Adaptive Estimator for Tracking a Maneuvering Target with Unknown Inputs

미지의 입력을 갖는 기동표적의 추적을 위한 적응 추정기

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Abstract

An adaptive state and input estimator for the tracking of a target with unknown randomly switching input is developed. In modeling the unknown inputs, it is assumed that the input sequence is governed by semi-Markov process. By incorporating the semi-Markov probability concepts into the Bayesian estimation theory, an effective adaptive state and input estimator which consists of parallel Kalman-type filters is obtained. Computer simulation results reveal that the proposed adaptive estimator have improved tracking performance in spite of the unknown randomly switching input.

요 약

임의로 변하는 미지의 입력을 갖는 표적의 추적을 위한 적응 상태 및 입력 추정기를 설계한다. 미지의 입력을 semi-Markov 프로세스로 모델링하고, 이를 Bayesian 추정이론에 접목함으로써 여러개의 Kalman 필터가 병렬로 구성된 효과적인 적응 상태 및 입력 추정기를 구한다. 컴퓨터 모사를 통하여, 제안된 적응추정기는 임의로 변하는 미지의 입력에도 불구하고 개선된 추적성능을 보임을 확인하였다.

Ⅰ. INTRODUCTION

The problem of tracking a maneuvering target has many military and civilian application areas and it still remains a great deal of debate to get the best solution[1]-[3]. The basic problem is that there exists a mismatch between the mathematically modeled target dynamics and the actual target dynamics. The system model of a target moving with constant velocity in a straight line is different from that of a target moving with acceleration or maneuver. If the system model is not correct, track loss may occur easily in the tracking process.

There have been many approaches in the literature to get around the dilemma of the model mismatch problem. In general, these approaches can be categorized into three classes. The IE(input estimation) algorithm [4], models the maneuver as constant unknown input, estimates its magnitude and onset time and then compensates the state esti-

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mate in accordance with the estimated input. In the VSD(variable state dimension) algorithm[5], the state dynamic model for the target is changed by introducing additional state components when the maneuver is detected. As pointed out in [6], the VSD algorithm has the undesirable feature of requiring reinitialization when the maneuver is detected and the lower order CV(constant velocity) target model is replaced by the higher order CA(constant acceleration) model. The IMM(interacting multiple model) algorithm[7]~[10] consists of parallel Kalman filters for each model which has different dimension and process noise intensity, a model probability evaluator, an estimate mixer at the input of each Kalman filters, and an estimate combiner at the output of the filters. In parallel to the above three classes, Moose et al.[11] present an efficient adaptive estimator for tracking a maneuvering target containing unknown or randomly switching biased measurements by using a semi-Markov concepts. Despite the large and randomly varying measurement biases, the adaptive estimator provides an accurate estimate of maneuvering target state.

In this paper, we have developed an adaptive state estimator algorithm for tracking a maneuvering target with unknown inputs. In modeling the unknown randomly switching maneuver inputs, it is assumed that the input sequence dynamics are governed by a semi-Markov process[12]. A semi-Markov process differs from a Markov process in that the duration of time in one state before switching to another state is itself a random variable. By inclusion the semi-Markov concepts into a Bayesian conditional probability theory, we obtained an adaptive estimator which is composed of parallel Kalman-type filters. In particular, we developed the input estimator for the unknown randomly varying maneuver inputs using the weights for each Kalman-type filter which is matched to a set of different possible input sequences.

The rest of this paper is organized as follows. Section Ⅱ describes the problem formulation for the maneuvering target which have unknown inputs. Adaptive state and input estimators are developed in section Ⅲ and Ⅳ, respectively. Computer simulation is provided in section Ⅳ and conclusion is described in the final section.

Ⅱ. PROBLEM FORMULATION

The dynamic model for the maneuvering target with randomly switching inputs can be described in the LTI(linear time-invariant) form as

$$x_{k+1} = \Phi x_k + \Gamma u^b + \Psi w_k \tag{1}$$

$$z_k = H x_k + v_k \tag{2}$$

where $x_k \in R^{n\times 1}$ is the state vector, $z_k \in R^{r\times 1}$ is measurement vector, $w_k \in R^{q\times 1}$ and $v_k \in R^{r\times 1}$ are zero-mean white Gaussian process and measurement noise sequences, respectively with known covariance matrices such that

$$E[w_k w_j^T] = Q\delta_{kj} \tag{3a}$$

$$E[v_k v_j^T] = R\delta_{kj} \tag{3b}$$

where δ_{kj} is the Kronecker delta which is equal to 1 if $k=j$, otherwise it is zero. And it is assumed that the process noise and measurement noise are uncorrelated each other such that

$$E[v_k w_j^T] = 0,\ \forall k \text{ and } j \tag{4}$$

The new term u^b is the maneuver input which is unknown to the estimator. It is assumed that u^b is governed by a semi-Markov process. A semi-Markov process differs from a Markov process in that the duration of time in one state before switching to another state is itself a random variable. Therefore, the maneuver input can take any one of N possible discrete values such as $\{u^1, u^2, \cdots, u^N\}$ for a random duration of time before switching to the other states. The block diagram of the dynamic model is depicted in Fig. 1.

Ⅲ. ADAPTIVE STATE ESTIMATOR

The optimal state estimate can be obtained from the conditional expectation as

$$\begin{aligned}\hat{x}_{k|k} &= E[x_k|Z_k] \\ &= \int_{-\infty}^{\infty} x_k p(x_k|Z_k) dx_k \end{aligned} \tag{5}$$

where, the Z_k is the measurement sequences up to t_k as $Z_k = [z_1\ z_2\ \cdots\ z_k]$. The conditional density function of (5) can be written using Bayes theory as

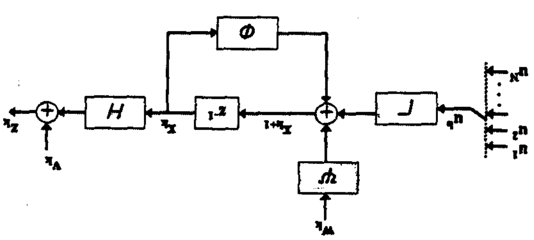

그림 1. 표적의 동적 모델에 대한 블럭선도

Fig. 1. Block diagram of the target dynamic model.

$$\begin{aligned} p(x_k|Z_k) &= \frac{\sum_{i=1}^{N} p(x_k, Z_k, u^i)}{p(Z_k)} \\ &= \sum_{i=1}^{N} p(x_k|Z_k, u^i)\ p(u^i|Z_k) \end{aligned} \tag{6}$$

Substituting (6) into (5), we obtain

$$\begin{aligned}\hat{x}_{k|k} &= \sum_{i=1}^{N} \int_{-\infty}^{\infty} x_k p(x_k|Z_k, u^i) dx_k\, p(u^i|Z_k) \\ &= \sum_{i=1}^{N} \hat{x}^i_{k|k}\, p(u^i|Z_k) \end{aligned} \tag{7}$$

where

$$\hat{x}^i_{k|k} \equiv \int_{-\infty}^{\infty} x_k\, p(x_k|Z_k, u^i) dx_k \tag{8}$$

The term $p(u^i|Z_k)$ in (7) means the weighting factor for the i-th estimator. Therefore the optimal state estimate can be obtained from the weighted sum of N individual state estimate which is conditioned on a different u^i maneuver input.

To obtain the i-th state estimator, the conditional density function of (8) can be rewritten as by using the Bayes theory

$$\begin{aligned} p(x_k|Z_k, u^i) &= p(x_k|Z_{k-1}, z_k, u^i) \\ &= \frac{p(z_k|x_k, u^i, Z_{k-1})\, p(x_k|u^i, Z_{k-1})}{p(z_k|u^i, Z_{k-1})} \end{aligned} \tag{9}$$

The first term of the numerator of (9) can be rearranged by

$$P(z_k|x_k, u^i, Z_{k-1}) = P(z_k|x_k, u^i) \tag{10}$$

Using the measurement equation of (2), the mean and the covariance of (10) are given by

$$E[z_k|x_k, u^i] = Hx_k \tag{11}$$

$$E[(z_k - Hx_k)\ (z_k - Hx_k)^T] = R \tag{12}$$

Also, by using the state equation of (1), the second term of the numerator of (9) has the following mean and covariance which constitute the time-update process of the i-th estimator.

$$\hat{x}^i_{k|k-1} \equiv E[x_k|u^i, Z_{k-1}] \\ =\Phi \hat{x}^i_{k-1|k-1} + \Gamma u^i \quad (13)$$

$$P^i_{k|k-1} \equiv E[(x_k - \hat{x}^i_{k|k-1})(x_k - \hat{x}^i_{k|k-1})^T] \\ = E[(x_k - \Phi \hat{x}^i_{k-1|k-1} - \Gamma u^i) \\ \times (x_k - \Phi \hat{x}^i_{k-1|k-1} - \Gamma u^i)^T] \\ = \Phi P^i_{k-1|k-1} \Phi^T + \Psi Q \Psi^T + D^i_b \quad (14)$$

where

$$D^i_b \equiv \Gamma E[(u^b - u^i)(u^b - u^i)^T]\Gamma^T \\ + \Gamma E[(u^b - u^i)(x_{k-1} - \hat{x}^i_{k-1|k-1})^T]\Phi^T \\ + \Phi E[(x_{k-1} - \hat{x}^i_{k-1|k-1})(u^b - u^i)^T]\Gamma^T \quad (15)$$

Remark 1; From extensive simulations, we have found the estimates of the target states to be close enough to allow the assumption that $E[x_{k-1} - \hat{x}^i_{k-1|k-1})] \simeq 0$. Thus D^i_b reduces to $\Gamma E[(u^b - u^i)(u^b - u^i)^T]\Gamma^T$ which can be determined by assuming that u^b is uniformly distributed between adjacent vectors u^i and u^{i+1}.

In view of (11)~(14), we could write the probability density function of (9) as

$$p(x_k|Z_k, u^i) = K \exp[-1/2\{(z_k - Hx_k)^T R^{-1}(z_k - Hx_k) \\ + (x_k - \hat{x}^i_{k|k-1}) P^i_{k|k-1} (x_k - \hat{x}^i_{k|k-1})\}] \quad (16)$$

where K is an appropriate normalizing constant which takes into account the denominator of (9). We differentiate the logarithm of (16) with respect to x_k and set this to zero to obtain the optimal state estimate of the i-th filter $\hat{x}^i_{k|k}$.

$$\hat{x}^i_{k|k} = \hat{x}^i_{k|k-1} + K^i_k[z_k - H\hat{x}^i_{k|k-1}] \quad (17)$$

where K^i_k is the Kalman gain of the i-th estimator and defined by

$$K^i_k \equiv P^i_{k|k-1} H^T (HP^i_{k|k-1} H^T + R)^{-1} \quad (18)$$

In addition, the covariance of estimation error is given by

$$P^i_{k|k} \equiv E[(x_k - \hat{x}^i_{k|k})(x_k - \hat{x}^i_{k|k})^T] \\ = [I - K^i_k H] P^i_{k|k-1} \quad (19)$$

It is noted that (17), (18) and (19) constitute the measurement-update process of the i-th estimator. If we consider the time and measurement update process of estimation error covariance matrices (14) and (19) and Kalman gain (18), we find that these equations are independent of the measurement data so that these can be precalculated. In addition, if we assume that D^i_b in (15) is equal to D_b for all i, the off-line computational load for (14), (18) and (19) can be reduced to $1/N$.

Let us consider the weighting term in (7) by splitting the measurement sequence $\{Z_k\}$. into $\{z_k, Z_{k-1}\}$.

$$p(u^i|Z_k) = p(u^i|z_k, Z_{k-1}) \\ = \frac{p(z_k|u^i, Z_{k-1})\, p(u^i|Z_{k-1})}{p(z_k|Z_{k-1})} \quad (20)$$

Assuming that the duration of time u^b remains in one state before switching to another state is much longer than the sampling interval, the first term of the numerator in (20) becomes Gaussian distribution with the following mean and covariance.

$$\bar{z}_k^i \equiv E[z_k | u^i, Z_{k-1}] = H\hat{x}^i_{k|k-1} \quad (21)$$

$$Q_z^i = E[(z_k - \bar{z}_k^i)(z_k - \bar{z}_k^i)^T] = HP^i_{k|k-1}H^T + R \quad (22)$$

Next, the second term of the numerator in (20) can be expressed as follows by using Bayes theory.

$$\begin{aligned} p(u^i|Z_{k-1}) &\equiv p(u_k^b = u^i | Z_{k-1}) \\ &= \frac{\sum_{i=1}^{N} p(u_k^b = u^i, u_{k-1}^b = u^i, Z_{k-1})}{p(Z_{k-1})} \\ &= \sum_{i=1}^{N} p(u_k^b = u^i | u_{k-1}^b = u^i, Z_{k-1}) \\ &\quad \times p(u_{k-1}^b = u^i | Z_{k-1}) \end{aligned} \quad (23)$$

Let us define the weighting factor and the Markov transition probability as follows:

$$\omega_{k-1}^i \equiv p(u_{k-1}^b = u^i | Z_{k-1}) \quad (24)$$

$$\theta_{ji} \equiv p(u_k^b = u^i | u_{k-1}^b = u^i, Z_{k-1}) \quad (25)$$

In view of (21)~(25), (20) can be expressed as the following vector-matrix recursive form:

$$\Omega_k = c_k P_k \Theta^T \Omega_{k-1} \quad (26)$$

where $\Omega_k \in {}^{NX1}$ is the weighting matrix whose i-th element is ω_k^i, $P_k \in R^{NxN}$ is diagonal matrix whose element is given by

$$p_{ii} = \exp[-1/2\,(z_k - \bar{z}_k^i)^T (Q_z^i)^{-1} (z_k - \bar{z}_k^i)] \quad (27)$$

where $\bar{z}_k^i$ and Q_z^i are given by (21) and (22), respectively. In addition, $\Theta \in R^{NxN}$ is predetermined Markov transition matrix whose element is θ_{ij} and c_k is a normalizing factor which is calculated at each iteration to satisfy the following:

$$\sum_{i=1}^{N} \omega_k^i = 1 \quad (28)$$

Remark 2; In general, the sample rate is set high enough to be much faster than the random switching of the maneuver input. Therefore the predetermined Markovian transition probability θ_{ji} defined in (25) is close to one if $j=i$, otherwise it is close to zero.

Ⅳ. ADAPTIVE INPUT ESTIMATOR

In some applications such as maneuvering target tracking, it is essential for the tracking filter to estimate the magnitude and onset time of the maneuver input in an efficient way. In order to estimate the maneuver input in an optimal sense, conditional expectation is introduced again.

$$\hat{u}_k^b = E[u_k^b | Z_k] = \int_{-\infty}^{\infty} u_k^b\, p(u_k^b | Z_k)\, du_k^b \quad (29)$$

The probability density function in (29) can be rewritten as follows by using Bayes theory:

$$\begin{aligned} p(u_k^b | Z_k) &= \frac{\sum_{i=1}^{N} p(u_k^b, Z_k, u^i)}{p(Z_k)} \\ &= \sum_{i=1}^{N} p(u_k^b | Z_k, u^i)\; p(u^i | Z_k) \end{aligned} \quad (30)$$

Substituting (30) into (29) and exchanging summation and integration gives

$$\begin{aligned} \hat{u}_k^b &= \sum_{i=1}^{N} \int_{-\infty}^{\infty} u_k^b p(u_k^b | Z_k, u^i) du_k^b\, p(u^i | Z_k) \\ &= \sum_{i=1}^{N} E[u_k^b | Z_k, u^i]\, p(u^i | Z_k) \end{aligned} \quad (31)$$

where $E[u_k^b | Z_k, u^i] = u^i$ and $p(u^i | Z_k) = \omega_k^i$ by considering (24). Thus we can obtain the input estimate as

$$\hat{u}^b_k = U^T \Omega_k \tag{32}$$

where $U \in R^{N\times 1}$ is an assumed N discrete input whose i-th element is u^i and $\Omega_k \in R^{N\times 1}$ is obtained from (26). It is pointed out that the estimate of maneuver input can be obtained quite effectively by using the weighting factor which is already calculated in adaptive state estimator. The block diagram of the adaptive state and the input estimator is illustrated in Fig. 2.

V. NUMERICAL EXAMPLE

We carried out numerous computer simulations to demonstrate the effect of the proposed algorithm. In this section, the 3rd-order dynamic target model is considered as in [11]. The coefficient matrices in (1) and (2) are given by

$$\Phi = \begin{bmatrix} -0.005 & -0.013 & 1.905 \\ 0.006 & 0.008 & -0.025 \\ 0.0 & 0.0 & 0.935 \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} -2.009 \\ -0.013 \\ 0.0 \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 15.588 \\ 1.9055 \\ 9.673 \end{bmatrix}, \quad H = [1.0 \quad 0.0 \quad 0.0]$$

It is assumed that the process noise covariance $Q=0.0015 I_3$, measurement noise covariance $R=200$ and $D^i_0 = I_3$ ($i=1, 2, \cdots, 5$). The generating data for the unknown input u^b is -17 for the first 250 sample times, and 17 for the next 250 sample times and the set of discrete states for $u^i (i=1, 2, \cdots, 5)$ is set to $\{-18, -8, 0, 8, 18\}$ to span the entire range of u^b. The predetermined Markov transition probability θ_{ij} is 0.95 for $i=j$ and 0.0125 for $i \neq j$. The necessary initial conditions are as follows : True and estimated state are $x_0 = [1\ 1\ 1]^T$ and $\hat{x}_{0|-1} = [0\ 0\ 0]^T$, respectively. Error covariance matrix $P_{0|-1} = I_3$ and weighting factor $\omega^i_0 = 0.2$ for $i=1, 2, \cdots, 5$.

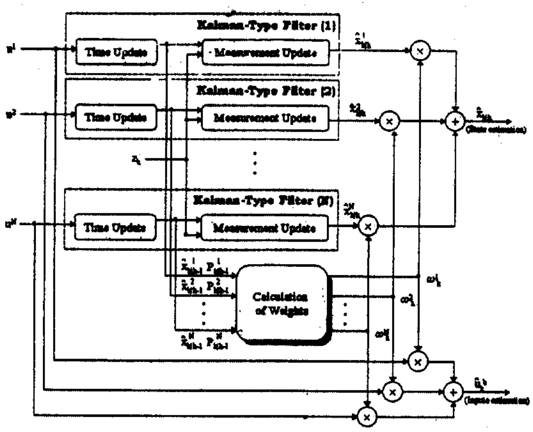

그림 2. 적응추정기의 블럭선도

Fig. 2. Block diagram of the adaptive estimator.

Figs. 3(a)-(c) show the true and estimated states of target. Figs. 4(a)-(c) show their associated root-mean square errors(RMSE) which are obtained from 50 Monte Carlo runs. As noticed, the proposed adaptive state estimator can track the target states very closely in steady-state in spite of the unknown randomly switching maneuver input. Fig. 5 shows the weighting factors for entire sample intervals. As can be seen, ω^1 is more dominant than the others for the first 250 sample intervals since u^b is close to u^1 for that period and the probability value of ω^5 is nearly to 1

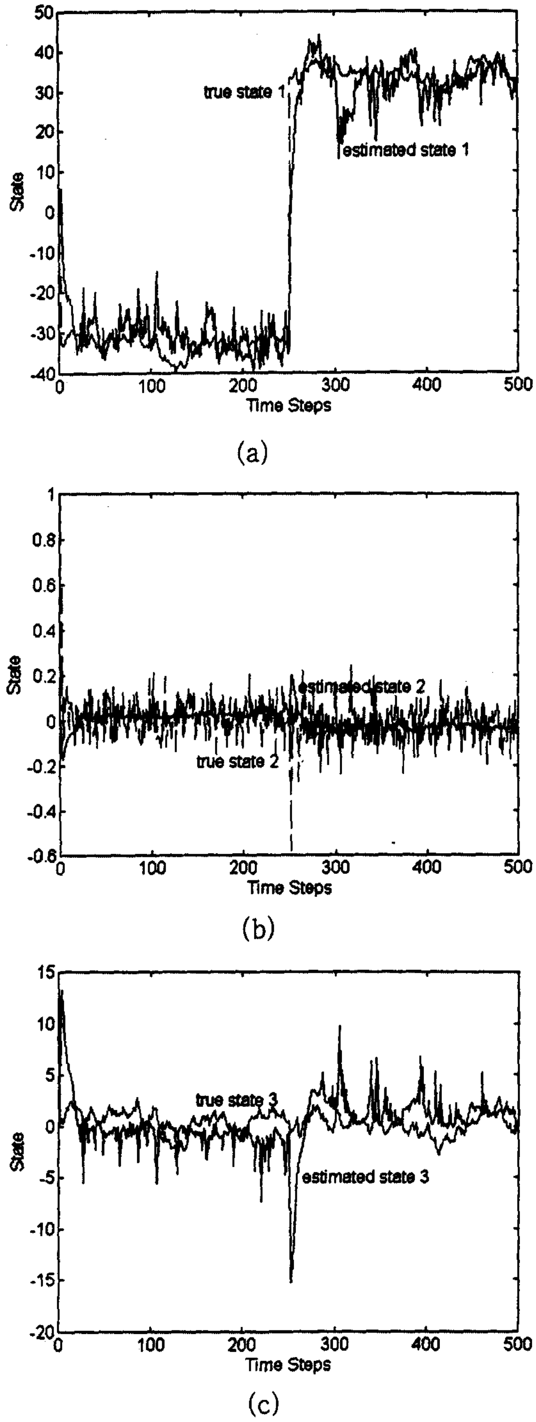

그림 3. 실제 및 추정된 상태
(a) 첫번째 상태 (b) 두번째 상태 (c) 세번째 상태
Fig. 3. True and estimated states.
(a) first state. (b) second state. (c) third state.

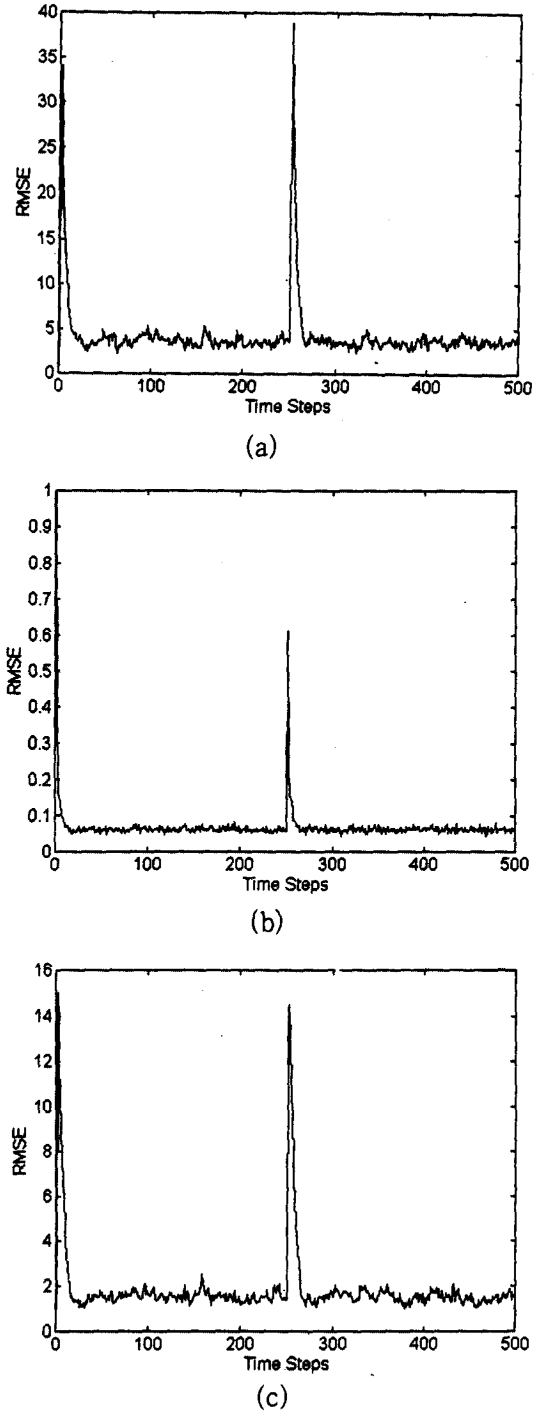

그림 4. 실제 및 추정된 상태에 대한 RMSE
(a) 첫번째 상태 (b) 두번째 상태 (c) 세번째 상태
Fig. 4. RMSE between true and estimated states.
(a) first state. (b) second state. (c) third state.

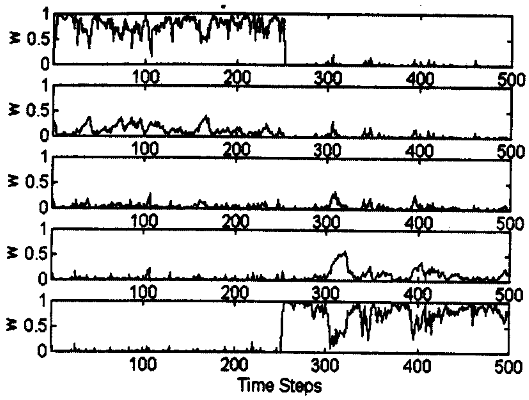

그림 5. 5개의 Kalman 필터에 대한 가중치

Fig. 5. Weighting factors for five Kalman-type filters.

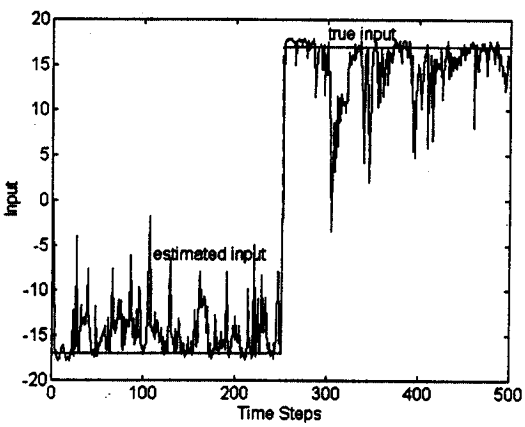

그림 6. 실제 및 추정된 기동입력

Fig. 6. True and estimated maneuver input.

for the next sample intervals for the same reason. Fig. 6 represents the true maneuver input and the estimated input. Finally, Fig. 7 shows its RMSE obtained from 50 Monte Carlo runs. It is worth noting that the unknown randomly switching input can be estimated very effectively by the adaptive input estimator.

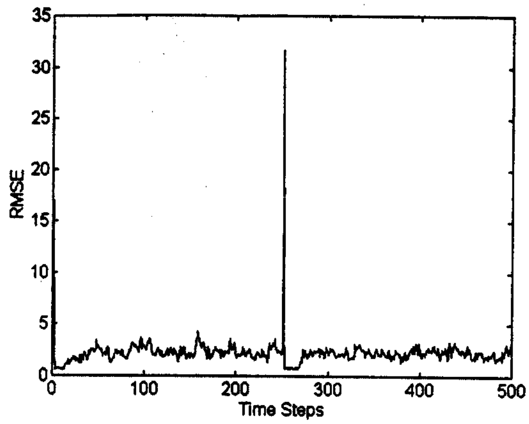

그림 7. 실제 및 추정된 기동입력에 대한 RMSE

Fig. 7. RMSE between true and estimated maneuver input.

V. CONCLUSION

The tracking problem of a target with unknown randomly switching maneuver input is considered. By incorporating the semi-Markov probability concepts into the Bayesian estimation theory, an effective adaptive state estimator which consists of parallel Kalman-type filters is obtained. In addition, an adaptive input estimator for the unknown randomly varying maneuver inputs is developed using the same weight terms as in the adaptive estimator. The exact estimation of the magnitude and onset time of the maneuver input is particularly useful in the tactical applications. From the results of computer simulations, it can be found that the adaptive state and the input estimator have improved tracking performance in spite of the unknown randomly switching input.

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