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CENTRAL SEPARABLE ALGEBRAS OVER REGULAR DOMAIN

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ABSTRACT. Over a field k, every Schur k-algebra is a cyclotomic algebra due to Brauer-Witt theorem. Similarly every projective Schur k-division algebra is itself a radical algebra by Aljadeff-Sonn theorem. We study the two theorems over a certain commutative ring, and prove similar results over regular domain containing a field.

1. Introduction

Let R denote a commutative ring and $U(R)$ denote the set of units in R. Let $B(R)$ be the Brauer group of classes of central separable R-algebras [2]. A central separable R-algebra which is a homomorphic image of a group ring RG for some finite group G is called a Schur algebra, and the set of similar classes of Schur algebras forms the Schur subgroup $S(R)$ of $B(R)$.

The Brauer (1961) and Witt (1963) theorem ([11, p. 31]) shows that if R is a field k of characteristic 0 then every Schur k-algebra is similar to a cyclotomic algebra. Thus if $C(k)$ is the set of all algebra classes of $B(k)$ which are represented by a cyclotomic algebra over k then $C(k) = S(k)$.

In [8], Lorenz and Opolka generalized Schur algebras by substituting group ring RG by twisted group ring RG^α for some 2-cocycle $\alpha \in Z^2(G, U(R))$ on which trivial action is defined. A central separable R-algebra that is a homomorphic image of RG^α is called a projective Schur algebra, and the set of similar classes of projective Schur algebras forms the projective Schur subgroup $PS(R)$. Aljadeff and Sonn proved in [1]

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that if $R = k$ is a field then a projective Schur division k-algebra is a radical algebra $(k(\Omega)/k, \alpha)$ which is a generalization of cyclotomic algebra. The Aljadeff-Sonn theorem is a counterpart of Brauer-Witt theorem with respect to projective Schur algebra.

The purpose of this paper is to study the Brauer-Witt and Aljadeff-Sonn theorems over a certain commutative ring. For any commutative ring R, the Brauer-Witt theorem may fail that a Schur R-algebra is not necessarily similar to a cyclotomic R-algebra [7]. One of our main theorem is that if R is a regular domain containing a field then every Schur R-algebra is similar to a cyclotomic algebra. In studying the Aljadeff-Sonn theorem with projective Schur R-algebras, we do not have a corresponding situation: for a projective Schur R-algebra A and for the field of quotient K of R, even though $K \otimes A$ is a radical K-algebra, A need not be a radical R-algebra. However we prove that there is a ring extension T of R such that $T \otimes A$ is a radical T-algebra.

In this paper we use the usual notations: for two central separable R-algebras A and B, we denote by $A \sim B$ if A and B are similar. The similar class of central separable R-algebras A is denoted by $[A] \in B(R)$. Let $U(R)$ be the set of units in R and ε_n be a primitive n-th root of unity for $n > 0$.

2. Schur Algebra over Ring

Let $\phi : R \to T$ be a homomorphism of commutative rings. Then there are induced homomorphisms over Brauer and Schur groups

$$B(\phi) : B(R) \to B(T) \quad \text{and} \quad S(\phi) : S(R) \to S(T)$$

which are defined by tensor product that $[A] \to [T \otimes A]$. In case that T is a quotient field of R, if $B(\phi)$ is a monomorphism then so is $S(\phi)$. However $S(\phi)$ need not always be a monomorphism [2]. Of course the map $B(R) \to B(T)$ may not be a monomorphism while $S(R) \to S(T)$ is a monomorphism [5].

LEMMA 1. ([9, (6.19)]) *Let R be a regular domain with quotient field K. Then $B(R) \to B(K)$ is a monomorphism. Thus so is $S(R) \to S(K)$.*

Suppose that T is a Galois ring extension of R with Galois group $\mathrm{Gal}(T/R) = G$ (refer to [6, (3.1.2)]). For the algebra $(T/R, \alpha) = \sum_{\sigma\in G} Tu_\sigma$ having T-basis $\{u_\sigma | \sigma \in G\}$ such that $u_\sigma x = \sigma(x)u_\sigma$ and $u_\sigma u_\tau = \alpha(\sigma, \tau)u_{\sigma\tau}$ for $x \in T$, $\sigma, \tau \in G$ where each $\alpha(\sigma, \tau) \in U(T)$, α is a 2-cocycle in $Z^2(T/R, U(T))$ on which natural Galois action is defined, and $(T/R, \alpha)$ is called a crossed product algebra.

Assume that the separable closure of R contains a primitive root of unity ε and that the cyclotomic extension $T = R(\varepsilon)$ of R is a Galois extension of R in the separable closure with Galois group $G = \mathrm{Gal}(T/R)$. Then the crossed product algebra $(T/R, \alpha)$ where $\alpha \in Z^2(T/R, U(T))$ has values in $\langle\varepsilon\rangle$ is called the cyclotomic algebra. And there is a central group extension H of $\langle\varepsilon\rangle$ by G determined by the cocycle α, and the natural homomorphism from RH onto $(R(\varepsilon)/R, \alpha)$, thus $[(R(\varepsilon)/R, \alpha)]$ belongs to $S(R)$.

Let $S'(R)$ denote the set of similar classes of algebras of $B(R)$ which are represented by a cyclotomic algebra over R. Then $S'(R)$ is a subgroup of $S(R)$ [7]. Due to Brauer-Witt theorem if $R = k$ is a field then $S'(k) = C(k) = S(k)$. However $S'(R)$ may be proper in $S(R)$ [7]. We now prove the Brauer-Witt theorem over regular domain containing a field.

THEOREM 2. *Let R be a regular domain containing a field. Then every Schur algebra over R is similar to a cyclotomic R-algebra.*

Proof. Let $[A]$ be any element in $S(R)$. If $\mathrm{char}R > 0$ then $S(R)$ is trivial (refer to [7]) thus we have nothing to do. We now assume $\mathrm{char}R = 0$.

If K is the field of quotient of R then $B(\phi) : B(R) \to B(K)$ is a monomorphism by Lemma 1. Thus as an element $[A]$ in $B(R)$, we have

$$B(\phi)[A] = [K \otimes A] \in B(K). \tag{1}$$

Since A is the homomorphic image of RG for some finite group G, $K \otimes A$ is a homomorphic image of $K \otimes RG = KG$ thus $[K \otimes A] \in S(K)$. Because of the Brauer-Witt theorem, we may write

$$K \otimes A \sim (K(\varepsilon_n)/K, \beta) \quad \text{for} \quad \beta \in Z^2(K(\varepsilon_n)/K, K(\varepsilon_n)^*)$$

where β has values in $\langle\varepsilon_n\rangle$ for a root of unity ε_n $(n > 0)$.

Since $\mathrm{char}R=0$, R contains the field of rational numbers Q thus any nonzero integer is unit in R, i.e., $n \in U(R)$. This means that $R(\varepsilon_n)$ is a separable extension of R. By restriction $\mathcal{G} = \mathrm{Gal}(K(\varepsilon_n)/K)$ can be considered as a group of automorphisms of $R(\varepsilon_n)$. And since R is integrally closed, $\mathcal{G}$ fixes exactly R, i.e., $R(\varepsilon_n)^{\mathcal{G}} = R$. Thus $R(\varepsilon_n)$ is a Galois extension of R with Galois group $\mathcal{G}$ by [6, (3.1.2)]. Now considering β as an element in $Z^2(R(\varepsilon_n)/R, U(R(\varepsilon_n)))$, we have a cyclotomic algebra $(R(\varepsilon_n)/R, \beta)$, thus $[(R(\varepsilon_n)/R, \beta)] \in B(R)$ and

$$B(\phi)[(R(\varepsilon_n)/R, \beta)] = [(K(\varepsilon_n)/K, \beta)] = [K \otimes A].$$

Since $B(\phi)$ is injective, comparing this with (1), we have $[A] = [(R(\varepsilon_n)/R, \beta)]$ and $A \sim (R(\varepsilon_n)/R, \beta)$. This completes the proof. □

This means that $S(R) = S'(R)$ if R is a regular domain containing a field.

COROLLARY 3. *Let R, K be as in* Theorem 2. *Then $S(R)$ is isomorphic to $S(K)$.*

Proof. Let $[A]$ be an element in $S(R)$. Due to Theorem 2, A is similar to a cyclotomic R-algebra $(R(\varepsilon)/R, \beta)$ for some root of unity ε, and $S(\phi)$ maps $[A] = [(R(\varepsilon)/R, \beta)]$ to $[(K(\varepsilon)/K, \beta)]$.

Conversely any Schur K-algebra B is similar to $(K(\varepsilon_n)/K, \alpha)$ for $n > 0$ by Brauer-Witt theorem. If we consider $(R(\varepsilon_n)/R, \alpha)$ then it is a central separable R-algebra as we saw in the proof of Theorem 2, hence is a Schur R-algebra. Moreover $S(\phi)[(R(\varepsilon_n)/R, \alpha)] = [(K(\varepsilon_n)/K, \alpha)]$ thus $S(\phi)$ is an isomorphism. □

For $[A] \in B(R)$ and for a ring extension T of R, we say T splits A if $[T \otimes A]$ is trivial in $B(T)$. Let $B(T/R)$ denote the subset of $B(R)$ whose elements are splitted by T. Then $B(T/R)$ is a subgroup of $B(R)$ and is indeed the kernel of $B(R) \to B(T)$. By substituting $B(R)$ by $S(R)$, we write $S(T/R)$ for the subgroup of $S(R)$ consisting of Schur R-algebras splitted by T.

Let K be the quotient field of R. For a ring extension T of R, let F be the quotient field of T. Then F is an extension of K, and we consider $B(K/R)$, $B(T/R)$, $B(F/R)$, $B(F/T)$ and $B(F/K)$. The first three are subgroups of $B(R)$, and $B(T/R)$, $B(K/R)$ are subgroups of $B(F/R)$.

THEOREM 4. *Let R be a commutative ring with ring extension T. Let K and F be the quotient fields of R, T respectively. Then $B(F/R)/B(T/R)$ is isomorphic to a subgroup of $B(F/T)$. Moreover if R is a regular domain then there are monomorphisms $B(T/R) \to B(F/K)$ and $B(F/R) \to B(F/K)$.*

Proof. Consider the following diagram

$$\begin{array}{ccc} B(T) & \xrightarrow{\psi} & B(F) \\ f\uparrow & & \uparrow g \\ B(R) & \xrightarrow{\phi} & B(K) \end{array}$$

where f, g, ψ and ϕ are homomorphisms naturally defined by tensor product. Then it is a commutative diagram because

$$\psi f[A] = [F \otimes T \otimes A] = [F \otimes A] = g\phi[A], \quad \text{for } [A] \in B(R).$$

Let $\mu : B(F/R) \to B(F/T)$ be a map defined by $[A] \mapsto [T \otimes A]$. Clearly if $[F \otimes A] = 1$ then $[F \otimes T \otimes A] = 1$ and μ is a homomorphism. Moreover

$$\begin{aligned} \ker \mu &= \{[A] \in B(F/R) |\ [T \otimes A] = 1 \in B(F/T)\} \\ &= \{[A] \in B(R) |\ [F \otimes A] = 1,\ [T \otimes A] = 1, [F \otimes T \otimes A] = 1\} \\ &= \{[A] \in B(T/R)\} = B(T/R). \end{aligned}$$

Thus $B(F/R)/B(T/R)$ is isomorphic to a subgroup of $B(F/T)$.

If $[A] \in B(T/R)$ then $[T \otimes A] = 1$ in $B(T)$ and $[K \otimes A] \in B(K)$, moreover $[K \otimes A] \in B(F/K)$ because $[F \otimes K \otimes A] = [F \otimes A] = g\phi[A] = \psi f[A] = \psi[T \otimes A] = [1]$. Thus we have a homomorphism $\chi : B(T/R) \to B(F/K)$ defined by $[A] \mapsto [K \otimes A]$. Now

$$\ker \chi = \{[A] \in B(T/R) |\ [K \otimes A] = 1 \in B(F/K)\} \subseteq B(K/R).$$

Since $B(K/R)$ is the kernel of $B(R) \to B(K)$ which is injective due to Lemma 1, $B(K/R)$ and $\ker \chi$ are trivial hence χ is a monomorphism. Similarly there is a homomorphism $B(F/R) \to B(F/K)$ with kernel $B(K/R)$ which is zero. □

COROLLARY 5. *Suppose that R is a Dedekind domain with the quotient field K. Let F be a finite dimensional extension of K and T be an integral closure of R contained in F. Then $B(K/R) \cong B(F/T) = 1$ and $B(F/R) \cong B(T/R)$.*

Proof. If R is a Dedekind domain and K is the quotient field of R then $B(R) \to B(K)$ is a monomorphism (refer to [6, Lemma V.2.2]). And it is a well known fact that the extension T of the Dedekind domain is itself a Dedekind domain having quotient field as F (refer to [10, (4.4)]). Thus $B(T) \to B(F)$ is also a monomorphism, hence $B(K/R)$ and $B(F/T)$ are trivial. Moreover since $B(F/R)/B(T/R)$ is isomorphic to a subgroup of $B(F/T)$ by Theorem 4, we have $B(F/R) \cong B(T/R)$. □

We remark that by substituting Brauer groups $B(K/R)$ and $B(F/R)$ by Schur groups $S(K/R)$, $S(F/R)$ respectively, we have similar results to Theorem 4 and Corollary 5 by Lemma 1. Furthermore we have the next corollary.

COROLLARY 6. *Let R be a regular domain containing a field and let T, K and F are the same as in Theorem 4. Then $S(F/R) \cong S(F/K) \cong S(T/R)$.*

Proof. Due to Corollary 3, $S(R) \cong S(K)$ thus $S(F/R) \cong S(F/K)$. □

Therefore under the same assumptions of Corollary 5, $S(F/K)$, $S(F/R)$ and $S(T/R)$ are all isomorphic.

3. Projective Schur Algebra over Ring

A central separable R-algebra is called a projective Schur R-algebra if it is a homomorphic image of twisted group ring RG^α for some finite group G and $\alpha \in Z^2(G, U(R))$. A set of similar classes of projective Schur R-algebras forms a group, called projective Schur group $PS(R)$ (refer to [3], [4]). We recall that RG^α is separable if $|G| \in U(R)$.

THEOREM 7. *Let $\phi : R \to T$ be a homomorphism of commutative rings. If ϕ maps unity of R to unity of T then there is an induced group homomorphism $PS(\phi) : PS(R) \to PS(T)$ defined by $[A] \mapsto [T \otimes_R A]$ for $[A] \in PS(R)$.*

Proof. For $[A] \in PS(R)$, there is a surjection $\psi : RG^\alpha \to A$ for some finite group G and $\alpha \in Z^2(G, U(R))$. Let $\{u_g | g \in G\}$ be a basis of RG^α satisfying $u_g u_x = \alpha(g,x) u_{gx}$ for $g, x \in G$. By setting $\beta = \phi\alpha$ defined by $\beta(x,y) = \phi(\alpha(x,y))$ for all $x, y \in G$, it is clear that β is a 2-cocycle in $Z^2(G, U(T))$. If we consider the twisted group ring TG^β with basis $\{v_g | g \in G\}$ such that $v_g v_x = \beta(g,x) v_{gx}$ then the algebra can be regarded as an R-module by the action $r \cdot \sum_{g\in G} t_g v_g = \sum_{g \in G} f(r) t_g v_g$ for $r \in R, t_g \in T$. And we can see that $T \otimes RG^\alpha$ is isomorphic to TG^β as R-modules where the tensor map $T \otimes RG^\alpha \to TG^\beta$ is defined by $t \otimes \sum_{g \in G} r_g u_g \mapsto \sum_{g\in G} t f(r_g) v_g$ for $t \in T$ and $r_g \in R$. This shows that there is a surjection $TG^{\beta} \to T \otimes_R A$, hence $[T \otimes A] \in PS(T)$. Moreover since $T \otimes_R (A \otimes_R B) = (T \otimes_R A) \otimes_T (T \otimes_R B)$ for $[A], [B] \in PS(R)$, it follows that $PS(\phi)$ is a homomorphism. □

We call $PS(\phi)$ a projective Schur homomorphism. In case of regular domain, the next corollary follows immediately from Lemma 1.

COROLLARY 8. *Let R be a regular domain and K be the quotient field of R. Then $PS(R) \to PS(K)$ is a monomorphism.*

The role of cyclotomic field extension in the theory of Schur algebra can be replaced by the radical field extension in the theory of projective Schur algebra. When we say $L = k(\Omega)$ is a radical extension of k, we mean that $\Omega < L^*$ and $\Omega k^*/k^*$ is a torsion group, that is for any $x \in \Omega$, x mod k^* is of finite order in $\Omega k^*/k^*$. A crossed product algebra $A = (L/k, \alpha)$ is called a radical k-algebra if $L = k(\Omega)$ is a finite radical Gal(L/k)-Galois extension over k (i.e., Ω is a Gal(L/k)-invariant subgroup of L^*) and $\bar{\alpha} \in H^2(L/k, L^*)$ is the image of some $\bar{\alpha}' \in H^2(L/k, \Omega)$ (refer to [1]). Moreover if the Galois group is abelian then A is called an abelian radical algebra. A radical algebra is a projective Schur k-algebra [1], and an analog of Brauer-Witt theorem was proved as follow.

LEMMA 9. [1] *Let k be any field. Then every projective Schur division k-algebra is an abelian radical algebra over k.*

Over a commutative ring R, a crossed product algebra $(T/R, \alpha)$ is a radical R-algebra if $T = R(\Omega)$ is a finite radical Gal(T/R)-Galois ring extension of R (i.e., Ω is a Gal(T/R)-invariant subgroup of $U(T)$ such that $\Omega U(R)/U(R)$ is finite) and $\bar{\alpha} \in H^2(T/R, U(T))$ is an image of some $\bar{\alpha}' \in H^2(T/R, \Omega)$. By considering the group extension Γ of Ω by Gal(T/R) determined by α', the algebra $(T/R, \alpha)$ is a projective Schur R-algebra.

We study similar property of Lemma 9 over a certain commutative ring.

THEOREM 10. *Let R be a regular domain of characteristic 0 containing a field and K be the quotient field of R. Let A be a projective Schur R-algebra such that $K \otimes A$ is division over K. Then there is a ring extension T of R such that $T \otimes A$ is similar to a radical T-algebra.*

Proof. For $[A] \in PS(R)$, there is a finite group G and $\alpha \in Z^2(G, U(R))$ such that A is a homomorphic image of RG^α. For quotient field K of R, the usual inclusion $\phi : R \to K$ produces monomorphisms

$$B(\phi): \ B(R) \to B(K) \quad \text{and} \quad PS(\phi): \ PS(R) \to PS(K)$$

defined by tensor product, and $K \otimes RG^\alpha = KG^\alpha$ represents $K \otimes A$.

Since $K \otimes A$ is a projective Schur K-algebra which is division, it is an abelian radical K-algebra by Lemma 9 thus we may write

$$K \otimes A \ = \ (K(\Omega)/K, \beta)$$

where $K(\Omega)$ is an abelian radical extension of K and $\bar{\beta} \in H^2(K(\Omega)/K, K(\Omega)^*)$ is the image of some $\bar{\beta}' \in H^2(K(\Omega)/K, \Omega)$ under the inclusion $\Omega \hookrightarrow (K(\Omega))^*$. For our convenience we write the abelian Galois group $\mathrm{Gal}(K(\Omega)/K)$ by $\mathcal{G}$.

Consider the ring extension $R(\Omega)$ over R. By restriction, $\mathcal{G}$ is the group of automorphisms of $R(\Omega)$. And since R is a regular domain, R is integrally closed [9, (6.23)] thus $\mathcal{G}$ fixes exactly R, i.e., $R(\Omega)^{\mathcal{G}} = R$. Moreover since $\mathrm{char} R = 0$, R contains the field of rational numbers Q hence the order $|\mathcal{G}|$ is unit in R. This implies that $R(\Omega)$ is a Galois extension of R with Galois group $\mathrm{Gal}(R(\Omega)/R)$ by [6, (3.1.2)]. Denoting the restriction of $\bar{\beta}' \in H^2(K(\Omega)/K, \Omega)$ by the same notation $\bar{\beta}' \in H^2(R(\Omega)/R, \Omega)$, the crossed product algebra $(R(\Omega)/R, \beta')$ is a central separable R-algebra. By regarding $\bar{\beta}$ as an image of $\bar{\beta}' \in H^2(R(\Omega)/R, \Omega)$ under the inclusion $\Omega \hookrightarrow U(R(\Omega)) \hookrightarrow (K(\Omega))^*$, we can consider $(R(\Omega)/R, \beta)$ with $\bar{\beta} \in H^2(R(\Omega)/R, U(R(\Omega)))$. Furthermore since $B(\phi)$ is a monomorphism and since

$$B(\phi)[(R(\Omega)/R, \beta)] = [(K(\Omega)/K, \beta)] = [K \otimes A] = B(\phi)[A],$$

it follows that the R-algebras A and $(R(\Omega)/R, \beta)$ are similar.

Let $T = R(\Omega) \cap K$. Then $R \subset T$ and $T(\Omega) = (R(\Omega) \cap K)(\Omega) = R(\Omega)$. Since $K(\Omega)$ is radical over K, there is $n > 0$ such that $x^n \in K$ for any $x \in \Omega$. Hence $x^n \in R(\Omega) \cap K = T$, this shows $T(\Omega)$ is a radical extension of T. Moreover the extension $T(\Omega)/T$ is finite and abelian because $\mathrm{Gal}(T(\Omega)/T) < \mathrm{Gal}(R(\Omega)/R)$. By using the same notation β abusively for the restriction of β to $\mathrm{Gal}(T(\Omega)/T)$, the crossed product algebra $(T(\Omega)/T, \beta)$ is a radical T-algebra hence is a projective Schur algebra in $PS(T)$.

If we consider the quotient field L of T then L is isomorphic to K thus $PS(L) = PS(K)$. Moreover since T is a regular domain, there is a monomorphism $\psi : PS(T) \to PS(L)$. And from the inclusion $R \to T$, we also have a homomorphism $f : PS(R) \to PS(T)$ where both ψ and f are defined by tensor products. Thus the following diagram

$$\begin{array}{ccc} PS(R) & \overset{PS(\phi)}{\longrightarrow} & PS(K) \\ f\downarrow & & \| \\ PS(T) & \overset{\psi}{\longrightarrow} & PS(L) \end{array}$$

is commutative because for any $[B] \in PS(R)$,

$$\psi f[B] = \psi[T \otimes B] = [K \otimes T \otimes B] = [K \otimes B] = PS(\phi)[B].$$

Since $PS(\phi)$ is injective, so is f. Now from $A \sim (R(\Omega)/R, \beta)$, we have $[T \otimes A] = f[A] = f[(R(\Omega)/R, \beta)] = [(T(\Omega)/T, \beta)]$, i.e., $T \otimes A \sim (T(\Omega)/T, \beta)$ which is a radical algebra. This completes the proof. □

REMARK. Although $K(\Omega)/K$ is a radical extension, $R(\Omega)$ is not necessarily radical over R because some powers of element x in Ω may not contained in R, thus $(R(\Omega)/R, \beta)$ need not be a radical algebra. Hence we take T as $R(\Omega) \cap K$ to make $T(\Omega)/T$ is a radical extension.

Theorem 10 implies that for $[A] \in PS(R)$ even if $K \otimes A$ is a radical K-algebra, A need not be a radical R-algebra. However there exists a ring extension T of R which makes $T \otimes A$ a radical T-algebra.

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