Journal of the Korea Society of Mathematical Education Series D: Research in Mathematical Education
Vol. 2, No. 1, July 1998, 13–19

韓國數學敎育學會誌 시리즈 D: <數學敎育硏究>
제 2 권 제 1 호 1998 년 7 월, 13–19

An Important Component on Using the What-If-Not Strategy

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The What-If-Not strategy as proposed by Brown & Walter (1969) is one of the most effective strategies for problem posing. However, it has focused only on the aspect of algorithms for generating problems. The aim of this strategy and how it is used to accomplish the aim of the challenging phase are not clear.

We need to clarify the aim of the What-If-Not strategy and to establish the process of the strategy for accomplishing the aim.

The purpose of this article is to offer a new What-If-Not strategy by clarifying the aim of the challenging phase.

1. INTRODUCTION

The What-If-Not strategy as proposed by Brown & Walter (1969) is one of the most effective strategies for problem posing. They divided the aspects of problem posing into two phases, Accepting and Challenging according to the aim. The strategy is used in the second, Challenging phase of problem posing. However, the What-If-Not strategy has meant many things to many researchers. It is not clear what the aim of the strategy is, or how it is to be used to accomplish the aim of the Challenging phase. Rather, it has focused only on the aspect of algorithms for generating problems; We need to clarify the aim of the What-If-Not strategy and to solidify the process of the strategy for accomplishing this aim.

The purpose of this article is to offer a new What-If-Not strategy by clarifying the aim of the challenging phase.

For this, we first investigate the content of the What-If-Not strategy as a key concept of problem posing. We then clarify the aim of problem posing and tasks in the challenging phase of their strategy. By analyzing the example used by What-If-Not, we will propose a new strategy involving comparative analysis.

2. The Aim of the What-If-Not Strategy

In Brown & Walter (1990, p.32) they mentioned,

> ··· so much of mathematical thinking begins with the assumption that we take the "given" for granted. We are trained to begin a proof by first stating and accepting what is given.

But they emphasize that we need a notion different from that of merely specifying and accepting the given as it is used in problem solving. They emphasize asking questions or posing problems rather than on answering questions. They describe a new perspective on viewing questions, and then propose two phases of problem posing:

1. Accepting phase:
   The given object is made clear.
2. Challenging phase:
   The essence and significance of the given is clarified from multiple perspectives.

In particular, they argue that it is challenging the given which frequently opens up new vistas in our thinking: "Only after we have looked at something, not as it 'is' but as it is turned inside out or upside down, do we see its essence or significance (Brown & Walter 1990, p.15)." The term "challenging" is rephrased as "What-If-Not?" This What-If-Not strategy can reveal to our students several very important points about mathematical thinking, thinking which is hindered by taking the "given" for granted as mentioned before. Taking the "given" for granted will lead our students to begin a proof. To the contrary, Brown and Walter argue that there is certainly much more to mathematics than proving things. They list several reasonable mathematics activities when using the What-If-Not strategy:

(1) Coming up with a new idea
(2) Finding an appropriate image to enable us to hold on to an old image
(3) Evaluating the significance of an idea we have already learned
(4) Seeing new connections

The four tasks above can be subdivided into two main categories: the first involves (1), and the other involves (2), (3) and (4). By subdividing the four tasks above into two groups, it is clear that accomplishment of the tasks in the second group becomes possible by reconsidering the given from the perspective gained through the first task. Each task in the second group clearly needs some comparative perspective. Finding a new image (2) causes us to clarify the main idea (1), and to become aware of the differences and similarities between the old image and the new one. To evaluate the significance of an old idea (3), we need to know that the old idea is useful in new situations which differ from those already known; this means recognizing the differences between the two situations. To see new connections (4), we must find a new connection between two objects that seem to be different. This means that some comparative process between the two is required.

Comparing the assumption of problems is one of the most powerful methods to generate new ideas or gain deeper understanding, especially in mathematical history. Thus, to obtain clearer insight into novel situations, we should analyze the results of solving a new problem with an original given situation.

Let us examine an example from the famous Pythagorean theorem using the What-If-Not strategy.

3. An Example

Brown & Walter illustrate the What-If-Not strategy by using the Pythagorean Theorem. Some parts of the process of the strategy, drawn from their explanation are as follows:

Level 0: Choosing a Starting Point

Pythagorean Theorem

Level 1: Listing Attributes

A1: It is a theorem.
A2: There are three variables.
A3: Variables are related by an "equal sign".

Level 2: What-If-Not-ing

(~3)1: related by < : $a^2 + b^2 < c^2$
(~3)2: related by > : $a^2 + b^2 > c^2$

Level 3: Problem Posing

(~3)1(a): Does $a^2 + b^2 < c^2$ have any geometrical significance?
(~3)1(b): For what numbers is the inequality, $a^2 + b^2 < c^2$ true?

Level 4: Analyzing the Problem

The above case (~3)1(a), seeking a geometric way of "seeing" the defect, that is $c^2 - (a^2 + b^2)$, follows Euclid's proof of the Pythagorean Theorem. We can see that the total defect $c^2 - (a^2 + b^2)$ is the sum of the two rectangles GCC'G' and CC"H'H and that certain points are collinear in the case of a right triangle and certain "convenient lines" form altitudes.

The above analytic problem, (~3)1(a) can be solved and analyzed to see some connections with a case in which ∠C is a right angle. This illustrates that the Pythagorean Theorem is a special case in which the defect is zero. In fact, the content of any deeper insight through Brown & Walter's process is not clear. We need to establish the content of the insight and the process of how to come up with it. If we reconsider that the What-If-Not strategy above should be used in the second phase of problem posing and according to the aim of the strategy, the phase involves important tasks as we have already noticed. This leads us to be unable to attain a deeper understanding of the nature of the Pythagorean Theorem itself merely by using the strategy, without reconsidering the aim and function of the strategy. We will try to analyze the process in Level 4 according to the four tasks already noted and then clarify the content of "the given", "idea", "image", "significance" and "connection" as follows.

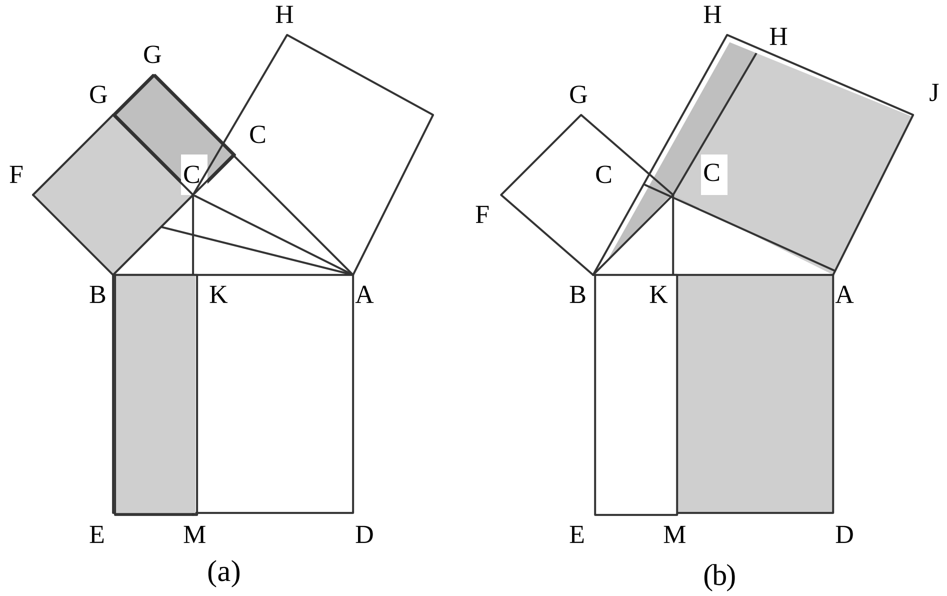

Figure 1. Pythagorean Theorem

At first, the What-If-Not strategy begins with challenging "something" taken for granted, "given". But it is very difficult to know what someone takes as "given" for the first time, before some new thinking would be clear. In fact, we can see a new idea in solving the new problem(If ∠C >∠R, does $c^2 > a^2 + b^2$ have any geometrical significance?) by using the Euclidean proof. Concretely, the new idea is to express the defect of two areas geometrically, using the square equation congruence of a triangle as in Euclid's idea for proving the Pythagorean Theorem. Here we know that the "given" was the Euclidean proof of the Pythagorean Theorem and that the new idea is to apply

Euclid's old idea in a right triangle into a non-right triangle, especially in order to solve the new problem.

In the second step, to apply the idea, we should manage an obtuse angle. However, when we look at △BFA and consider the base to be BF, the altitude is no longer CB since ∠C is not a right angle and ACG is no longer a line segment and therefore neither parallel to BF nor perpendicular to BC. To use Euclid's proof in a new problem situation, we need to make a triangle which is congruent to △BFA. Thus, we can draw AC'G' perpendicular to BC produced as indicated in Figure 1(a) and then the area of △BFA=1/2 area BFC'B=1/2 area BFG'C'. Using the fact that the area of △BEC equals the area of △BAF, we can conclude that the area of rectangle BEMK equals the area of rectangle BFG'C'; thus the defect contributed by BEMK consists of the shaded area GCC'G'. Similarly, MDAK contributes CC"H'H in Figure 1(b). That is, here, to mimic Euclid's proof, we make a new line AC'G' instead of ACG, and focus on the rectangle FBC'G' not the square FBCG. Thus finally, the total defect $c^2-(a^2+b^2)$ is seen to be the sum of the two rectangles GCC'G' and CC"HH'.

Through the process, we can gain some insights that certain points are collinear in the case of right triangles or that certain "convenient lines" form altitudes. Then, we can see that the Pythagorean Theorem is a special case in which the defect is zero. Reconsidering the process, I have clarified that we can acquire some new connection between right triangles and non-right triangles and understand some significance of parts of the Pythagorean proof by rethinking it in a new situation with the same idea. For this, there exists some comparative perspective in the process and we cannot have a deeper understanding of something without comparative analysis, merely by using the What-If-Not strategy.

Furthermore, there is the possibility to understand the relationship between angles and lengths of the triangle as the property of a general triangle, not only a right triangle, by comparing assumptions and results between the new and the original problem. For example, from the results of problem solving, we may form a hypothesis that in not only right triangles but also in non-right triangles, there is a certain relationship between a^2+b^2 and c^2. That is, without comparing the results, for example:

If ∠C >∠R, then $c^2 > a^2 + b^2$
If ∠C =∠R, then $c^2 = a^2 + b^2$

It is difficult to recognize the relationship between angles and lengths of the triangle, even if we pose a problem, "If ∠C <∠R, then $c^2 < a^2 + b^2$?".

4. A New What-If-Not Strategy

So far, I have described how comparative analysis can be used to accomplish the four tasks in the example. When we remind that the What-If-Not strategy is for challenging the given, the components of the strategy are not merely an algorithm of making a new problem and solving it. Rather, the new problem is a place to apply the given idea and solving it is a process of applying the given idea to the new problem situation. Since we cannot use the given idea to a new situation entirely, some parts of the idea are needed to be changed appropriately, these are included in the tasks, especially the first and the second, as we commented above. So, solving the new problem means not just acquiring the answer, but having some new insights, namely, new connections and the significance of some ideas.

Now, I propose a new What-If-Not strategy which clarifies where we consider the given and divide the solving process into two: applying the given idea and finding some insights by analyzing the results and the process. The five levels are as follows:

Level 0 starts by concluding what is given; Level 1 lists some attributes of the given. In Level 2, the selected attributes are modified by asking "if each attribute is not so" what could it be then? Level 3 consists of questioning the modified attributes and posing the problem. Level 4 has two stages; in level 4-1, the problem posed in level 3 is solved. In level 4-2, by comparing the unified problem of level 4-1 with the given, it is possible to pose a new problem.

Original What-If-Not strategy	New What-If-Not strategy
L 0: Choosing a Starting Point	L 0: Starting Point Concluding What is Given
L 1: Listing Attributes	L 1: Listing Attributes of the Given
L 2: What-If-Not-ing	L 2: Selecting Attributes and What-If-Not-ing
L 3: Problem Posing	L 3: Problem Posing about Attributes modified
L 4: Analyzing the Problem	L 4-1: Solving the Problem
	L 4-2: Comparing the Problem and the Given

It should be emphasized that the What-If-Not strategy is not the most effective algorithm for problem generation; however, the strategy is effective for acquiring deeper understanding of some original situations. If we use the strategy for greater understanding, we are prepared to do some mathematical tasks and solve the new problem, which involves finding a new idea for solving, making the appropriate image of an old one, evaluating the significance of an idea, and seeing new connections. In each task, we need a comparative perspective. So, when we want to provide a deeper understanding through the What-If-Not strategy, we need to clarify in which level of the strategy comparative perspective is used. In the example, we followed proof of the Phythagorean Theorem for which $a^2 + b^2 = c^2$. Analyzing the variation of a proof by mimicking the original, that is, to use some comparative perspectives, sometimes pays off in our understanding of the original situation.

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