j) = \min_{u_k}\{u_k(a_i) - u_k(a_j)\},$$
$$k = 1, \ldots, N.$$

Then, $(a_i, a_j) \in \Omega$ iff $z_{\min}(a_i, a_j) \geq 0$. To construct Ω, it is required to solve M(M-1)(1+N) linear programs.

2.4 Consistency checking

The aforementioned types of preference information elicited from decision maker inescapably reduce inconsistent (infeasible) case, although it reduces the information articulation burden imposed by decision maker. So, it is necessary to feedback the most "plausible" violating constraints which violate the mentioned constraints set. To this end, several approaches have been proposed to tackle this unwanted situation.

There may exist 'critical' constraints that DM does not want to change his/her preference structure. For example, in the case of attribute weights specified incompletely, let us denote index set as the set of 'critical' constraints that DM would not change his/her preference and as 'uncritical' constraints that DM would change his/her preference in attribute weight set, W (we can rewrite it as $W = \{\sum_j a_{ij} w_j \geq 0,\ \text{for } i \in S \cup S^c\}$). We want to minimize the number of uncritical constraints which violate critical region comprised of critical constraints. This model can be formulated as follows (Kim and Ahn, 1997a, 1997b):

$$Min \sum_{i \in S^c} \delta_i$$
$$\begin{aligned} s.t \quad & \textstyle\sum_j a_{ij} w_j \geq 0 && \text{for } i \in S \\ & \textstyle\sum_j a_{ij} w_j + M\delta_i \geq 0 && \text{for } i \in S^c \\ & w_j \geq 0, \delta_i = 0 \ or\ 1. \end{aligned}$$

M is a large number greater than $\max_i \sum_j |a_{ij}|$, for $i \in S^c$ and this model can be solved by a mixed integer programming code. The minimization results say that the constraints corresponding to $\delta = 1$ violate the space and, for further process of our procedure, we ask the DM in what direction (e.g., change of inequality sign in Form 1 to 5, change of multiplier in Form 3, change of the constants in Form 2, 4, retraction of constraints, etc.) those constraints can be changed or removed to construct feasible space.

2.5 The Imprecise Information Integration Method (IIIM)

The pairwise dominance relationships applied to prioritize alternatives sometimes result in isolated subsets of alternatives in which strict dominance relation among alternatives are made, because of the fuzziness of preference. So, if the purpose of decision aiding is to find out the most preferred alternative or ranking alternatives, the method suggested in Section 2.3 sometimes fails to this end unless further preference information is gathered or modification of preference information is made. To circumvent this problem, we consider the DM's preference strength between an alternative over the others and the others over the alternative, instead of considering only strict dominance result which delivers that, for example, an alternative strictly dominates another if the pairwise dominance relationship results in nonnegative value. In other words, the differential degree of preference of each alternative can be used to prioritize alternatives under incomplete information. This concept can be found in outranking-related research areas. An outranking relation is conceived so as to represent in a preference model, the particular situation in which two actions (alternatives) are incomparable. Obviously in several cases of decision problems, the DM does not know or does not wish to compare two actions. In this case, it is said that these two actions are incomparable. This situation is often due to phenomena such as imprecision of data, and unreliability linked with the personality of the DM (Siskos, 1982).

Roy (1973) suggested that concordance and discordance indices, respectively, can be viewed as measures of satisfaction and dissatisfaction that a decision maker feels on choosing one alternative over the other under single decision situation. He introduced the outranking concept that alternative a_i outranks a_j if there is a sufficiently strong argument in favor of the assertion that a_i is at least as good as a_j from the decision maker's point of view. Brans, et. al. (1986) suggested modification to Roy's ELECTRE, intended to be very simple and easy to understand, assuming that the intensity of preference of one alternative over the other follows specified normalized function on each criterion.

Let us consider two nondominated alternatives, $a_i, a_j \in A$. Through the pairwise dominance relations between a_i and the other alternatives except a_i, we can compute the 'leaving' dominance value by adding pairwise dominance values between a_i and other alternatives respectively. Similarly, through the pairwise dominance among the other alternatives and a_i (the order is reversed), we can compute the 'entering' dominance value. The difference between the leaving and entering dominance value implies the net dominance value which a_i has over the other alternatives. By comparing the magnitude of net dominance values of alternatives, we can conclude that the alternative which has the largest value is the most preferred alternative in the sense of outranking dominance relationship. The aforementioned method can be outlined by the following procedure.

Procedure

Step 1. Using z_{Min} values obtained in individual optimization stage, calculate aggregated intensity of alternative a_i dominating all the other alternatives except alternative a_i.

$$\varphi^+(a_i) = \sum_{a_j \in A-\{a_i\}} z_{Min}(a_i, a_j) \ \forall a_i \in A.$$

This measure implies the intensity by which the DM prefers alternative a_i to all the others. Similarly, calculate aggregated intensity of alternative a_i being dominated by all the other alternatives except a_i.

$$\varphi^-(a_i) = \sum_{a_j \in A-\{a_i\}} z_{Min}(a_j, a_i) \ \forall a_i \in A.$$

Step 2. Calculate the difference between dominating degree and dominated degree of each alternative. The net preference intensity of alternative a_i being preferred to all the other alternatives is defined as difference between the degree of alternative a_i being preferred to the others and that of the others being preferred to alternative a_i.

$$\varphi^N(a_i) = \varphi^+(a_i) - \varphi^-(a_i) \ \forall a_i \in A.$$

Step 3. Alternative a_i is preferred to alternative a_j if aggregated net degree of alternative a_i is greater than that of alternative a_j, that is

$$\begin{cases} a_i P a_j & \text{iff } \varphi^N(a_i) > \varphi^N(a_j) \\ a_i I a_j & \text{iff } \varphi^N(a_i) = \varphi^N(a_j) \\ a_j P a_i & \text{iff } \varphi^N(a_i) < \varphi^N(a_j) \end{cases}$$

If an alternative a_i is the most preferred one, then net preference of alternative a_i, $\varphi^N(a_i) > 0$ because $z_{Min}(a_i, a_j) \geq 0$, $z_{Min}(a_j, a_i) < 0$, and $\varphi^N(a_i) = \sum_{a_j \in A-\{a_i\}} \{z_{Min}(a_i\ a_j) - z_{Min}(a_j\ a_i)\} \geq 0$. Similarly, if an alternative a_i is the least preferred alternative, then $\varphi^N(a_i) < 0$. The above statement states only sufficient condition, not necessary condition.

3. A Numerical Example

This section illustrates features of the proposed procedure in the context of an modified example (Salo and Hamalainen, 1992). The aim of the example is to help graduating students evaluate alternative employment options. There are three employment alternatives: a government office (x), a large enterprise with a good reputation (y), a small company finance by venture capital (z). In the original example, there are two-leveled hierarchical attributes classified under the attributes job security, income, and career opportunities, but in this paper, we consider stability of the firm (w_1), annual starting salary (w_2), future salary increases (w_3), educational opportunities (w_4). In our example, we consider the hierarchical structure single level representation because linear inequalities in terms of hierarchical weights or weight ratios become linear inequalities in

〈Table 2〉 Incomplete information about utility

	Incomplete information on utility
Stability of the firm	$u_S(x) \geq u_S(y), u_S(y) \geq 3u_S(z),\ 0.6 \leq u_S(x) \leq 0.7$
Annual starting salary	$1 = u_A(z) \geq u_A(y) \geq u_A(x),\ 0.7 \leq u_A(y) \leq 0.8,$ $0.1 \leq u_A(x) \leq 0.2$
Future salary increase	$u_F(z) \geq 1.2u_F(y),\ u_F(y) \geq 2u_F(x),\ u_F(y) + u_F(z) \geq 0.8$
Educational opportunity	$u_E(y) \geq 1.5u_E(y),\ u_E(z) \geq 5u_E(x),\ u_E(y) + u_E(z) \geq 0.8$

terms of single level weights (Sage and White, 1984). The utilities articulated from the DM are, for example, as following Table 2.

For example, the incompletely specified utility, $u_s(x) \geq 3u_s(y)$ means that the DM feels that government office is three times as stable as large enterprise. Furthermore, suppose that the DM makes the following preference statements between attributes: 1) annual starting salary is more important than the sum of stability of the firm and educational opportunity, and future salary increase is more important than annual starting salary 2) stability of the firm is twice as important as educational opportunity 3) educational opportunity is between 0.1 and 0.2. These statements can be summarized by the following weight space,

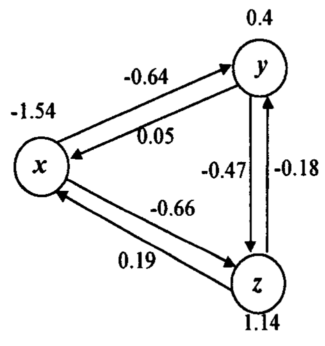
$$\Phi_w = \{w_2 \geq w_1 + w_4,\ w_3 \geq w_2,\ w_1 \geq 2w_4,\ 0.1 \leq w_4 \leq 0.2\}$$

DM's optimization results by pairwise dominance relationship suggested in Section 2.3 can be obtained through 30 LPs in Table 3.

〈Table 3〉 The pairwise dominance results (strict/weak)

	x	y	z	φ^+
x		-0.64	-0.66	-1.3
y	0.05		-0.47	-0.42
z	0.19	-0.18		0.01
φ^-	0.24	-0.82	-1.13	

These pairwise dominance results can be depicted by the directed graph shown in following Figure 1.



[Figure 1] Preference strength between alternatives

The number of outgoing arc implies the dominance strength of emanating alternative having directed alternative. The number appeared on the each circle (alternative) means the net preference strength which each alternative has over all the other alternatives. By comparing the magnitude of net preference strength of each alternative, we come to conclusion that alternative z is most preferred, y secondly, and x thirdly. If we use only the information of strict dominance results, it is possible to obtain partial preference order as mentioned previously that demanding the strict dominance condition may not be fulfilled in most decisions with incomplete information.

4. Concluding Remarks

Multi-criteria decision support is conceived to prioritize alternatives under the conflicting nondominated alternatives. Sometimes, DM faces the situation where s/he can not express preferences in exact way because some attribute is intangible and s/he has limited cognitive constraints about the problem domain. To resolve this situation, we suggest general incomplete information which takes any combination of 5 forms of linear inequalities. Under some practical assumption (functional independence), the difference between expected value for the pairwise dominance check can be break down into individual linear programs because the same attribute weight should be applied to. However the multi-criteria decision problem with incomplete information on both utility and trade-off weight sometimes does not result in prioritizing all alternatives because of the fuzziness of preference information. Further, there exists information loss which strict dominance checks do not take into account. To handle these problems, we suggest decision aiding based on outranking concept which considers pairwise strict and weak dominance results among alternatives.

Further research should be done on decision support system (DSS) for interactive preference elicitation from DMs and automated solving tool because the number of linear programs to be solved increase in multiplicative fashion as the number of alternative and attributes increase.

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