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Fuzzy Identification by Means of an Auto-Tuning Algorithm and a Weighted Performance Index

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ABSTRACT

The study concerns a design procedure of rule-based systems. The proposed rule-based fuzzy modeling implements system structure and parameter identification in the efficient form of "IF···, THEN···" statements, and exploits the theory of system optimization and fuzzy implication rules. The method for rule-based fuzzy modeling concerns the form of the conclusion part of the the rules that can be constant. Both triangular and Gaussian-like membership function are studied. The optimization hinges on an autotuning algorithm that covers as a modified constrained optimization method known as a complex method. The study introduces a weighted performance index (objective function) that helps achieve a sound balance between the quality of results produced for the training and testing set. This methodology sheds light on the role and impact of different parameters of the model on its performance. The study is illustrated with the aid of two representative numerical examples.

1. Introduction

In the early 1980, linguistic approach[1,2] and fuzzy relationship equation-based approch[3,4] were proposed as identification methods of fuzzy models. In the linguistic approach, Tong identified gas furnace process by means of logical examination of data[7]. B. Li *et al.* obtained good results through the modification of Tong's method[6] and also proposed the modified algorithm of adaptive model based on decision table. But the algorithm has some problems due to the computer capacity and computation time which is important, when it was applied to the high-order multivariable systems[5]. Pedrycz analyzed the identification of fuzzy system from the viewpoint of linguistic implication rule modeling, using the referential-fuzzy-set concept[2]. T. Li *et al.* presented a self-learning algorithm for the simple SISO fuzzy model[5]. In the fuzzy relationship equation-based approch, Pedrycz identified fuzzy systems, using the referential fuzzy set and Zadeh's conditional possibility distribution, that is, the new composition rule which were made by the fuzzy relationship equations[3]. Xu constructed and identified the fuzzy relationship model using the referential fuzzy set theory and the self-learning algorithm[5,6]. The direct inference utilized by two methods did not perform better than the linear inference. Sugeno identified the structure of systems through the standard least square methods[10], but the structure of premises of the rules was determined more heuristically through the experience and iterative fuzzy partitioning of the input space. Sugeno also applied his method to the fuzzy identification of gas furnace process, using fuzzy c-means clustering[11,12], but the method did not produce the identification of good performance; this could be alleviated to the use of direct linear inference[8].

In this paper, the simplified reasoning model is considered. In the fuzzy inference, we consider three types of membership functions, namely Gaussian membership functions with modifiable or fixed slope, and triangular membership functions. According to the proposed autotuning algorithm-the improved complex method, the parameters of such membership function can be easily adjusted. Furthermore we introduce an aggregate objective function that deals with training data and testing data, and elaborate on its optimization to produce a meaningful balance between approximation and generalization abilities of the model. The proposed ruled-based fuzzy modeling is carried out for time series data for gas furnace

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process[9] and traffic route choice. The performance of the proposed rule-based fuzzy modeling is presented from the viewpoint of the identification errors.

2. System Modeling by Means of Fuzzy Inference

The identification algorithm of fuzzy model is divided into the identification activities of premise and consequence parts of the rules. The identification at the premise level 1) selects the input variables x_1, x_2, $\cdots$ x_n of the rules, and 2) determines the fuzzy partitions (*Small, Large*, etc.) of fuzzy spaces. This means the determination of the number of the optimal fuzzy space partitions, that is, fuzzy subspaces that determine the number of fuzzy implication rules. The premise identification has to determine the membership values of fuzzy variables. The consequence identification embraces the following phases 1) selection of the consequence variables of the fuzzy implication rules, 2) determination of the consequence parameters.

In this paper, in order to identify the premise structure and parameters of fuzzy linguistic rules, two essential input variables of process influenced are considered and the improved complex method which is a powerful auto-tuning algorithm is used. Furthermore, we restrict ourselves to some types of membership function such as Gaussian-like and triangular ones. The parameters of the membership functions are tuned with the help of the autotuning method. The parameters of the consequence part of the rules are determined using the standard least square method (Gaussian elimination with maximal pivoting algorithm). We also discuss a modified performance index (objective function) that aims at achieving a balance between approximation and prediction capabilities of the fuzzy model.

3. An Algorithm of Fuzzy Identification

In this section we elaborate on algorithmic details of the identification method discussing the optimization problem to the antecedent (condition part) of the rules as well as an enhancements of their conclusions.

3.1 Premise Identification

In the premise part of the rules we confine ourselves to Gaussian-like and triangular type function. This selection, even though looks somewhat limited, embraces a broad range of cases that could be covered by modifying the parameters of these functions. For the triangular membership functions we have either 2 or 3 parameters, see Fig. 1, whose points can be autotuned (adjusted). The Gaussian type of the membership function assumes the form

$$f(x) = e^{\frac{-s(x-a)^2}{b^2}}$$

Furthermore we consider Gaussian membership functions involving fixed slope, and assume several levels of their parametric flexibility such as

- fixed slope of the functions
- the slopes are modified but kept the same for all the functions
- the slopes of the membership function can vary and will be adjusted (optimized), refer to Fig. 2.

In the case of the same slope and different slope, these mean the same and different slope in each input variable, and the slope parameter of each case is auto-tuned according to the proposed optimization.

3.2 Consequence Identification

The identification of the conclusion parts of the the rules deals with a selection of their structure and a determination of parameters therein.

The consequence part of the simplified inference mechanism where the rules have constant conclusion

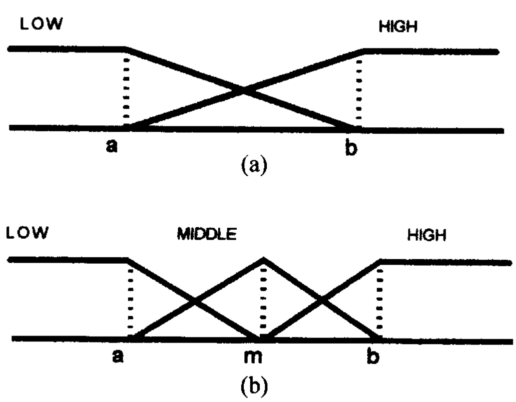
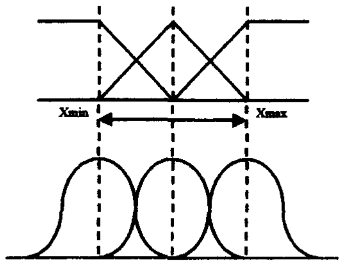
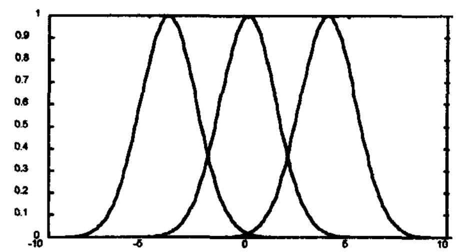


Fig. 1. Triangular membership function.



(a) Parameters of Gaussian type membership function



(b) 3 fuzzy variables with 3 modifiable parameters(s, a, and b)

Fig. 2. Gaussian membership function.

part is given as follows.

$$R^i: \text{If } x_1 \text{ is } A_{i1}, \cdots, \text{and } x_k \text{ is } A_{ik}, \text{then } y = a_i \quad (1)$$

The calculations of the numeric output of the model are carried out in the well-known form,

$$y^* = \sum_{i=1}^{n} \mu_i a_i / (\sum_{i=1}^{n} \mu_i) = \sum_{i=1}^{n} \hat{\mu}_i a_i$$

where R^i is the i-th fuzzy rule, x_j is input variables, A_{ij} is a membership function of fuzzy sets, a_i is a constant, n is the number of the fuzzy rules, y^* is the infered value, μ_i is the premise fitness matching of R^i (activation level) and $\hat{\mu}_i$ is the normalized premise fitness of R^i. If the input variables of the premise and parameters are given in consequence parameter identification, the optimal consequence parameters which minimize the assumed performance index can be determined. In what follows, we define the performance index as a sum of squared errors.

$$I = (1/m) \cdot \sum_{i=1}^{m} (y(k) - y^o(k))^2 \quad (2)$$

where y^o is the output of the fuzzy model, k denotes the number of the input variables, and "m" stands for the total number of data. Furthermore $x_{1i}, x_{2i}, \cdots, x_{ki}, y_i$ (i=1, 2, $\cdots$, m) are pairs of input-output data set. The consequence parameters ai can be determined by the standard least-square method. In the fuzzy model of Type 1, the parameters can be estimated by solving the optimization problem.

$Min_a\ V(a, m)$

Where

$$V(a,m) = 1/2 \cdot \sum_{i=1}^{m} \varepsilon_i^2 = 1/2 \cdot \sum_{i=1}^{m} (y_i - y^o)^2$$
$$= 1/2 \sum_{i=1}^{m} [y_i - \sum_{j=1}^{m} a_j u_{ji}]^2$$
$$= 1/2 \cdot \sum_{i=1}^{m} [y_i - x_i^T a]^2 = 1/2 \cdot \| E \|^2 \quad (3)$$

(Moreover

$$u_{ji} = \frac{A_{ki}(x_{1i})^* \cdots {}^*A_{jk}(x_{ki})}{\sum(j=1,m) A_{ji}(x_{1i})^* \cdots {}^*A_{jk}(x_{ki})}$$

where j- rule no., i- data no., m- total no. of data, k- no. of input fuzzy variable)

Due to the form of the performance index in Eq. (3), the minimal value produced by the least-square method is determined as follows.

$$\hat{a} = (X^T X)^{-1} X^T Y \quad (4)$$

Where $x_i^T = [u_{1i}, \cdots, u_{ni}]$, $a^T = [a_1, \cdots, a_n]$, $Y = [y_1, \cdots, y_m]^T$, $E = [\varepsilon_1, \cdots, \varepsilon_m]^T$, $X = [x_1^T, \cdots, x_m^T]^T$

3.3 The Objective Function with Weighting Factor

We elaborate on the performance index. The objective function for the training data and testing data assumes the form

$$f = (PARA1 \times PI + PARA2 \times E\_PI)/2$$

and is utilized as a cost function of the fuzzy model. Where, $PARA1$ and $PARA2$ are two weighting factors for PI and E_PI, respectively. PI and E_PI denote the values of the performance index for the training data and testing data, respectively. For the purpose of minimization of this objective function, all parameters of the premise membership functions such as Gaussian-like and triangular function are modified (optimized).

Depending upon the values of the weighting factor, let us discuss several specific cases of the objective function.

1) If $PARA1=1$ and $PARA2=0$ then the model becomes optimized based on the training set. No testing set is not taken into consideration.

2) Both $PARA1=1$ and $PARA2=1$ is the case the training and testing data set are taken into account.

3) The case both $PARA1=\alpha$ and $PARA2=1-\alpha$ where $\alpha\in[0, 1]$ embraces both the cases stated above. The choice of establishes a certain tradeoff between the approximation and generalization aspects of the fuzzy model.

4) In general, $PARA1$ and $PARA2$ can be selected and adjusted independently.

The performance index used in the ensuing numerical experiment will be as an Euclidean and Hamming distances, that is,

$$PI = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2 \tag{5}$$

$$PI = \sum_{i=1}^{N}|y_i - \hat{y}_i| \tag{6}$$

The variables of a cost function to be optimized come as the parameters of the membership functions, fuzzy rules, and weighting factors of the performance index. Based upon a selection of sound fuzzy reasoning type, specification of the membership function type, and weighting factors we can design an optimal fuzzy model.

3.4 Autotuning by Improved Complex Method

Usually, by combining these optimization tasks we end up with a problem that is highly nonlinear and may not fit well to the domain of gradient-based techniques. To alleviate the problem, we propose to use an autotuning algorithm that is an adaptation of the improved complex method.

We realize the algorithm by augmenting the simplex concept to the complex method [2] - constrained optimization technique. The proposed optimal autotuning algorithm known as the improved complex method, is the constrained complex method of the form:

Minimize $f(x)$

Subject to $g_j(x)\leq 0, j=1, 2, \cdots, m$

$x_i^{(l)}\leq x_i \leq x_i^{(u)} \quad i = 1, 2, \cdots, n$

where the superscripts l and u denote the lower and upper bound of the corresponding variable.

<step 1>

The parameters to be optimized include the elements of the fuzzy model. They include the slope and center parameter of Gaussian type membership function, and each parameter (a and b) of the triangular membership function. They are defined as $X_k=(x_1^k, x_2^k, \cdots, x_n^k; k=1, 2, \cdots n, n+1, \cdots, m)$ and form the points in an "n" dimensional space. In general, the value of "m" is selected as being equal 2n (where n is the number of the initial vertices).

<step 2>

The initial values of α, γ and β is specified using the Reflection, Expansion and Contraction of simplex concept as follows:

i) Reflection: $X_r = X_o + \alpha(X_o - X_h)$ (7)

ii) Expansion: $X_e = X_o + \gamma(X_r - X_o)$ (8)

iii) Contraction: $X_c = X_o + \beta(X_h - X_o)$ (9)

<step 3>

X_h and X_l are the vertices corresponding to the maximum function value $f(X_h)$ and the minimum function value $f(X_l)$. X_0 is the centroid of all the points X_i except $i=h$. The reflection point X_r is given by (7), with X_h=max $f(X_i)$, $(i=1, \cdots, k)$, $X_0=(1/(m-1))((\Sigma_{i=1}^n X_i)-X_h)$ and $\alpha=\|X_r-X_o\|/\|X_h-X_o\|$.

If X_r may not satisfy the constraints, a new point X_r is generated by $X_r=(X_0+X_r)/2$. This process is repeated until X_r satisfies the constraints. A new simplex is started.

<step 4>

If a reflection process gives a point X_r for which $f(X_r)<f(X_l)$, i.e. if the reflection produces a new minimum, we expand X_r to X_e by (8), with $\gamma=\|X_e-X_o\|/\|X_r-X_o\|>1$.

If X_e does not satisfy the constraints, a new point X_e is generated by $X_e=(X_0+X_e)/2$. This process is repeated until X_e satisfies the constraints. If $f(X_e)<f(X_l)$, we replace the point X_h by X_e and restart the process of reflection. On the other hand, if $f(X_e)>f(X_l)$, we replace the point X_h by X_r, and start the reflection process again.

<step 5>

If the reflection process produces a point X_r for which $f(X_r)>f(X_i)$, for all i except $i=h$. and $f(X_r)<f(X_h)$, then we replace the point X_h by X_r. In this case, we

contract the simplex as in (9), with $\beta=\|X_e-X_o\|/\|X_h-X_o\|$.

If $f(X_r)>f(X_h)$, we use X_c without changing the previous point X_h. If X_c does not satisfy the constraints, a new point X_c is generated with $X_c=(X_0+X_c)/2$. This process is conducted repeatedly until X_e satisfies the constraints. If the contraction process produces a point X_c for which $f(X_c)<\min[f(X_h), f(X_r)]$, we replace the point X_h by X_c. and proceed with the reflection again. On the other hand, if $f(X_c)>=\min[f(X_h), f(X_r)]$, we replace all X_i by $(X_i+X_l)/2$, and start the reflection process again.

<step 6>

The method is assumed to have converged whenever the standard deviation of the function at the vertices of the current simplex is smaller than some prescribed small quantity as ε follows:

$$Q=\left\{\sum_{i=1}^{n+1}\frac{[f(X_i)-f(X_o)]^2}{n+1}\right\}^{1/2}\leq\varepsilon \qquad (10)$$

If Q may not satisfy (10), we go to step 3.

4. Experimental Studies

Once the identification methodology has been established, one can proceed with intensive experimental studies. In this section, we report on the experiments using some well-known data sets used in fuzzy modeling. These include gas furnace data and traffic control data.

4.1 Gas Furnace Process

In this section, the proposed rule-based fuzzy modeling is applied to the time series data of gas furnace utilized by Box and Jenkins[9]. We try to model the gas furnace using 296 pairs of input-output data. The flow rate of metane gas, $U_m(t)$ used in laboratory changes from -2.5 to 2.5, the control $U(t)$ used in real process, ranges from 0.5 to 0.7 following the expression.

$$U(t)=0.60-0.048\,U_m(t) \qquad (11)$$

U denotes the flow rate of methane as input, the output stands for the carbon dioxide density i.e., the outlet gas.

The structure and parameter identification of premise are performed using the improved complex method. The improved complex method extracts the optimal fuzzy rules and upgrades the performance by auto-tuning parameters of premise membership function. The reflection, expansion and contraction coefficients which are the initial parameters of the improved complex method are set as α=1, γ=2 and β=0.5, respectively. The consequence parts of two kinds of types are used. Table 1 shows the performance

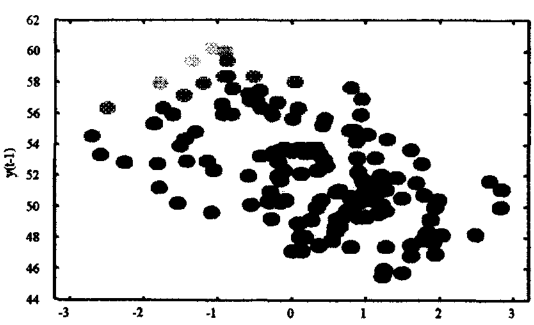


Fig. 3. Data points induced by I/O data set ($u(t-1)$, $y(t-1)$, $y(t)$).

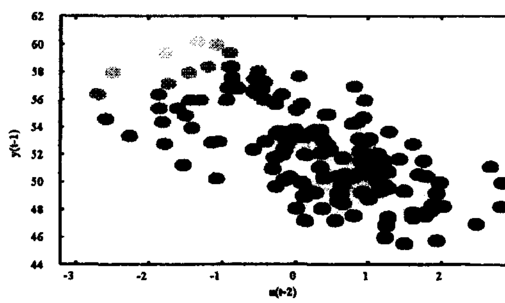


Fig. 4. Data points induced by I/O data set ($u(t-2)$, $y(t-1)$, $y(t)$).

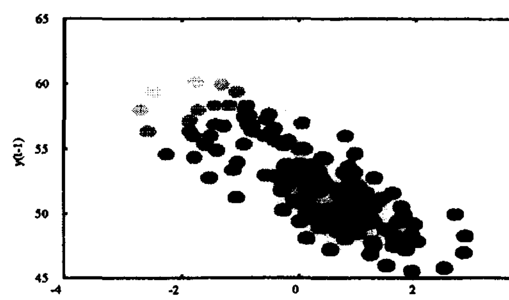


Fig. 5. Data points induced by I/O data set ($u(t-3)$, $y(t-1)$, $y(t)$).

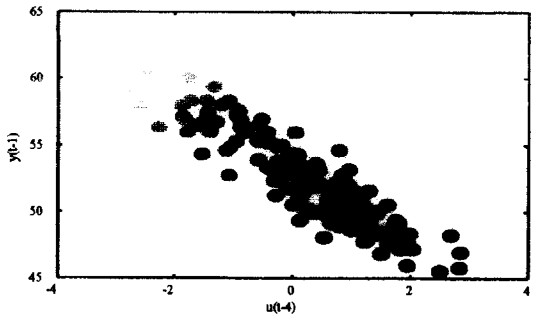


Fig. 6. Data points induced by I/O data set ($u(t\text{-}4)$, $y(t\text{-}1)$, $y(t)$).

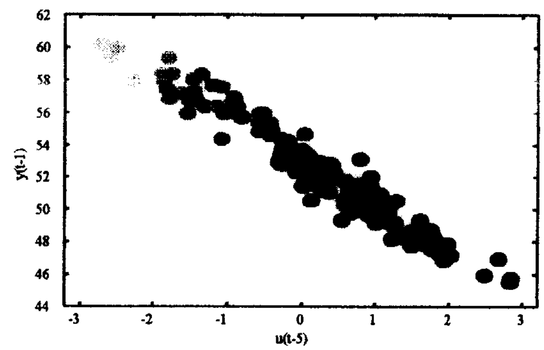


Fig. 7. Data points induced by I/O data set ($u(t\text{-}5)$, $y(t\text{-}1)$, $y(t)$).

index of the optimal rules obtained using the improved complex method for each fuzzy model consisted of the consequence types of simplified and linear inference, and the premise types of Gaussian type function with fixed slope, same slope and

Table 1. Optimal performance index for the fuzzy model by means of the adjustment of weighting factors

(a) Simplified fuzzy reasoning method with Gaussian type membership function

Model No.	Model Name & No. of Data	Weighting Factor (PARA1,	Con-sequence Structure	Premise Structure (Gauss.,	Input Variable & No. of Input (M×N)	No. of Rule	FIXED SLOPE		SAME SLOPE		DIFFERENT SLOPE	
							PI	E_PI	PI	E_PI	PI	E_PI
1	GAS(145+145)	1,0	Simplified	Gaussian	u(t-3),y(t-1),2×2	4	0.023	0.337	0.022	0.335	0.022	0.334
2	GAS(145+145)	1,1	Simplified	Gaussian	u(t-3),y(t-1),2×2	4	0.036	0.274	0.035	0.275	0.038	0.278
3	GAS(145+145)	1,3	Simplified	Gaussian	u(t-3),y(t-1),2×2	4	0.044	0.269	0.044	0.270	0.052	0.271
4	GAS(145+145)	1,5	Simplified	Gaussian	u(t-3),y(t-1),2×2	4	0.048	0.268	0.049	0.269	0.049	0.271
5	GAS(145+145)	1,10	Simplified	Gaussian	u(t-3),y(t-1),2×2	4	0.095	0.262	0.055	0.267	0.059	0.269
6	GAS(145+145)	1,30	Simplified	Gaussian	u(t-3),y(t-1),2×2	4	0.121	0.260	0.058	0.269	0.079	0.252
7	GAS(145+145)	1,50	Simplified	Gaussian	u(t-3),y(t-1),2×2	4	0.125	0.260	0.073	0.266	0.063	0.262
8	GAS(145+145)	3,1	Simplified	Gaussian	u(t-3),y(t-1),2×2	4	0.027	0.289	0.027	0.291	0.028	0.278
9	GAS(145+145)	5,1	Simplified	Gaussian	u(t-3),y(t-1),2×2	4	0.025	0.303	0.025	0.302	0.025	0.306
10	GAS(145+145)	10,1	Simplified	Gaussian	u(t-3),y(t-1),2×2	4	0.023	0.313	0.022	0.332	0.023	0.320
11	GAS(145+145)	30,1	Simplified	Gaussian	u(t-3),y(t-1),2×2	4	0.023	0.325	0.022	0.333	0.022	0.331
12	GAS(145+145)	50,1	Simplified	Gaussian	u(t-3),y(t-1),2×2	4	0.023	0.330	0.022	0.333	0.022	0.334

(b) Simplified fuzzy reasoning method with triangular type membership function

Model No.	Model Name & No. of Data	Weighting Factor (PARA1, PARA2)	Consequence Structure	Premise Structure (Gauss., Tria.)	Input Variable & No. of Input (M×N)	No. of Rule	PI	E_PI
1	GAS(145+145)	1,0	Simplified	Triangular	u(t-3),y(t-1),2×2	4	0.022	0.335
2	GAS(145+145)	1,1	Simplified	Triangular	u(t-3),y(t-1),2×2	4	0.024	0.328
3	GAS(145+145)	1,3	Simplified	Triangular	u(t-3),y(t-1),2×2	4	0.055	0.316
4	GAS(145+145)	1,5	Simplified	Triangular	u(t-3),y(t-1),2×2	4	0.085	0.308
5	GAS(145+145)	1,10	Simplified	Triangular	u(t-3),y(t-1),2×2	4	0.095	0.307
6	GAS(145+145)	1,30	Simplified	Triangular	u(t-3),y(t-1),2×2	4	0.108	0.306
7	GAS(145+145)	1,50	Simplified	Triangular	u(t-3),y(t-1),2×2	4	0.112	0.306
8	GAS(145+145)	3,1	Simplified	Triangular	u(t-3),y(t-1),2×2	4	0.023	0.331
9	GAS(145+145)	5,1	Simplified	Triangular	u(t-3),y(t-1),2×2	4	0.023	0.331
10	GAS(145+145)	10,1	Simplified	Triangular	u(t-3),y(t-1),2×2	4	0.022	0.334
11	GAS(145+145)	30,1	Simplified	Triangular	u(t-3),y(t-1),2×2	4	0.0228	0.335
12	GAS(145+145)	50,1	Simplified	Triangular	u(t-3),y(t-1),2×2	4	0.0228	0.335

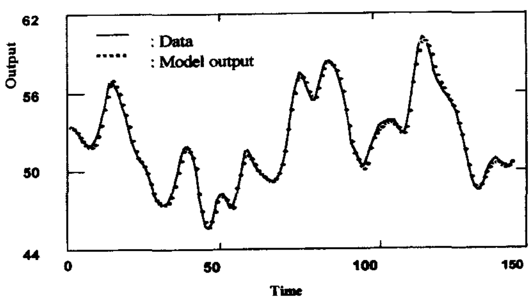
Table 2. Performance index in the fuzzy reasoning method by means of the change of no. of fuzzy variables

(a) Simplified fuzzy reasoning method with input variables u(t-3) and y(t-1)

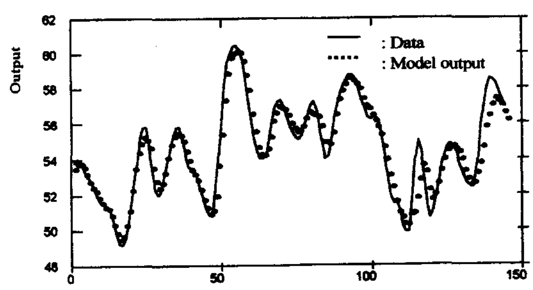
			u(t-3): No. of Fuzzy variables									
			2		3		4		5		6	
			PI	E_PI	PI	E_PI	PI	E_PI	PI	E_PI	PI	E_PI
y(t-1): No. of Fuzzy variables	2	Gaussian (fixed-slope)	0.036	0.274	0.040	0.252	0.092	0.215	0.038	0.261	0.034	0.272
		Gaussian (same-slope)	0.035	0.275	0.040	0.252	0.092	0.213	0.037	0.261	0.032	0.267
		Gaussian (different-slope)	0.038	0.276	0.040	0.252	0.092	0.213	0.037	0.259	0.036	0.262
		Triangular	0.024	0.328	0.022	0.333	0.022	0.332	0.021	0.327	0.021	0.330

(b) Simplified fuzzy reasoning method with input variables u(t-4) and y(t-1)

			u(t-4): No. of Fuzzy variables					
			2		3		4	
			PI	E_PI	PI	E_PI	PI	E_PI
y(t-1): No. of Fuzzy variables	2	Gaussian (fixed-slope)	0.090	0.265	0.093	0.208	0.039	0.257
		Gaussian (same-slope)	0.092	0.266	0.093	0.208	0.037	0.260
		Gaussian (different-slope)	0.101	0.259	0.093	0.209	0.037	0.260



(a) In the case of training data



(b) In the case of testing data

Fig. 8. The comparison of original data and output data for fuzzy model No. 2 (Table 1a).

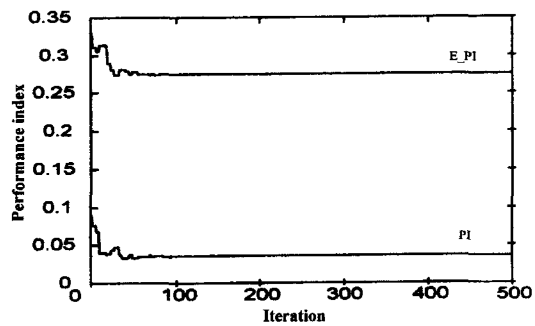
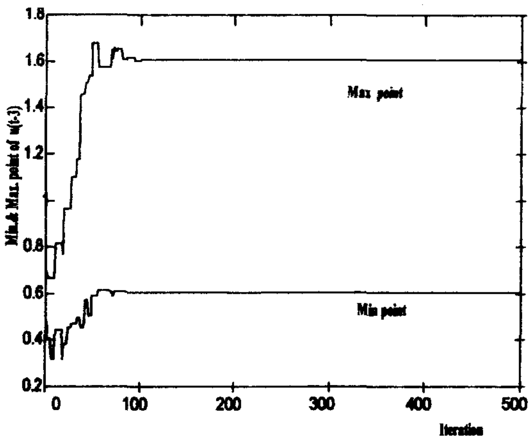


Fig. 9. Convergence procedure to optimal value of *PI* & *E_PI* for fuzy model No. 2 (Table 1-a).

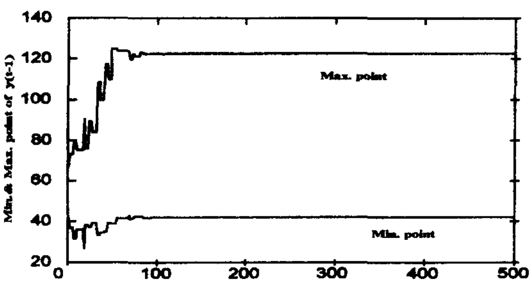
different slope versions, and triangular type function. From the two-dimensional plot of the data set shown in Fig. 3~7, in the case of the training data, the data sets ($u(t$-3), $y(t$-1), $y(t)$) and ($u(t$-4), $y(t$-1), $y(t)$) exhibit more uniform and less sparse distribution than any other data set. Therefore we can anticipate that the fuzzy model structure for the fuzzy partition of data set ($u(t$-3), $y(t$-1), $y(t)$) and ($u(t$-4), $y(t$-1), $y(t)$) could perform a little better than in the remaining sceneries.

4.2 Traffic Route Choice Process

Lately, researchers and officials who work in the field of traffic planning and engineering have expressed a lot of interest about the assignment of traffic load in road network. From 1950s, many highways have been planned and constructed for transportation of men and materials due to the economic development of the advanced industrial



(a) In the case of ($u_{min}(t$-3), $u_{max}(t$-3))



(b) In the case of ($y_{min}(t$-1), $y_{max}(t$-1))

Fig. 10. Convergence procedure to optimal value of maximal and minimal point of input fuzzy membership functions, ($u_{min}(t$-3), $u_{max}(t$-3)) and ($y_{min}(t$-1), $y_{max}(t$-1)) for fuzzy model No. 2 (Table 1a).

nations such as United State, Europe and Japan. It has triggered an important issue on how to assign traffic load and set up effectively the harmonious transportation system from two roads or a few roads all being in a competitive relationship. To solve such problems, lots of research is being carried out to get the optimal model of traffic transition and traffic route related to the traffic planning.

The selection of a certain traffic route is an example of a complex driver's decision making. Ambiguity (or fuzziness) inherently associated with traffic route problem exhibits the implicit meaning such as (1) Ambiguity of model's input data (2) Fuzziness of human cognition (3) Ambiguity of human decision making.

The usage of the knowledge data based on information mentioned above, modeling method was proposed to make traffic route choice model[15]. Generally, almost of these methods perform modeling from valid infomations which are obtained by human's knowledge. This method is different from some other modeling methods which are primarily based upon statistical approches.

We can think of many cases of route choice model. But, we confine our interest to assignment problem between road network and rail system or route choice problem between road and highway. As shown below, Fig. 11 illustrates a simple example of a route structure from O to D. Even though we can consider various types of cost related to many decision criterion for evaluation about each route for generalization, in this paper, we consider just time cost and traffic cost as the essential components of the objective function.

We consider simple problem related to traffic route choice which is based on two routes mentioned above. The model was considered as conventional example for traffic route choice model[14]. Then, we assume that the object for traffic routes is driver and our interest is focused on how to express traffic behavior at the individual level. The performance index is defined as shown in Eq. (6). The inputs T1 and T2 denote a traffic and time cost associated with route 1 and route 2, respectively. The output P_{1n} is choice probability ratio of route 1.

The performance index of each model is compared with other fuzzy modeling methods and the results

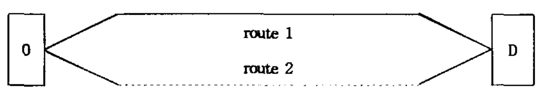


Fig. 11. Simple Example of Route Choice.

Table 3. Example Data For Logit Model

No	route	T1	T2	No	route	T1	T2
1	2	52.9	4.4	12	1	18.5	84.0
2	2	4.1	28.5	13	1	82.0	38.0
3	1	4.1	86.9	14	2	8.6	1.6
4	2	56.2	31.6	15	1	22.5	74.1
5	2	51.8	20.2	16	1	51.4	83.8
6	1	0.2	91.2	17	2	81.0	19.2
7	1	27.6	79.7	18	1	51.0	85.0
8	2	89.9	2.2	19	1	62.2	90.1
9	2	41.5	24.5	20	2	95.1	22.2
10	2	95.0	43.5	21	1	41.6	91.5
11	2	99.1	8.4				

Table 4. Optimal performance index for the fuzzy model by means of the adjustment of weighting factors

(a) Simplified fuzzy reasoning method with Gaussian type membership function

Model No.	Model Name & No. of Data	Weighting Factor (PARA1, PARA2)	Con-sequence Structure	Premise Structure (Gauss., Tria.)	Input Variable & No. of Input (M×N)	No. of Rule	FIXED SLOPE		SAME SLOPE		DIFFERENT SLOPE	
							PI	E_PI	PI	E_PI	PI	E_PI
1	TR(16+16)	1,1	Simplified	Gaussian	2×2	4	1.696	8.633	1.71	8.71	1.690	8.655
2	TR(16+16)	1,3	Simplified	Gaussian	2×2	4	1.712	8.685	1.50	7.50	1.691	8.657
3	TR(16+16)	1,5	Simplified	Gaussian	2×2	4	2.296	8.214	1.55	7.50	1.704	8.657
4	TR(16+16)	1,10	Simplified	Gaussian	2×2	4	1.712	8.685	1.36	7.02	4.037	6.830
5	TR(16+16)	1,30	Simplified	Gaussian	2×2	4	1.712	8.685	1.39	7.05	3.925	6.892
6	TR(16+16)	1,50	Simplified	Gaussian	2×2	4	1.712	8.685	1.35	7.01	3.934	6.918
7	TR(16+16)	3,1	Simplified	Gaussian	2×2	4	1.696	8.663	1.71	8.71	1.689	8.655
8	TR(16+16)	5,1	Simplified	Gaussian	2×2	4	1.696	8.663	1.50	7.50	1.706	8.721
9	TR(16+16)	10,1	Simplified	Gaussian	2×2	4	1.664	9.031	1.05	9.87	1.657	9.227
10	TR(16+16)	30,1	Simplified	Gaussian	2×2	4	1.469	9.696	1.33	7.00	1.651	9.312
11	TR(16+16)	50,1	Simplified	Gaussian	2×2	4	1.491	9.753	1.50	7.58	1.650	9.325

(b) Simplified fuzzy reasoning method with triangular type membership function

Model No.	Model Name & No. of Data	Weighting Factor (PARA1, PARA2)	Consequence Structure	Premise Structure (Gauss., Tria.)	Input Variable & No. of Input (M×N)	No. of Rule	PI	E_PI
1	TRAF(16+16)	1,0	Simplified	Triangular	2×2	4	1.408	9.744
2	TRAF(16+16)	1,1	Simplified	Triangular	2×2	4	1.768	8.709
3	TRAF(16+16)	1,3	Simplified	Triangular	2×2	4	1.768	8.709
4	TRAF(16+16)	1,5	Simplified	Triangular	2×2	4	1.768	8.709
5	TRAF(16+16)	1,10	Simplified	Triangular	2×2	4	2.170	9.659
6	TRAF(16+16)	1,30	Simplified	Triangular	2×2	4	2.442	9.642
7	TRAF(16+16)	1,50	Simplified	Triangular	2×2	4	2.442	9.642
8	TRAF(16+16)	3,1	Simplified	Triangular	2×2	4	1.427	9.616
9	TRAF(16+16)	5,1	Simplified	Triangular	2×2	4	1.427	9.617
10	TRAF(16+16)	10,1	Simplified	Triangular	2×2	4	1.422	9.626
11	TRAF(16+16)	30,1	Simplified	Triangular	2×2	4	1.395	9.713
12	TRAF(16+16)	50,1	Simplified	Triangular	2×2	4	1.406	9.652
13	TRAF(16+16)	1,1	Simplified	Triangular	3×2	6	1.631	8.551
14	TRAF(16+16)	1,1	Simplified	Triangular	4×2	8	1.605	7.855
15	TRAF(16+16)	1,1	Simplified	Triangular	3×3	9	1.690	8.534
16	TRAF(16+16)	1,1	Simplified	Triangular	5×2	10	1.144	9.746

are summarized in Table 6. This comparison reveals that performance index for the training data only; the testing data are not studied. In other words, the weighting factors assume the values *PARA*1=1 and *PARA*2=0, respectively.

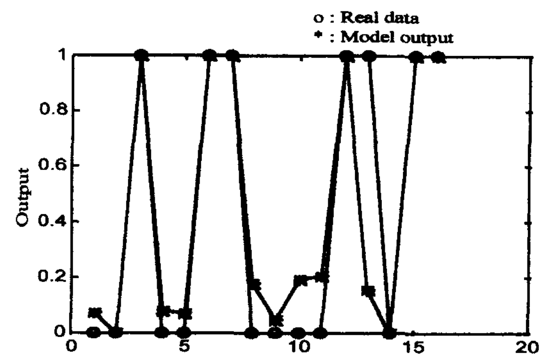
The analysis of the experimental data allows us to draw some general conclusions:

4.2.1 Gas Furnace Process

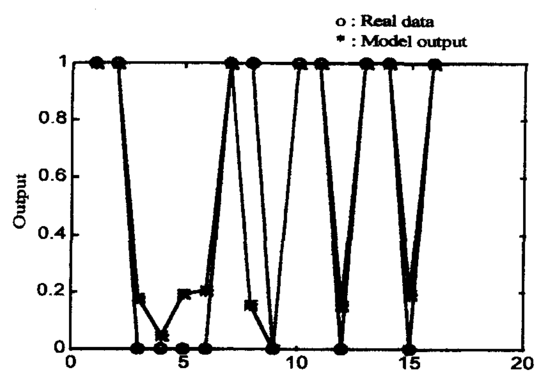
The most suitable design option is to optimize the second weighting parameter (*PARA*2) of the objective function and use the the Gaussian membership function. As Table 2 reveals, the increase in the number of the linguistic terms for the input space improves the performance of the model.

Table 5. Performance index in the simplified fuzzy reasoning method by means of the change of no. of fuzzy variables

			T1: No. of Fuzzy variables					
			2		3		4	
			PI	E_PI	PI	E_PI	PI	E_PI
T2: No. of Fuzzy variables	2	Gaussian (fixed-slope)	1.696	8.663	1.585	8.540	1.383	7.129
		Gaussian (same-slope)	1.710	8.710	1.361	7.030	1.357	7.020
		Gaussian (different-slope)	1.690	8.655	1.488	7.254	1.422	7.107
		Triangular	1.768	8.709	1.631	8.551	1.605	7.855
	3	Gaussian (fixed-slope)	1.913	8.682	0.751	7.977	0.114	7.314
		Gaussian (same-slope)	0.00009	7.000	0.002	7.003	0.107	7.305
		Gaussian (different-slope)	0.028	7.207	0.009	7.028	0.164	7.245
		Triangular	1.466	9.587	1.690	8.533		
	4	Gaussian (fixed-slope)	1.626	8.388	0.098	8.629	0.0011	7.333
		Gaussian (same-slope)	0.000004	7.993	0.002	7.039	0.0002	7.351
		Gaussian (different-slope)	0.00002	7.998	0.046	7.261	0.0003	7.570
		Triangular	1.603	8.792	1.179	8.797		



(a) In the case of the training data



(b) In the case of the testing data

Fig. 12. The comparison of original data and output data for fuzzy model No. 1 (Table 4a).

The optimal fuzzy rules obtained from model 6 with different slope in Table 1 are as follows

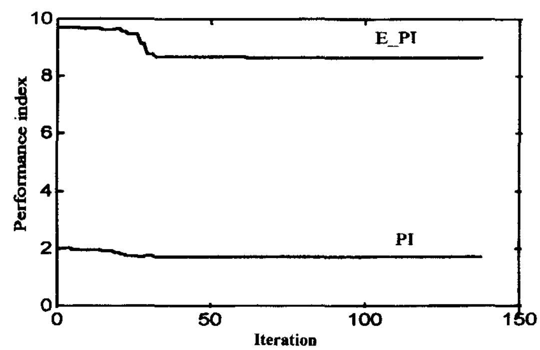


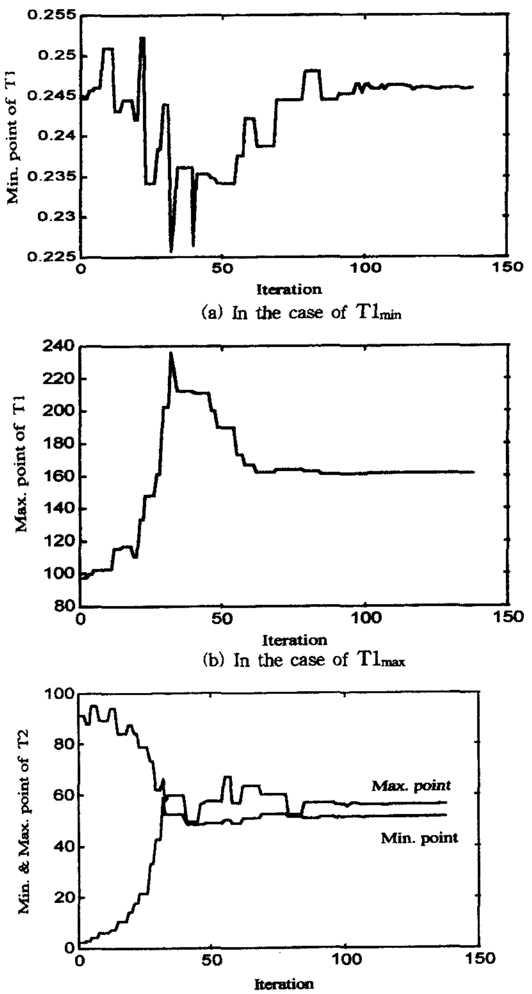
Fig. 13. Convergence procedure to optimal value of PI & E_PI for fuzzy model No. 1 (Table 4a).

R^1: If $u(t\text{-}3)$ is $Small_1$ & $y(t\text{-}1)$ is $Small_2$, then $y(t)=a_1$
R^2: If $u(t\text{-}3)$ is $Small_1$ & $y(t\text{-}1)$ is Big_2, then $y(t)=a_2$
R^3: If $u(t\text{-}3)$ is Big_1 & $y(t\text{-}1)$ is $Small_2$, then $y(t)=a_3$
R^4: If $u(t\text{-}3)$ is Big_1 & $y(t\text{-}1)$ is Big_2, then $y(t)=a_4$

The initial and final tuned values of each parameter associated with the fuzzy rules mentioned above are like Table 7. Those values of Gaussian membership functions of fuzzy input variables are shown as Fig. 15. The dotted line in Fig. 15 represents the final tuned values.

4.2.2 Traffic Route Choice Process

As mentioned in gas furnace and sewage treatment process, the quantity and sparse distribution of data set affect the structure of the optimal fuzzy model such as the reasoning method, number of



(a) In the case of $T1_{min}$

(b) In the case of $T1_{max}$

(c) In the case of ($T2_{min}$,$T2_{max}$)

Fig. 14. Convergence procedure to optimal value of maximal and minimal point of input fuzzy membership functions, ($T1_{min}$, $T1_{max}$) and ($T2_{min}$, $T2_{max}$) for fuzzy model No. 1 (Table 4a).

membership function of input fuzzy variables and membership function type.

Table 6. Comparison of identifiction error with previous models

Model Type		PI	Shooting ratio (%)
1) BL (Binary Logit) model [14]		5.452	90.4
2) PS (Production System) model[15]		2.0	85.7
3) Neural networks model[15]		0.497	95.2
4) Fuzzy neural networks model[15]		1.178	90.4
Proposed model	Gaussian (fixed-slope)	0.000002	99.9
	Gaussian (same-slope)	0.000001	99.9
	Gaussian (different-slope)	0.000001	99.9
	Triangular	0.01	99.7

The optimal fuzzy rules obtained from model 4 with different slope in Table 4(a) are as follows.

R^1: If $T1$ is $Small_1$ & $T2$ is $Small_2$, then route=a_1

R^2: If $T1$ is $Small_1$ & $T2$ is Big_2, then route=a_2

R^3: If $T1$ is Big_1 & $T2$ is $Small_2$, then route=a_3

R^4: If $T1$ is Big_1 & $T2$ is Big_2, then route=a_4

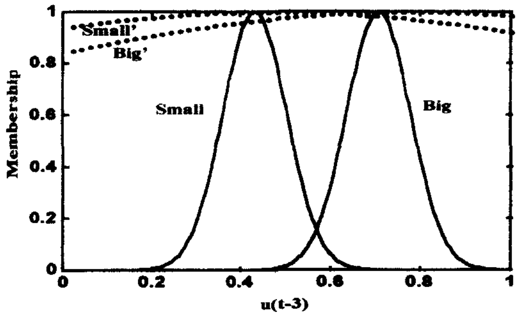
The initial and final tuned values of each parameter associated with the fuzzy rules mentioned above are like Table 8. Those values of Gaussian membership functions of fuzzy input variables are shown as Fig. 16.

Again, as in the previous experiments we worked with various optimization, scenaries utilizing all parameters.

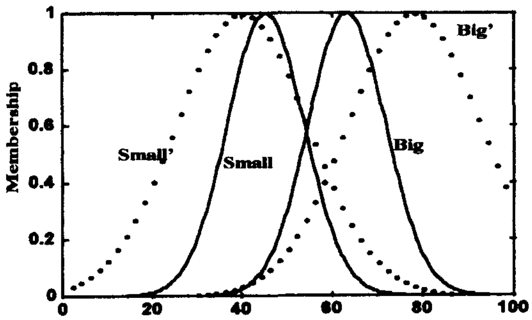
When we apply three types of Gaussian functions and triangular function to premise structure as the membership function of fuzzy input variables, we can compare and analyze the characteristics and the

Table 7. The initial & final tuned values of the membership functions of the premise fuzzy variables, consequence parameters, and performance index

	Premise parameter												Consequence parameter				Performance index	
	$Small_1$			Big_1			$Small_2$			Big_2			a_1	a_2	a_3	a_4	PI	E_PI
	a	b	s	a	b	s	a	b	s	a	b	s						
Initial value	0.431	0.166	2.7	0.707	0.166	2.7	45.25	10.80	0.8	63.20	10.80	0.8	44.91	58.87	47.81	61.91	0.204	0.392
Final tuned value	0.464	0.187	0.01	0.776	0.187	0.01	39.68	22.93	1.23	77.75	22.93	1.23	-358.2	170.0	454.8	-34.1	0.079	0.252

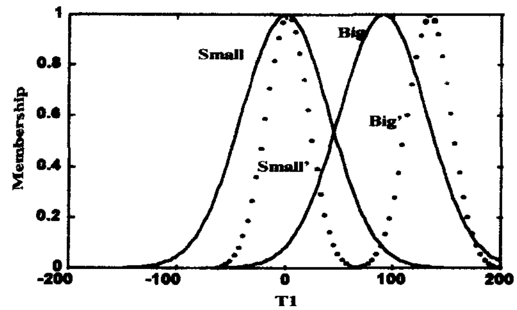


(a) In the case of membership function of fuzzy variable u(t-3)

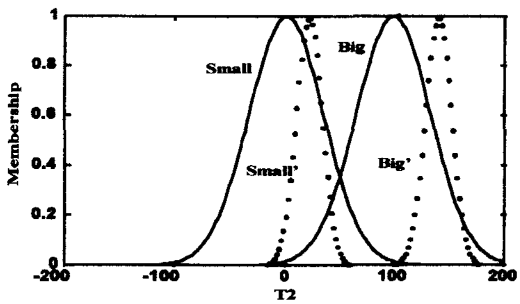


(b) In the case of membership function of fuzzv variable y(t-1)

Fig. 15. The initial and final tuned values of Gaussian-like membership functions for model 6 with different slope in Table 1a.



(a) In the case of membership functions of fuzzy variable T1



(b) In the case of membership functions of fuzzy variable T2

Fig. 16. The initial and final tuned values of Gaussian-like membership functions for model 4 with different slope in Table 4a.

output performance of fuzzy model according to each process. As shown in the Table 1~2 and 4~5, the output performance index for testing data in the case of Gaussian membership function is better than that produced in the case of triangular membership function.

By increasing the number of the membership functions in each of input fuzzy variables, in the simplified inference method we can find the optimal fuzzy model with better output performance for both training data and testing data. From the observation and characteristic analysis of fuzzy inference method, type of membership functions, no. of membership function of fuzzy input variables, and the weighting factors of the objective function shown in this paper, it is available and feasible to design the optimal fuzzy model structure with better performance output.

5. Conclusions

In this paper, the efficient identification technique is presented which automatically extract the optimal fuzzy rules, using a auto-tuning algorithm and the

Table 8. The initial & final tuned value of the membership functions of the premise fuzzy variables, consequence parameters, and performance index

	Premise parameter												Consequence parameter				Performance index	
	$Small_1$			Big_1			$Small_2$			Big_2			a_1	a_2	a_3	a_4	PI	E_PI
	a	b	s	a	b	s	a	b	s	a	b	s						
Initial value	0.25	54.54	0.90	90.79	54.54	0.90	1.56	58.81	1.50	99.19	58.81	1.50	-0.132	1.063	0.036	1.097	1.78	9.64
Final tuned value	0.59	79.25	7.75	132.1	79.25	7.75	21.19	70.97	18.14	139.0	70.97	18.14	0.180	1.075	0.246	-4.142	4.03	6.83

weighting factors of objective function. The improved complex method, which is a powerful auto-tuning, is used for auto-tuning of parameters of the premise membership functions in consideration of the overall structure of fuzzy rules. The simplified fuzzy reasoning method is used and we consider three types of Gaussian membership functions such as fixed, same slope and different slope, and triangular membership function. By means of the adjustment of weighting factors of objective function for both training and testing data, and the auto-tuning of the parameters of each membership function, we can get better performance of fuzzy model. According to the increase of no. of membership function of each input variable of process system, generally the *PI* (Performance Index) for fuzzy model using training data is improved, but the *PI* for fuzzy model using testing data gets worse. In comparison of the performance of fuzzy model of Gaussian type membership function with fixed slope, same slope, and different slope, it depends on no. of data, a certain degree of nonlinearity and weighting factor as shown in previous section. Generally, in the case of same-slope, we can get better performance.

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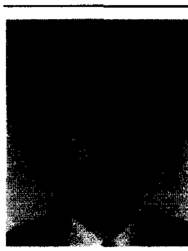

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