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신 스플라인보간법의 퍼포먼스 가설검정

Hypothesis Tests For Performances of a New Spline Interpolation Technique

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要 旨

벡터 GIS에서 자연선형체는 통상 일련의 직선분(line segments)에 의해 표시되나 그 대안으로 곡선분(curve segments) 역시 사용될 수 있다. 곡선분은 스플라인보간법에 의해 생성가능하며 이를 위해 Bezier방법과 신보간법(유기윤, 1998)이 사용될 수 있는데 본 연구에서는 신보간법의 퍼포먼스를 테스트해 보았다. 테스트는 두가지에 촛점을 두었는데 (1) 새보간법에 의해 생성된 선형분이 직선분 보다 정확하게 자연선형체를 표현할 수 있는지 여부와 (2) 새보간법에 의해 생성된 선형분이 Bezier방법에 의해 생성된 선형분 보다 자연선형체를 정확하게 표현할 수 있는지 여부에 대한 검정이다. 이를 위해 t-테스트에 의한 가설검정법이 이용되었으며 자료로는 미 지질조사국의 7.5분 지형도가 이용되었다. 테스트결과 새보간법과 Bezier방법에 의해 생성된 선형분이 직선분 보다 자연선형체를 정확하게 표현하였으며 새보간법에 의해 생성된 선형분이 Bezier방법에 의해 생성된 선형분 보다 정확하게 표현하였다.

ABSTRACT

In vector GIS, natural linear entities (called linear entities) are usually represented by a set of line segments. As an alternative of the line segments, curve segments can be used to represent the linear entities. The curve segments, as one-dimensional spatial objects, are generated by spline interpolation technique such as Bezier technique. In an effort to improve its accuracy in resembling the linear entities, the Bezier technique was modified generating a new technique (called New technique) (Kiyun, 1998). In this paper, validity of the New technique was tested. Test focused on answering two questions: (1) whether or not the curve segments from the New technique replace line segments so as to enhance the accuracy of representations of linear entities, and (2) whether or not the curve segments from the New technique represent the linear entities more accurately than curve segments from the Bezier technique. Answering these two questions entailed two hypothesis tests. For test data, a series of hydrologic lines on 7.5-minute USGS map series were selected. Test were done using t-test method and statistical inferences were made from the results. Test results indicated that curve segments from both the Bezier and New techniques represent the linear entities more accurately than the line segments do. In addition, curve segments from the New technique represent the linear entities more accurately than the line segments from the Bezier technique do at probability level 69% or higher.

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1. Introduction

Spline interpolation technique is generally used to generate curve segments from digitized linear entities. Such curve segments, as one-dimensional spatial objects, can be a more accurate alternative than straight line segments in representing the linear entities. This is especially possible when the source linear entities are flowing cartographic lines without sharp corners (Burrough, 1988) and digitization is done satisfying some conditions (Kiyun, 1998). Such cartographic lines are found customarily within natural resources boundaries such as soil type boundaries, hydrologic lines, and geologic lines.

In generating the curve segments, the Bezier spline interpolation technique is widely used. In an effort to improve its accuracy, a New technique was developed (Kiyun, 1998) of which mathematical framework was the Bezier technique. In using the New technique, two questions are arising: (1) whether or not curve segments from the New technique represent the linear entities more accurately than line segments, and (2) whether or not curve segments from the New technique represent the linear entities more accurately than curve segments from Bezier technique. Answering these two questions entails two hypothesis tests. Following discussions start with introducing concepts of spline interpolation techniques.

2. Spline Interpolation Techniques

2.1 Concepts of Spline Interpolations

Mathematically, a spline curve is a collection of piecewise polynomial curves of the same degree.

Using parametric representation, a spline curve that passes through n+1 points is normally formulated by

$$p(t)=f(t_i, p_0, p_1, \dots p_n)$$

where, $p_0, p_1, \dots p_n$ are position vectors of given data points and t_i is a corresponding local parameter. There are two approaches to constructing a spline curve : (1) interpolation and (2) approximation. Interpolation spline curves pass exactly through each data point, while approximation spline curves pass close to data points without actually going through them. The approximation spline curves include Bezier spline curves. By some additional computation, approximation spline curves can be changed so as to pass through each data point (Burger and Gilles, 1989). Thus, the Bezier spline curves can interpolate all the data points.

Generally, low degree spline curves are preferred for easy formulation. As the degree of curve decreases, computational requirements and numerical instability decrease as well. Usually, third degree spline curves -- or cubic spline curves -- are used, for this is the lowest degree allowing inflection and twisting of curve shapes (Rogers and Adams, 1990).

2.2 Bezier Spline Interpolation Technique

The Bezier spline interpolation basically approximates given data points. Accordingly, only the first and last points actually lie on the curve ; other points define derivatives at first and last points, degree, and shape of the curve. For n+1 given points $b_0, b_1, b_2, \dots b_n$ and t, the Bezier curve can be explicitly defined by Bernstein polynomials (Farin, 1993)

$$B_i^n(t)=\binom{n}{i} t^i (1-t)^{n-i} \qquad (2.1)$$

which have important properties that satisfy the following recursion :

$$B_i^n(t) = (1-t)B_i^{n-1}(t) + tB_{i-1}^{n-1}(t) \quad (2.2)$$

with

$$B_0^0(t) \equiv 1 \quad (2.3)$$

and

$$B_j^n(t) \equiv 0 \text{ for } j \not\in \{1,2,...n\} \quad (2.4)$$

For a Bezier curve of any degree, the parameter value t varies in the range 0 to 1. The corresponding de Casteljau point on the Bezier curve is given by

$$b^n(t) = \sum_{j=0}^{n} b_j B_j^n(t) \quad (2.5)$$

The Bezier curve is a blend of influences from n points. Each point is weighted by the corresponding Bernstein polynomial.

With some additional computation, the Bezier curve can be changed to interpolate all the data points. For this, two additional intermediate points must be developed for each pair of given points.

When they are connected, these four points, two given points and two intermediate Bezier points, constitute a control polygon. Provided two intermediate Bezier points, b_{i+b} and b_{i-b}, are at left and right sides of a given point b_i (a common joint between two Bezier curves). Now, because the direction of this tangent line at b_i is unknown, it is usually assumed to be parallel to the line through b_{i-1} and b_{i+1} (Overhauser's Central Difference Theorem). Therefore, if the tangent line (or tangent vector) is expressed by m_i, it can be estimated as (Farin, 1993)

$$m_i = \frac{b_{i+1} - b_{i-1}}{\| b_{i+1} - b_{i-1} \|} \quad (2.6)$$

$$i = 1,2,...n-1$$

Once the tangent vector m_i is found, the locations of two intermediate Bezier points, b_{i+b} and b_{i-b}, are calculated from

$$b_{i+b} = b_i + \alpha_i m_i$$
$$b_{i-b} = b_i - \beta_i m_i \quad (2.7)$$

There are no values fixed for these two coefficients, α_i an β_id, and they are assumed in practice. One simple solution is to set them as (Farin, 1993)

$$\alpha_i = \beta_i = 0.4 \| \Delta_i \| \quad (2..8)$$

where $\| \Delta_i \|$ is absolute length of b_i, b_{i+1} span.

2.3 New Spline Interpolation Technique

New spline interpolation technique focused on enhancing flexibility of the control polygons, in so doing enhancing flexibility in managing the spline curves. For this, several methods were developed (Kiyun, 1998): (1) a method to generate tangent vectors at given points, and (2) methods to generate two coefficients, α_i and β_i.

The method to generate tangent vectors at given points was developed as follow.

$$m_i = C\left(\frac{b_i - b_{i-1}}{\| b_i - b_{i-1} \|}\right) + (1-C)\left(\frac{b_{i+1} - b_i}{\| b_{i+1} - b_i \|}\right) \quad (2.9)$$

where, m_i indicates tangent vectors at a point I, and C value is empirically 0.480.

The methods to generate two coefficients, α_i and β_i, were developed as follow.

$$\alpha_i = \frac{3}{2\tan \rho_i}(2h_{1i} - h_{2i})$$

$$\beta_{i+1} = \frac{3}{2\tan \rho_{i+1}}(2h_{2i} - h_{1i}) \qquad (2.10)$$

where, tangent angle, ρ_i and ρ_{i+1} are,

$$\rho_i = C\theta_i$$

$$\rho_{i+1} = C\theta_{i+1} \qquad (2.11)$$

where, θ_i and θ_{i+1} are angular changes between successive line segments, and calculated from the digitized points,

$$\theta_i = \frac{b_i - b_{i-1}}{\|b_i - b_{i-1}\|} - \frac{b_{i+1} - b_i}{\|b_{i+1} - b_i\|}$$

$$\theta_{i+1} = \frac{b_{i+1} - b_i}{\|b_{i+1} - b_i\|} - \frac{b_{i+2} - b_{i+1}}{\|b_{i+2} - b_{i+1}\|} \qquad (2.12)$$

and the curve heights, h_{1i} and h_{2i}, at the ith line segment are

$$h_{1i} = A_1\theta_i + B_1\theta_{i+1}$$

$$h_{2i} = A_2\theta_i + B_2\theta_{i+1} \qquad (2.13)$$

where, four coefficients, A_1, B_1, A_2, and B_2 are empirically,

$$A_1 = 0.066, \quad B_1 = 0.045$$
$$A_2 = 0.042, \quad B_2 = 0.069$$

End Conditions

Now we concentrate on the two end points of a set of line segments which present a special case. Because there is only one neighboring point for each end point, the calculations for m_i and α_i, β_i, are invalid. The new calculation for m_i at these points relies on the rationale that the curve follows the direction of the end line segment to guarantee the convergence of the curve's tangential line to the end point smoothly. Hence, the calculations at these points are

$$m_0 = \frac{b_1 - b_0}{\|b_1 - b_0\|}$$

$$m_n = \frac{b_n - b_{n-1}}{\|b_n - b_{n-1}\|} \qquad (2.14)$$

where m_0 is for the first point and m_n is for the last point.

At these two end line segments, the calculations for α_i, and β_i, are

$$\alpha_i = \beta_i = 0.3 \qquad (2.15)$$

3. Acquisition of Source Linear Entities

For these two spline interpolation techniques, a series of tests were performed to examine the accuracy of curve segments from these techniques.

Before discussing about the tests, we select the source linear entities for test data. There can be many candidates for source linear entities, which are usually found in various boundaries of natural resources. Hydrologic lines are one example, and may include lines of streams, lakes, and rivers. In this paper, hydrologic lines were selected for source linear entities. They are found in the 7.5-minute USGS topographic map series from which the sample for tests was taken.

3.1 Sampling of Hydrologic Lines in USGS Map

The elmentary sampling unit is a fraction of hydrologic lines, which falls in a line segment span.

To sample this unit, a length of hydrologic lines was randomly selected among many on a map, and the map was randomly selected from all such maps of the United States. Thus, the sampling is a hierarchical kind of random sampling. In this case, the elementary unit was sampled from a random, two-stage cluster sampling. Here, two-stage signifies that the elementary unit is removed two steps from the primary sampling stage. This type of random two-stage cluster sampling is widely used when test samples cover large geographic area (Levy and Lemeshow, 1991).

At the first stage, sampling USGS maps, it was important to note that the hydrologic lines were not equally distributed over the United States. Rather, these lines emerged in clusters of various densities that varied by geographic regions. While stratified sampling might be useful if we could determine the exact distribution, obtaining such information was prohibitive. Thus, simple random sampling was substituted. For this sampling, the United States was divided into five geographic sub-areas in a manner most convenient: north, south, west, central, and east. Such divisions best permitted a generally equal distribution because only a small number of lists could be selected. Five USGS maps were selected; one from each sub-area. After the first sampling stage, sampling at the second stage included a list of hydrologic lines from within each selected map, and parts of hydrologic lines were randomly selected from each map. Finally, the elementary units were compiled by fragmenting these hydrologic lines by digitization. Table 3.1 shows the 25 hydrologic lines sampled.

Table 1.1 Sampled hydrologic lines in USGS maps

Geographic Sub-Area	First-Stage Sampled USGS Maps	Second-Stage Sampled Hydrologic Lines
North West	Bremerton West, Washington Bremerton West 7.5-min quad N4730-W12237.5/7.5	Kitsap Lake, Sinclair Inlet, Dickerson Creek, Oyster Bay, Dyes Inlet
North Central	Albion Center, Minnesota NE/4 Cokato 15-min quad N4507.5-W9400/7.5	Rock Lake, Granite Lake, North Fork Crow River, Eduards Lake, Willima Lake
North East	North Whitefield, Maine NE/4 Wiscasset 15-min quad N4407.5-W6930/7.5	Clary Lake, Sheepscot River, Dyer Long Pond, Deer Meadow Pond, Little Dyer Pond
South West	Laurel, California SW/4 Los Gatos 15-min quad N3700-W12152.5/7.5	Aptos Creek, Soquel Creek, Amaya Creek, Hinckley Creek
South East	Bean Station, Tennessee Bean Station 7.5-min quad N3615-W8315/7.5	Cherokee Lake, Holston River, German Creek

3.2 Conversion of Sampled Hydrologic Lines

The sampled hydrologic lines in USGS maps were converted from paper format to vector format. Conversion was done in two steps : rasterization and vectorization. For rasterization, the hydrologic lines were scanned with a 500 dot per inch (dpi) resolution using a Hewlett Packard ScanJetII-cx scanner. Width of the hydrologic lines is typically 0.1mm (0.0039inch) or more, depending on the drawing device used. Thus, a 500 dpi resolution allowed scanning hydrologic lines with a 0.1 mm (0.0039 inch) line width with 2 pixels. On the scanned images, vectorization was done by extracting coordinates of points in two to ten-pixel intervals using the ArcInfo (version 7.0.4) from ESRI Inc., Redland, CA, in a Unix environment.

3.3 Manual Digitization of Acquired Hydrologic Lines

Acquired hydrologic lines in vector format could be displayed on screen and heads-up digitized using the ArcInfo to produce line segments. Here, the accuracy of the spline curves may be affected by how the points are digitized, i.e., what points are selected. This is defined as the digitization pattern. Though a commonality exists among digitizer operators on their digitization patterns -- for example the local extreme points are usually chosen -- there are unavoidable differences. In an effort to quantify the digitization pattern, Kiyun (1998) proposed two indices. These indices are named number of intersection (NIT) and number of inflection (NIF).

NIT indicates how many times a hydrologic line within a line segment span crosses over the line segment. NIF indicates how many times a hydrologic line within a line segment span changes its direction. By utilizing these two numeric indices, various digitization patterns of digitizer operators are quantified. Empirical test results indicated that three combinations of NIT and NIF, (NIT-0,NIF-0), (NIT-0,NIF-1), and (NIT-1,NIF-1), allowed for relatively higher accuracy of curve segments than other combinations (Kiyun, 1998). For a better understanding of what these three combinations mean, see Figure 3.1 Considering this, digitization was done on all 25 sampled hydrologic lines, thus producing 25 sets of digitized points and 25 sets of corresponding line segments.

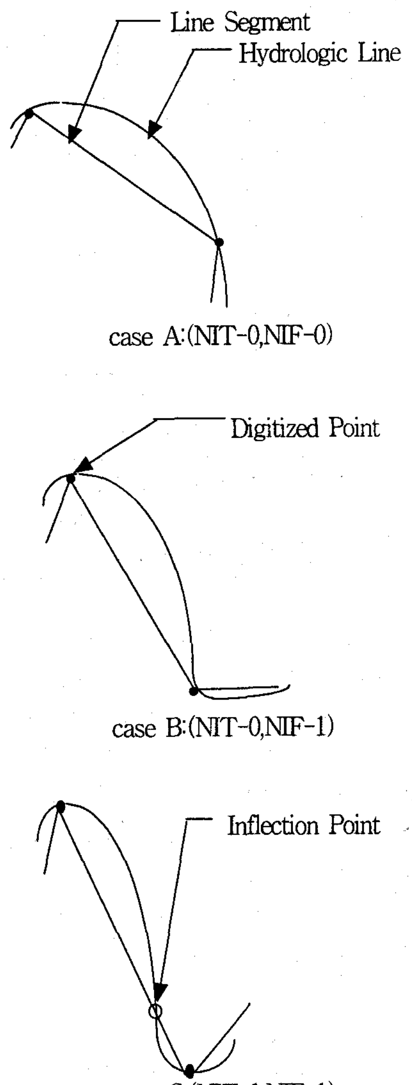

Figure 3.1 Geometric explanation of three combinations of NIT and NIF

4. Examination Methods of Accuracy of Curve Segments

To examine accuracy of the generated curve segments, they were overlaid on the hydrologic lines. Any deviations between the curve segments and the hydrologic lines were measured and defined as raw deviations, which were assumed positive.

Measurements taken utilized two numerical indices, maximum vector displacement (MVD) and area displacement (AD), methods also used by other researchers (Ping, 1994; McMaster and Veregin, 1997). The MVD is the maximum deviation between two lines, the curve segments and the hydrologic lines, and there is an MVD for each line segment span. The AD is the sum of the areas of sliver polygons in a line segment span, where the sliver polygons are created from deviations between the two lines. These indices were used to measure the level of closeness of the curve segments to the hydrologic lines, which indicate accuracy of the curve segments. After examining accuracy of the curve segments, accuracy of the line segments was measured using the same method.

To examine relative deviations of the curve segments compared to the line segments, the closeness ratio (CR) was calculated. Calculation entailed dividing MVD (or AD) of the line segments by MVD (or AD) of the curve segments,

$$CR_{AD} = \frac{AD\ of\ line\ segments}{AD\ of\ curve\ segments} \qquad (4.1)$$

$$CR_{MVD} = \frac{MVD\ of\ line\ segments}{MVD\ of\ curve\ segments} \qquad (4.2)$$

If the calculated CR is greater than 1, the curve segments are more accurate than the line segments; if less than 1, the line segments are more accurate than the curve segments; if 1, the line segments are as accurate as the curve segments.

5. Analysis of Accuracy of Curve Segments

Analysis of accuracy of curve segments consisted of two phases : (1) phase 1 -- spline interpolations were applied to digitized points to generate curve segments, and (2) phase 2 -- accuracy of the curve segments was examined and compared to that of line segments. In phase 1, each spline interpolation technique was applied to each set of digitized points, thus generating 25 sets of curve segments for each technique.

In phase 2, accuracy of both curve segments and line segments was measured by MVD and AD, which measure raw deviation of the curve segments and line segments from the hydrologic lines respectively. Lastly, the relative deviation was measured by calculating the CR_{MVD} and CR_{AD}.

Results showed that sum of raw deviations of curve segments from the Bezier technique and the New technique were smaller than the line segments both for MVD and AD (Table 5.1). This implies that the line segments are not more accurate than the curve segments from the Bezier technique and the New technique. The conclusion was the same when the relative deviations were used. The curve segments from the Bezier technique and the New technique had larger sum of CR_{MVD} and CR_{AD} than did the line segments.

5.1 Hypothesis Tests 1 : Comparison of Curve Segments and Line Segments

The previous conclusion on accuracy of curve segments from the two techniques was tested for any statistical significance using the t-test. In this

Table 5.1 Results of accuracy analysis of curve segments

	Sum of Raw Deviation		Sum of Relative Deviation	
	MVD(in)	AD(in2)	CR_{MVD}	CR_{AD}
Line Segments	16.4492	1.8135	1457	1457
Curve Seg from Bezier Tech	11.0466	1.1504	2555.9830	3046.3530
Curve Seg from New Tech	10.8424	1.1134	2725.4250	3277.7750

test, to make the distributions normal, the measured deviations were grouped and averaged (Kachigan, 1991; Johnson and Wichern, 1992). Thus, every 30 measured deviations were averaged to constitute a case resulting a total of 94 cases. The null hypothesis under test was,

H_0: Line segments represent hydrologic lines more accurately than curve segments from the two spline interpolation techniques.

The hypothesis was tested for curve segments from the two techniques using both raw deviation measures, MVD and AD, and relative deviation measures, CR_{MVD} and CR_{AD}. Using MVD, the statement could be rewritten as,

H_0: $\mu_p < \mu_s$

where, μ_p indicates the mean MVD of the line segments

μ_s indicates the mean MVD of the curvesegments from one of the two techniques.

The test statistic appropriate for this purpose was (Ramakant, 1990),

$$\frac{\mu_p - \mu_s}{\sqrt{\dfrac{s_p^2 + s_s^2}{n}}} \qquad (4.3)$$

where,

μ_p and s_p^2 are the mean and variance of MVD of the line segments

μ_s and s_s^2 are the mean and variance of MVD of the curve segments from one of the two techniques

n is doubled total number of MVD.

When AD was used instead of MVD, the previous statistic was calculated using AD.

Likewise, when CR_{MVD} or CR_{AD} was used, the previous statistic was calculated using CR_{MVD} or CR_{AD} respectively. In this case, the null hypothesis under test was,

H_0: $\mu_p > \mu_s$

The null hypothesis and decision rule is summarized

Table 5.2 Null hypothesis and decision rule

Measure Used	Null Hypothesis	Decision Rule to Reject H_0
MVD(or AD)	H_0: $\mu_p < \mu_s$	Computed value is equal to or greater than $t_{n-2,\alpha}$
CR_{MVD}(or CR_{AD})	H_0: $\mu_p > \mu_s$	Computed value is equal to or less than $t_{n-2,\alpha}$

in (Table 5.2).

For the test t values (of one tail test), $t_{92,0.05} = 1.664$ and $t_{92,0.01} = 2.373$ were used for MVD (or AD), and $t_{92,0.05} = -1.664$ and $t_{92,0.01} = -2.373$ were used for CR_{MVD} (or CR_{AD}), which correspond to 95% and 99% probability levels, respectively.

Table 5.3 T-test result using MVD (or AD). Results for the MVD is not underlined ; for the AD is underlined

	Mean (in/in^2)	t-value	Degree of Freedom	p-level	Standard Deviation
Line Segment	1.0000 1.0000	N/A	N/A	N/A	.0000 .0000
Curve Seg from Bezier Tech	1.5687 1.7170	-40.8301 -39.8347	92	.0000 .0000	.0955 .1234
Curve Seg from New Tech	1.6839 1.7902	-60.0718 -57.1942	92	.0000 .0000	.0781 .0947

When MVD (or AD) were used, resulting t-values exceeded test t-values for curve segments from the two techniques (Table 5.3) Thus, these results strongly suggested that the null hypothesis should be rejected for curve segments from the two techniques; this implies that these techniques produce curve segments that represent the hydrologic lines equally or more accurately than the line segments do.

The same conclusions were achieved when the CR_{MVD} (or CR_{AD}) was used (Table 5.4). In this test, the calculated t-values were smaller than the test t-values for curve segments from the two techniques (Table 5.4)

5.2 Hypothesis Test2 : Comparison of Bezier Technique and New Technique

The second null hypothesis under test was,

H_0: Curve segments from the Bezier technique represent hydrologic lines more accurately than curve segments from the New technique.

Table 5.4 T-test result using CR_{MVD} (or CR_{AD}). Results for the MVD is not underlined; for the AD is underlined

	Mean (in/in^2)	t-value	Degree of Freedom	p-level	Standard Deviation
Line Segment	.0143 .0021	N/A	N/A	N/A	.0030 .0006
Curve Seg from Bezier Tech	.0103 .0015	7.1235 5.0721	92	.0000 .0000	.0024 .0005
Curve Seg from New Tech	.0098 .0014	8.4176 6.0860	92	.0000 .0000	.0022 .0005

The hypothesis was tested for curve segments from the New technique using both raw deviation measures and relative deviation measures. Using MVD, the null hypothesis could be rewritten as,

$$H_0: \mu_s < \mu_{ns}$$

Where, μ_{ns} indicates the mean MVD of the curve segments from the New technique,

μ_s indicates the mean MVD of the curve segments from the Bezier technique.

The test statistic appropriate for this purpose was,

$$\frac{\mu_s - \mu_{ns}}{\sqrt{\frac{s_s^2 + s_{ns}^2}{n}}} \quad (4.4)$$

Where,

μ_{ns} and s_{ns}^2 are the mean and variance of MVD of the curve segments from the New technique.

μ_s and s_s^2 are the mean and variance of MVD of the curve segments from the Bezier technique.

n is doubled total number of MVD.

When AD was used instead of MVD, the previous statistic was calculated using AD. Likewise, when CR_{MVD} or CR_{AD} was used, the previous statistic was calculated using CR_{MVD} or CR_{AD} respectively. In this case, the null hypothesis under test was,

$$H_0: \mu_{ns} > \mu_s$$

The null hypothesis and decision rule is summarized in (Table 5.5)

The test results are in (Table 5.6) From the (Table 5.6), when MVD (or AD) were used, the resulting t-values were smaller than the test t-values. Corresponding p-levels were 0.22 for MVD and 0.31 for AD. Such results suggested that the null hypothesis should not be rejected at probability level 99% but should be rejected at probability level 78% (for MVD) or 69% (for AD).

This means the curve segments from the New technique represent the hydrologic lines more accurately than the curve segments from the Bezier technique do at probability level 78% (for MVD) and 69% (for AD).

Compare to this, when CR_{MVD} or (CR_{AD}) were used, the t-values were smaller than the test t-values and the p-level was 0.0032 and 0.0017 respectively (Table 5.7). Thus, the null hypothesis was rejected at probability level 99%, This implies that the New technique significantly improves the performance over the Bezier technique. Such results do not conform to the results for MVD (or AD).

Consequently, a confusion regarding which conclusion is more reliable was arising. In this paper, we gave more credit to the conclusion using the raw deviation measures. This is because the computed values of CR_{MVD} or (CR_{AD}), in its nature, can be extremely high or low for a few special geometric cases and such a few extreme values may affect to the conclusions significantly.

From this point, the comparison using MVD (or AD) was decided to be more reliable.

As a conclusion, the New technique improves the performance of the Bezier technique at probability level 78% (for MVD) and 69% (for AD).

Table 5.5 Null hyothesis and decision rule

Measure Used	Null Hypothesis	Decision Rule to Reject H_0
MVD(or AD)	H_0: $\mu_s < \mu_{ns}$	Computed value is equal to or greater than $t_{n-2,\alpha}$
CR_{MVD}(or CR_{AD})	H_0: $\mu_{ns} > \mu_s$	Computed value is equal to or less than $t_{n-2,\alpha}$

Table 5.6 T-test result using MVD (or AD). Results for the MVD is not underlined; for the AD is underlined

	Mean	t-value	Degree of Freedom	p-level	Standard Deviation
Curve Seg from New Tech	.0098 .0014	N/A	N/A	N/A	.0022 .0005
Curve Seg from Bezier Tech	.0103 .0015	1.2332 1.0243	92	.2207 .3084	.0024 .00 05

Table 5.7 T-test result using CR_{MVD} or (CR_{AD}). Results for the MVD is not underlined; for the AD is underlined.

	Mean	t-value	Degree of Freedom	p-level	Standard Deviation
Curve Seg from New Tech	1.6839 1.7902	N/A	N/A	N/A	.0781 .0947
Curve Seg from Bezier Tech	1.5687 1.7170	-6.4027 -3.2252	92	.0032 .0017	.0955 .1234

6. Conclusions

Throughout this paper, a series of tests were performed focusing on answering to two questions: (1) whether or not curve segments from the New technique replace line segments so as to enhance the accuracy of spatial representations of linear entities, and (2) whether or not curve segments from the New technique represent the linear entities more accurately than curve segments from the Bezier technique. Answering these two questions entailed two hypothesis tests. The linear entities selected for test data were hydrologic lines on 7.5-minute USGS maps. For curve segment generation, Bezier spline interpolation technique and New spline interpolation technique were used. Statistical inferences were made from the t-test results for the two hypothesis tests. Test results showed that curve segments from both the Bezier and New techniques represent the linear entities more accurately than the line segments do at probability level 99%. In addition, it was revealed that curve segments from the New technique represent the linear entities more accurately than the curve segments from the Bezier technique do at probability level 69% or higher.

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