

# 비트-플레인 영상을 이용한 프랙탈 부호화 기법

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## 요 약

본 논문에서는 탐색영역 제한방식과 스케일링 방식을 이용하여 프랙탈 영상 부호화시에 걸리는 시간을 줄이는 새로운 프랙탈 영상 부호화 알고리즘을 제안한다. 먼저, 원영상을 스케일링 방식을 이용해 1/2과 1/4의 크기를 가지는 영상으로 축소한다. 이어서, 1/2 크기를 가지는 레인지 블록들 중에 1/4의 크기를 가지는 도메인 블록과 가장 유사한 블록을 탐색영역 제한방식을 이용해 검색하게 된다. 이와같이 검색된 블록이 부호화에 사용된다. 또한, 비트 플레인 영상을 이용하여 압축율을 향상시키기 위한 알고리즘도 제안한다. 실험 결과, 제안된 알고리즘은 Jacquin의 방법에 비해 복원된 영상의 화질은 약간 떨어졌으나, 압축율이 향상되고 부호화 시간을 대폭 감소시킬 수 있었다.

## Fractal Coding Method using Bit-plane Image

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## ABSTRACT

This paper describes a new fractal image coding algorithm to shorten time to take on fractal image encoding by using limited search area method and scaling method. First, the original image is contracted respectively by half and by quarter with the scaling method. And then, the corresponding domain block of the quarter-sized image which is most similar with one range block of the half-sized image is searched within the limited area in order to reduce the encoding time extremely. This searched block is used in encoding. Also, we propose an algorithm to enhance compression ratio by using bit-plane image. As the result of the evaluation, the proposed algorithm provided much shorter encoding time and better compression ratio with a little degradation of the decoded image quality than Jacquin's method.

### 1. Introduction

Today, image compression techniques such as predictive coding, transform coding, block truncation coding, vector quantization coding, subband coding, hierarchical coding, wavelet coding and fractal coding have been developed

and used in several application areas. Especially, the fractal coding has an attraction because of scalable image magnification.

A word "fractal" means that a partial shape represents the whole shape, and Mandelbrot[1] established fractal theory for the first time. In 1980's, Michael Barnsley[2-4] applied the fractal theory to practical image codings. Although his coding method provided very high compression, it

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required a lot of time on encoding because manual operation was required. Jacquin[5-8] suggested a new image coding algorithm to remove the manual operation used by Barnsley's fractal image coding algorithms. Jacquin's method divides an original image and the corresponding contracted image into range blocks and domain blocks respectively and then the corresponding domain block which is most similar to the range block of the original image is searched in the contracted image. Jacquin's method requires the large amount of computation during encoding process because of searching the whole area of the contracted image to find the corresponding domain block for each range block. Also, Monro[9] suggests the method which performs encoding process in the unit of independent block. Although his method reduces encoding time, discontinuity on the boundary of the reconstructed image exists. The proposed method in this paper first converts a 8 bit-plane image into a 4 bit-plane image[10] and uses the 4 bit-plane image as an original image in order to improve compression ratio. And then, the 4 bits original image is reduced by half with scaling method[11]. Finally, the corresponding domain block which is most similar to the range block of the reduced original image is searched in the limited area[12] of the corresponding contracted image in order to reduce encoding time considerably.

The contents of this paper are as follows. In section II, we describe theoretical background of fractal coding. In section III, we explain the proposed algorithm of the fractal image coding. In section IV, we compare the proposed method with the traditional Jacquin's method and analyze the results. Finally, we finish section V with concluding remarks.

## 2. Theoretical Background

### 2.1. Fundamental Concept

Fractal coding adopts affine transformations with rotation, scaling, reflection and translation and can be expressed by following equation (1).

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = W \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} \quad (353)$$

where  $W$  is two-dimensional affine transformation. Therefore, Equation (1) implies that coordinate  $(x, y)$  on plane is replaced with new coordinate  $(x', y')$  by parameters  $a, b, c, d, e$  and  $f$ . The parameters  $a, b, c$  and  $d$  indicate rotation, scaling and reflection, while the parameters  $e$  and  $f$  indicate translation. The parameters  $a, b, c$  and  $d$  are such that the transformation acts in the  $xy$  plane according to the symmetries in Table 1.

<Table 1> 8 symmetries

Symmetry	Matrix	Description
0	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Identity
1	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	Reflection in y-axis
2	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Reflection in x-axis
3	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	180° rotation
4	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Reflection in line $y = x$
5	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	90° rotation
6	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	270° rotation
7	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	Reflection in line $y = -x$

Also, the affine transformations have to be contractive in order to ensure convergence.

and the conditional equation can be expressed by following equation (2).

$$d(W(A), W(B)) \leq s \cdot d(A, B) \quad (354)$$

$$\forall A, B \in U, 0 \leq s \leq 1$$

where U is metric space, d is a metric in metric space. And s is contraction ratio for transformation W.

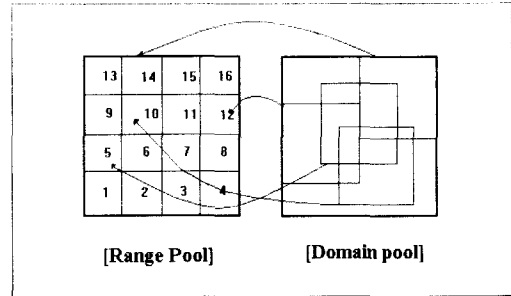
### 2.2. Fractal Image Coding

Since a gray level or a color image has its pixel value, the general affine transformation can be rewritten in 3-dimensional model.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = w \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & P \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} e \\ f \\ Q \end{bmatrix} \quad (355)$$

where parameters a, b, c, d, e and f are equal to those of equation (1), while parameter P is chosen a positive number with  $0 < P < 1$ . P and Q represent modified pixel value.

The fractal coding system is based on the construction, for any given original image to encode, of an image transformation of a special kind which, when iterated on any initial image, produces a sequence of images that converges to a fractal approximation of the original. The requirements on the transformation are (i) it is contractive in the metric space of images endowed with L2 metric[5-8], (ii) it leaves the original image approximately invariant, and (iii) its complexity is smaller than that of the original image. The mapping between domain blocks of the contracted image with range blocks of an original image is illustrated in Fig. 1. The detail encoding scheme can be found in references [5,6,7,8].



(Fig. 1) The mapping between some range blocks and the corresponding domain blocks.

### 3. The Proposed Fractal Algorithm

The proposed fractal algorithm is performed as follows :

- (i) Bit-plane image is constructed to enhance compression ratio.
- (ii) Corresponding domain block which is most similar with one range block is searched within limited area using the proposed limited search area method.
- (iii) Image divided into shade blocks and edge blocks using the proposed block classifier.
- (iv) Bit allocation : 6 bits for shade blocks and 15 bits for edge blocks.
- (v) Encoding is perform by using the result of bit allocation
- (vi) Decoding is carried out using transmitted fractal parameter values.
- (vii) Estimation of the reconstructed image uses the root mean square error(RMSE) and peak signal-to-noise ratio(PSNR).

#### 3.1. Construction of Bit Plane Image

Firstly, bit-plane image is constructed to enhance compression ratio. Each pixel in a gray image is represented by 8 bits. The image is composed of eight 1-bit planes, ranging from plane 0 for the least significant bit to plane 7 for the most significant bit. The level of a

gray image can be described for  $256 \times 256$  pixels like the following equation :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \quad (356)$$

where  $n=m=256$ .

Each pixel in equation (4) can be described 8 bits like the following equation :

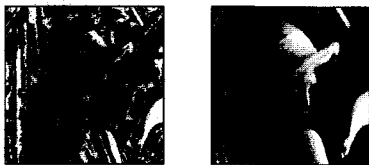
$$[b_7 \ b_6 \ b_5 \ b_4 \ b_3 \ b_2 \ b_1 \ b_0] \begin{bmatrix} 2^7 \\ 2^6 \\ 2^5 \\ 2^4 \\ 2^3 \\ 2^2 \\ 2^1 \\ 2^0 \end{bmatrix} \quad (357)$$

Using the equation (5), the  $n$ -bit plane image can be constructed with an bit of each pixel in the image.

Fig. 2 shows several bit plane images and a composite bit plane image for Lenna image.



(a)  $b_7$  bit-plane image      (c)  $b_5$  bit-plane image  
(b)  $b_6$  bit-plane image



(d)  $b_4$  bit-plane image      (e)  $b_7+b_6+b_5+b_4$  bit-plane image

(Fig. 2) Bit-plane images for Lenna image.

### 3.2. Limitation of Search Area

The proposed algorithm uses Fig. 2 (e) as reconstructed original image. First, the original image is contracted respectively by half and by quarter with the scaling method. And then, the corresponding domain block of the quarter-sized image which is most similar with one range block of the half-sized image is searched within the limited area in order to reduce the encoding time extremely. The idea is illustrated in Fig. 3. Fig. 3 shows classification of search area in the domain pool. The limitation of search area is as follows :

#### Case 1 : Area a, c, g and i

Search range of these areas is 8 pixels in the arrow direction in Fig.3

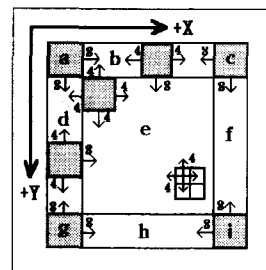
#### Case 2 : Area b, d, f and h

Search range of these areas is 4 or 8 pixels in the arrow direction in Fig.3

#### Case 3 : Area e

Search range of these areas is 4 pixels in the arrow direction in Fig.3

If the selected block is an edge block, the block is divided into 2 by 2 size blocks, and the limited search area is the same as before.



(Fig. 3) Classification of search area.

3.3. Block Classifier

In this paper, we use a block classifier to satisfy the following relationships in order to classify the block type (shade or edge).

$$\text{Edge block if } \sum_{i=1}^{B^2} (P_i - P_{\text{avg}}) >= \text{Th} \quad (6)$$

$$\text{Shade block if } \sum_{i=1}^{B^2} (P_i - P_{\text{avg}}) < \text{Th} \quad (7)$$

where one block size is B x B (B=4 for parent block and B=2 for child block), Pi is value of each pixel in the block, and Pavg is the average pixel value of the block. Threshold value, Th, has been determined by considering compression ratio and image quality through experiments with more than 20 image samples. We have assigned 10 as the threshold value for both parent block and child block.

If a fractal coding algorithm does not contain any block classification process, the algorithm has to search the whole area of the domain image for all range blocks in order to find a domain block corresponding to a range block. It requires a lot of encoding time. For example, when we tested a fractal coding algorithm without block classification for a 256 x 256 sized Lenna image, it took more than 15 hours in encoding the image on a 486 IBM PC. If the image can be divided into shade blocks and edge blocks, the edge blocks need the search of the whole domain area as before but the shade blocks need no search in the domain area because the shade block needs its mean pixel value for encoding. Therefore, the block classification process reduces fractal encoding time significantly.

The fractal block coding algorithm is a lossy algorithm by nature. The fractal algorithm tries to find the most similar domain block to a range block. However, if the range block is exactly shade type, the mean pixel value for

the range block will provide better reconstructed block image on decoding than the most similar domain block information. Through experiments, in most cases we found that Jacquin's method using its block classifier achieved higher PSNR on decoding than a fractal block coding method without a block classifier. However, poor block classifiers considerably drop the quality of the reconstructed image. Therefore, the selection of the block classifier determines the quality of the reconstructed image. (i.e. PSNR)

3.4. Bit Allocation

For improvement of compression ratio and reduction of search time, each block in an image is classified into two types : shade type and edge type. Bit allocation for the proposed fractal algorithm is as follows : 6 bits for shade blocks and 15 bits for edge blocks. The details are shown in Table 2.

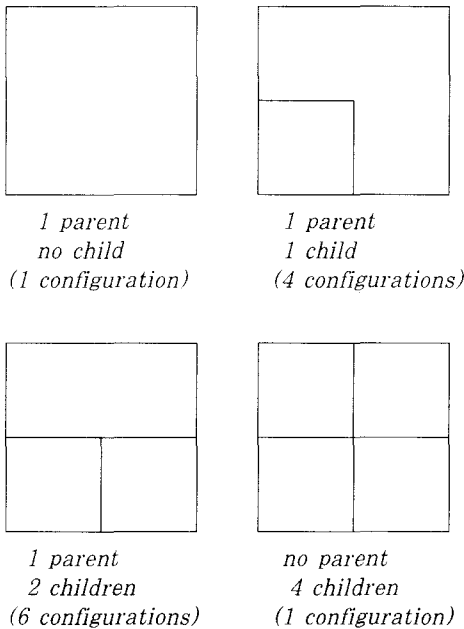
<Table 2> Bit allocation

Block type	Parameters	Information in bits	
		Details	Total
Shade	block type	2	6
	mean pixel value	4	
Edge	block type	2	15
	mean pixel value	4	
	coordinates	6	
	isometry	3	

The computation of bit rates is as follows. Let Is and Ie denote the total bits required for expressing the shade block and the edge block respectively. Let Ns and Ne denote the total number of the shade block and edge block respectively. If the parent block has size of B x B and the child block has size of B/2 x B/2, the bit rate is given by :

$$\frac{N_p I_c + N_s I_s + N_e I_e}{N_p B^2} \quad \text{bits/pixel} \quad (36)$$

where  $I_c$  is the bits for twelve possible coding configurations of a parent block. The twelve coding configurations are shown in Fig. 4.



(Fig. 4) Coding configurations of a parent range block

The configurations can thus be encoded with  $I_c=4$  bits.  $N_p$  is the total number of parent blocks in the image.

### 3.5. Decoding

Decoding is carried out using transmitted fractal parameter values. Shade type blocks are decoded with  $B$  size while edge type blocks are decoded with  $B/2$  size. Of course, the decoding process should be recursively iterated to get a reconstructed image. In most cases, 8 iterations for decoding are enough for tolerable reconstructed image.

### 3.6. Estimation of the Reconstructed Image

The estimation of the reconstructed image uses the root mean square error(RMSE) and peak signal-to-noise ratio(PSNR). If the original image of size  $B \times B$  is  $A$  and the reconstructed image is  $\hat{A}$ , RMSE can be calculated by following equation :

$$(9) \quad RMSE = \sqrt{\frac{1}{B^2} \sum_{i=1}^B \sum_{j=1}^B (A(i, j) - \hat{A}(i, j))^2}$$

PSNR for 256 gray level image can be calculated by following equation :

$$(10) \quad PSNR = 20 \log_{10} \left( \frac{255}{RMSE} \right)$$

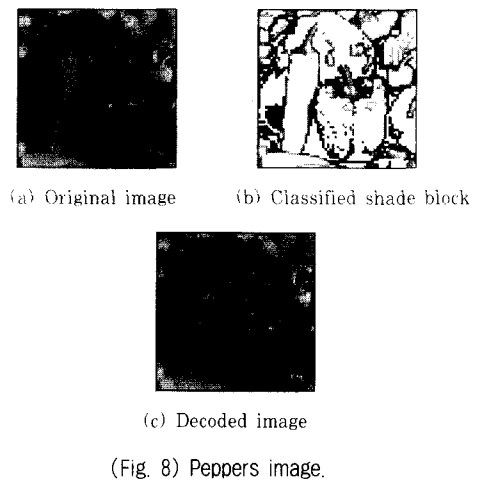
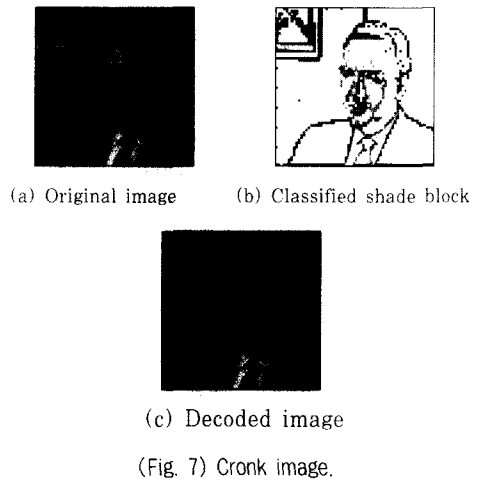
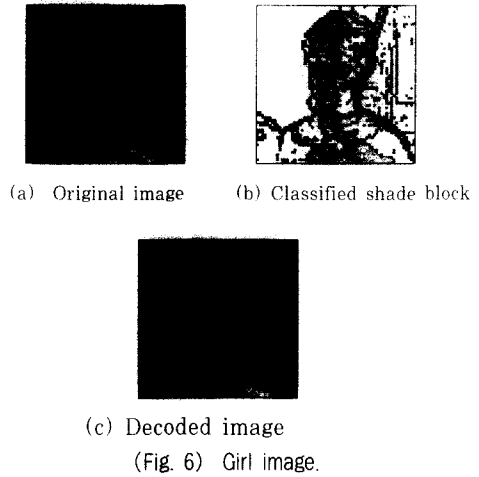
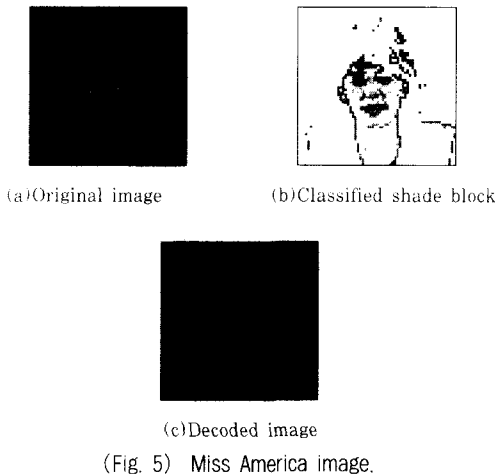
## 4. Simulation Results

We simulated Miss America, Girl, Cronk, Lenna and Peppers images with 256 gray level and 256 by 256 size on 486DX-66 IBM PC. Figures 5, 6, 7, 8 and 9 show original images, classified shade images and decoded images. Although Jacquin's algorithm decreases encoding time by using block classification, it takes a few hours. However, as shown in Table 3, the proposed algorithm not only reduces encoding time extremely but also improves compression ratio with a little degradation of the decoded image. When PSNR falls in range from maximum 5.28dB to minimum 2.58dB, there is a little degradation of image quality. But in the light of subjective vision, there is almost no difference. According to simulation results, although not stated in the table, it is interesting that 4 bit-plane image is better than 3 bit-plane image in terms of compression ratio and encoding time.

There are two factors that have an effect on the achieved compression results: bit plane reduction method and limited search area method.

The bit plane reduction method increases the compression ratio by using upper 4 bits-plane image instead of 8 bits-plane image. This method uses 4 bits instead of 8 bits to represent a pixel. Therefore, in this case, bit allocation for mean pixel value field of each shade block and difference of mean pixel value field of each edge block is reduced by 4 bits. (refer to Table 2) Jacquin also uses vector quantized 64 grey level image (that is, vector quantized 6 bits-plane image) instead of original 256 grey level image in order to reduce encoding bits.

The limited search area method increases the compression ratio by reducing search area of domain image. Assume that there is a domain image of size 64 x 64. If we search the whole area of the domain image in order to find a domain block corresponding to a range block, we need 12 bits (6 bits for X direction and Y direction respectively) to represent the coordinates of the matched domain block. If we limit the search area of the domain image to size 8 x 8, we need only 6 bits to represent the coordinates of the matched domain block. Therefore, in this case, bit allocation for coordinates field of each edge block is reduced by 6 bits. (refer to Table 2)





(a) Original image



(b) Classified shade block



(c) Decoded image

(Fig. 9) Lena image.

<Table 3> Performance comparison between proposed method and Jacquin's method

Methods	Proposed method			Jacquin's method		
	Encoding time(sec)	bpp	PSNR [dB]	Encoding time(sec)	bpp	PSNR [dB]
Images						
<b>Miss America</b>	35	<b>0.35</b>	<b>33.30</b>	9180	0.71	37.04
<b>Girl</b>	58	<b>0.57</b>	<b>28.81</b>	14040	0.82	32.87
<b>Cronk</b>	46	<b>0.48</b>	<b>30.13</b>	7020	0.54	34.76
<b>Peppers</b>	62	0.67	27.29	11279	0.85	32.57
<b>Lenna</b>	80	0.57	24.66	20640	0.89	27.24

### 5. Conclusions

The proposed fractal algorithm has reduced encoding time and increased compression ratio. We not only considerably reduce encoding time by using limited search area method but also improve compression ratio by using bit-plane. Compared with Jacquin's algorithm, the proposed algorithm increases compression ratio in range from 0.36 bpp to 0.06 bpp and reduces remarkably encoding time in range from maximum 20560 seconds to minimum 6974

seconds for five different images on 486DX-66 IBM PC. In future, we will concentrate on developing new algorithms to increase PSNR with little loss of encoding time and compression ratio.

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