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REMARKS ON WEAK HYPERMODULES

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ABSTRACT. H_v-rings first were introduced by Vougiouklis in 1990. Then Darafsheh and the present author defined the H_v-ring of fractions $S^{-1}R$ of a commutative hyperring. The largest class of multivalued systems satisfying the module-like axioms is the H_v-module. In this paper we define H_v-module of fractions of a hypermodule. Some interesting results concerning this H_v-module is proved.

1. Basic Definitions of Hyperstructures

The concept of a hyperstructure first was introduced by Marty in [2]. A hyperstructure is a set H together with a function $\cdot : H \times H \longrightarrow \mathcal{P}^*(H)$ called hyperoperation, where $\mathcal{P}^*(H)$ denotes the set of all non-empty subsets of H. If $A, B \subseteq H$, $x \in H$ then we define

$$A \cdot B = \bigcup_{a \in A,\, b \in B} a \cdot b, \quad x \cdot B = \{x\} \cdot B, \quad A \cdot x = A \cdot \{x\}.$$

DEFINITION 1.1. A hyperstructure $(H, \cdot)$ is called a hypergroup if

(i) $x \cdot (y \cdot z) = (x \cdot y) \cdot z$, $\forall x, y, z \in H$,

(ii) $a \cdot H = H \cdot a = H$, $\forall a \in H$.

DEFINITION 1.2. A multivalued system $(R, +, .)$ is a hyperring if

(i) $(R, +)$ is a hypergroup,

(ii) $(R, \cdot)$ is a semihypergroup,

(iii) $(\cdot)$ is distributive with respect to $(+)$, i.e., for all x, y, z in R we have

$$x \cdot (y + z) = (x \cdot y) + (x \cdot z), \qquad (x + y) \cdot z = (x \cdot z) + (y \cdot z).$$

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A hyperring may be commutative with respect to $(+)$ or $(\cdot)$. If R is commutative with respect to both $(+)$ and $(\cdot)$, then we call it a commutative hyperring. If there exists $u \in R$ such that $x \cdot u = u \cdot x = \{x\}$, $\forall x \in R$, then u is called the scalar unit of R and is denoted by 1.

DEFINITION 1.3. M is a left hypermodule over hyperring R (R-hypermodule) if $(M, +)$ is a commutative hypergroup and there exists a map $\cdot : R \times M \longrightarrow \mathcal{P}^*(M)$ denoted by $(r, m) \longrightarrow rm$ such that for all r_1, r_2 in R and m_1, m_2 in M, we have

$$\begin{aligned} r_1(m_1 + m_2) &= r_1m_1 + r_1m_2, \\ (r_1 + r_2)m_1 &= r_1m_1 + r_2m_1, \\ (r_1r_2)m_1 &= r_1(r_2m_1). \end{aligned}$$

There are generalizations of the above hyperstructures (hypergroup, hyperring and hypermodule) where axioms are replaced by the weak ones. That is instead of the equality on sets one has non-empty intersections.

H_v-structures first introduced by Vougiouklis in the Fourth AHA congress (1990) [5]. In this paper we are interested in H_v-rings and H_v-modules.

DEFINITION 1.4. A multivalued system $(R, +, \cdot)$ is called an H_v-ring if the following axioms hold:

(i) $(R, +)$ is an H_v-group, i.e.,
$(x + y) + z \cap x + (y + z) \neq \emptyset, \quad \forall x, y, z \in R,$
$a + R = R + a = R, \quad \forall a \in R,$

(ii) $(x \cdot y) \cdot z \cap x \cdot (y \cdot z) \neq \emptyset, \quad \forall x, y, z \in R,$

(iii) $(\cdot)$ is weak distributive with respect to $(+)$, i.e., for all $x, y, z \in R$,

$$x \cdot (y + z) \cap (x \cdot y + x \cdot z) \neq \emptyset, \qquad (x + y) \cdot z \cap (x \cdot z + y \cdot z) \neq \emptyset.$$

DEFINITION 1.5. M is a left H_v-module over an H_v-ring R if $(M, +)$ is a weak commutative H_v-group and there exists a map $\cdot : R \times M \longrightarrow \mathcal{P}^*(M)$ denoted by $(r, m) \longrightarrow rm$ such that for all r_1, r_2 in R and m_1, m_2 in M, we have

$$\begin{aligned} r_1(m_1 + m_2) \cap (r_1m_1 + r_1m_2) &\neq \emptyset, \\ (r_1 + r_2)m_1 \cap (r_1m_1 + r_2m_1) &\neq \emptyset, \\ (r_1r_2)m_1 \cap r_1(r_2m_1) &\neq \emptyset. \end{aligned}$$

DEFINITION 1.6. Let M_1 and M_2 be two H_v-modules over an H_v-ring R. A mapping $f : M_1 \longrightarrow M_2$ is called an $R - H_v$-homomorphism if, $\forall x, y \in M_1$ and $\forall r \in R$, the following relations hold:

$$f(x+y) \cap (f(x)+f(y)) \neq \emptyset, \quad f(rx) \cap rf(x) \neq \emptyset.$$

f is called an inclusion R-homomorphism if, $f(x+y) \subseteq f(x)+f(y)$, $f(rx) \subseteq rf(x)$.
Finally f is called a strong R-homomorphism if, $f(x+y) = f(x) + f(y)$, $f(rx) = rf(x)$.

If there exists a strong one to one homomorphism from M_1 onto M_2, then M_1 and M_2 are called isomorphic.

In this paper, we shall work over a commutative hyperring R with scalar unit, and we shall assume that M is an R-hypermodule. We remark that according to [1], a non-empty subset S of R is a strong multiplicatively closed subset (s.m.c.s) if the following conditions hold:

(i) $1 \in S$,

(ii) $a \cdot S = S \cdot a = S$, $\forall a \in S$.

Darafsheh and the present author in [1] defined the H_v-ring of fractions $S^{-1}R$ of a commutative hyperring. The construction of $S^{-1}R$ can be carried through with an R-hypermodule M in place of the hyperring R. In section 2 of this paper we introduce the set of fractions $S^{-1}M$ and define addition and multiplication by elements of $S^{-1}R$, then we get that $S^{-1}M$ is an $S^{-1}R - H_v$-module as well as some interesting results with this respect.

2. H_v-module of Fractions

Let X be the set of all ordered pairs (m, s) where $m \in M$, $s \in S$. For $A \subseteq M$ and $B \subseteq S$, we denote the set $\{(a,b) \mid a \in A, b \in B\}$ by (A, B). The relation $\sim$ is defined on $\mathcal{P}^*(X)$ as follows:
$(A, B) \sim (C, D)$ iff there exists a subset T of S such that $T \cdot (B \cdot C) = T \cdot (D \cdot A)$.

LEMMA 2.1. *$\sim$ is an equivalence relation on $\mathcal{P}^*(X)$.*

If we restrict the relation $\sim$ on X, and identify $(m, x) \in X$ with the subset $\{(m, x)\}$ of X, then we obtain the following two lemmas.

LEMMA 2.2. *For (m_1, s_1), $(m_2, s_2) \in X$, we have $(m_1, s_1) \sim (m_2, s_2)$ iff there exists $T \subseteq S$ such that $T \cdot (s_1 \cdot m_2) = T \cdot (s_2 \cdot m_1)$.*

LEMMA 2.3. *$\sim$ is an equivalence relation on X.*

The equivalence class containing (m, s) is denoted by $[m, s]$ and we let $S^{-1}M$ to be the set of all the equivalence classes.

In $\mathcal{P}^*(X)$, the equivalence class containing (A, B) is denoted by $[[A, B]]$. We define

$$\langle\langle A, B\rangle\rangle = \bigcup_{(C,D)\in[[A,B]]} \{[c, d] \mid c \in C, d \in D\}.$$

LEMMA 2.4. *For all $m \in M$, $s \in S$, we have $\langle\langle m, s\rangle\rangle = \langle\langle sm, ss\rangle\rangle$.*

Now we define addition and multiplication by elements of $S^{-1}R$, as follows:
$[m_1, s_1] \oplus [m_2, s_2] = \bigcup\{[a, b] \mid a \in A, b \in B\} = \langle\langle s_1m_2 + s_2m_1, s_1s_2\rangle\rangle$, where the union is over $(A, B) \in [[s_1m_2 + s_2m_1, s_1s_2]]$
$[r, s] \odot [m_1, s_1] = \bigcup\{[a, b] \mid a \in A, b \in B\} = \langle\langle rm_1, ss_1\rangle\rangle$, where the union is over $(A, B) \in [[rm_1, ss_1]]$
In both cases $[m_1, s_1]$, $[m_2, s_2] \in S^{-1}M$ and $[r, s] \in S^{-1}R$.

THEOREM 2.5. *$\oplus$ and $\odot$ defined above are independent of the choices of representatives $[m_1, s_1]$, $[m_2, s_2]$ and $[r, s]$ and that $S^{-1}M$ satisfies the axioms of an $S^{-1}R - H_v$-module.*

If we define

$$r \odot [m_1, s_1] = \langle\langle rm_1, s_1\rangle\rangle$$

then $S^{-1}M$ becomes an $R - H_v$-module.

Proof. The proof is straightforward and omitted. □

DEFINITION 2.6. Hypersubmodule U of M is called a hyperisolated submodule if it satisfies the following axiom:
For all $A \subseteq U$, $B \subseteq S$ if $(X, Y) \in [[A, B]]$ then $X \subseteq U$.

LEMMA 2.7. *Let U be a hyperisolated submodule of M then the set $S^{-1}U = \{[u,s] \mid u \in U,\ s \in S\}$ is an $R - H_v$-submodule of $S^{-1}M$.*

Proof. First, we prove that $(S^{-1}U, \oplus)$ is an H_v-subgroup of $(S^{-1}M, \oplus)$. For every $[m_1, s_1]$, $[m_2, s_2] \in S^{-1}U$, we have

$$[m_1, s_1] \oplus [m_2, s_2] = \bigcup_{(A,B)\in[[s_2m_1+s_1m_2,\ s_1s_2]]} \{[m,s] \mid m \in A,\ s \in B\}.$$

Since $m_1,\ m_2 \in U$ then we get $s_2m_1 + s_1m_2 \subseteq U$ and since U is a hyperisolated submodule of M, then $A \subseteq U$. Therefore $[m_1, s_1] \oplus [m_2, s_2] \subseteq S^{-1}U$.

Now we prove the equality $S^{-1}U = [m_1, s_1] \oplus S^{-1}U$, for all $[m_1, s_1] \in S^{-1}U$. Suppose $[m, s] \in S^{-1}U$, $m \in U$. Since $s, s_1 \in S$, by definition of S there exists $s_2 \in S$ such that $s \in s_1s_2$. And since U is a hypersubmodule, we have $s_2m_1 + (s_1 + 1)U = U$. Now $m \in U$ and $s_2m_1 + (s_1 + 1)U = U$ imply that there exists $m_2 \in U$ such that $m \in s_2m_1 + s_1m_2 + m_2$ hence $m \in s_2m_1 + s_1(m_2 + s_3m_2)$ where $1 \in s_3s_1$. So there exists $x \in m_2 + s_3m_2$ such that $m \in s_2m_1 + s_1x$, therefore $[m, s] \in [m_1, s_1] \oplus [x, s_2]$ implying $S^{-1}U \subseteq [m_1, s_1] \oplus S^{-1}U$.

It remains to prove that $R \odot (S^{-1}U) \subseteq S^{-1}U$. To do this suppose that $[u, s] \in S^{-1}U$ and $r \in R$, then

$$r \odot [u,s] = \bigcup_{(A,B)\in[[ru,s]]} \{[x,y] \mid x \in A,\ y \in B\}.$$

Since $u \in U$ we have $ru \subseteq U$ and since U is a hyperisolated submodule of M we get $A \subseteq U$. Consequently $r \odot [u, s] \subseteq S^{-1}U$. Therefore the lemma is proved. □

THEOREM 2.8. *Let M_1 and M_2 be two R-hypermodules and let $f : M_1 \longrightarrow M_2$ be a strong R-homomorphism. Then the map $S^{-1}(f) : S^{-1}M_1 \longrightarrow S^{-1}M_2$ defined by $S^{-1}(f)[m, s] = [f(m), s]$, is an $S^{-1}R - H_v$-homomorphism.*

Proof. Suppose that $[m_1, s_1]$, $[m_2, s_2] \in S^{-1}M_1$ and $[r, s] \in S^{-1}R$. First we show that $S^{-1}(f)$ is well-defined. If $[m_1, s_1] = [m_2, s_2]$ then there exists $T \subseteq S$ such that $T \cdot (s_1 \cdot m_2) = T \cdot (s_2 \cdot m_1)$ which implies $f(T \cdot (s_1 \cdot m_2)) = f(T \cdot (s_2 \cdot m_1))$ and so $T \cdot (s_1 \cdot f(m_2)) = T \cdot (s_2 \cdot f(m_1))$ or

$[f(m_1), s_1] = [f(m_2), s_2]$. Therefore $S^{-1}(f)$ is well-defined.

Moreover, $S^{-1}(f)$ is an $S^{-1}R - H_v$-homomorphism because, we have

$$\begin{aligned}
&S^{-1}(f)([m_1, s_1] \oplus [m_2, s_2]) \\
&= S^{-1}(f)\left(\bigcup_{(A,B)\in[[s_1m_2+s_2m_1,s_1s_2]]} \{[a,b] \mid a \in A, b \in B\}\right) \\
&= \bigcup_{(A,B)\in[[s_1m_2+s_2m_1,s_1s_2]]} S^{-1}(f)(\{[a,b] \mid a \in A, b \in B\}) \\
&= \bigcup_{(A,B)\in[[s_1m_2+s_2m_1,s_1s_2]]} \{[f(a),b] \mid a \in A, b \in B\}
\end{aligned}$$

and

$$\begin{aligned}
&S^{-1}(f)([m_1, s_1]) \oplus S^{-1}(f)([m_2, s_2]) \\
&= [f(m_1), s_1] \oplus [f(m_2), s_2] \\
&= \bigcup_{(A,B)\in[[s_1f(m_2)+s_2f(m_1),s_1s_2]]} \{[a,b] \mid a \in A, b \in B\} \\
&= \bigcup_{(A,B)\in[[f(s_1m_2+s_2m_1),s_1s_2]]} \{[a,b] \mid a \in A, b \in B\}.
\end{aligned}$$

Therefore we have

$$\begin{aligned}
&\{[f(a),b] \mid a \in s_1m_2 + s_2m_1, b \in s_1s_2\} \\
&\qquad\qquad \subseteq S^{-1}(f)([m_1, s_1] \oplus [m_2, s_2]), \\
&\{[a,b] \mid a \in f(s_1m_2 + s_2m_1), b \in s_1s_2\} \\
&\qquad\qquad \subseteq S^{-1}(f)([m_1, s_1]) \oplus S^{-1}(f)([m_2, s_2]).
\end{aligned}$$

And so

$$S^{-1}(f)([m_1, s_1] \oplus [m_2, s_2]) \cap S^{-1}(f)([m_1, s_1]) \oplus S^{-1}(f)([m_2, s_2]) \neq \emptyset.$$

Similarly, we get

$$\{[f(a),b] \mid a \in rm_1, b \in ss_1\} \subseteq S^{-1}(f)([r,s] \odot [m_1, s_1]),$$

$$\{[a,b] \mid a \in rf(m_1), b \in ss_1\} \subseteq [r,s] \odot S^{-1}(f)([m_1, s_1]).$$

And so

$$S^{-1}(f)([r,s] \odot [m_1, s_1]) \cap [r,s] \odot S^{-1}(f)([m_1, s_1]) \neq \emptyset,$$

which proves that $S^{-1}(f)$ is an $S^{-1}R - H_v$-homomorphism. □

LEMMA 2.9. *The natural mapping* $\Psi : M \longrightarrow S^{-1}M$ *defined by* $\Psi(m) = [m, 1]$ *is an inclusion* R*-homomorphism.*

Proof. For every $m_1, m_2 \in M$, we have

$$\begin{aligned} \Psi(m_1 + m_2) &= \{[\alpha, 1] \mid \alpha \in m_1 + m_2\} \\ &\subseteq \bigcup_{(A,B)\in[[m_1+m_2,1]]} \{[a, b] \mid a \in A, b \in B\} \\ &= [m_1, 1] \oplus [m_2, 1] \\ &= \Psi(m_1) \oplus \Psi(m_2). \end{aligned}$$

And for every $r \in R$ and $m \in M$ we have

$$\Psi(rm) = \{[\alpha, 1] \mid \alpha \in rm\} \subseteq \langle\langle rm, 1\rangle\rangle = r \odot \Psi(m).$$

Therefore Ψ is an inclusion R-homomorphism. □

DEFINITION 2.10. Let M_1 and M_2 be two R-hypermodules and $f, g : M_1 \longrightarrow M_2$ be R-homomorphisms. We define the mapping $S^{-1}(f + g)$ from $S^{-1}M_1$ into $\mathcal{P}^*(S^{-1}M_2)$ as follows:

$$S^{-1}(f + g)([m, s]) = \langle\langle f(m) + g(m),\ s\rangle\rangle.$$

PROPOSITION 2.11. *Let* M_1, M_2 *and* M_3 *be* R*-hypermodules and* $f, g : M_1 \longrightarrow M_2$ *and* $h : M_2 \longrightarrow M_3$ *be* R*-homomorphisms. Then*

(i) $S^{-1}(f + g) = S^{-1}(f) + S^{-1}(g)$,
(ii) $S^{-1}(hof) = S^{-1}(h)oS^{-1}(f)$,
(iii) $S^{-1}(id_{M_1}) = id_{S^{-1}M_1}$.

PROPOSITION 2.12. *Let* $\mathcal{C}_1$ *be the category of* R*-hypermodules and strong* R*-homomorphisms and let* $\mathcal{C}_2$ *be the category of* $R - H_v$*-modules and* $R - H_v$*-homomorphisms. Then the mapping* $S^{-1} : \mathcal{C}_1 \longrightarrow \mathcal{C}_2$ *is a functor.*

DEFINITION 2.13. Let M_1, M_2 and M_3 be R-hypermodules ($R - H_v$-modules respectively) and let U be a subhypermodule (H_v-submodule) of M_3. The sequence of strong R-homomorphisms $M_1 \xrightarrow{f} M_2 \xrightarrow{g} M_3$ is said to be U-exact if $\mathrm{Im}f = g^{-1}(U)$.

THEOREM 2.14. *Let U be an H_v-isolated submodule of M_3 and the sequence $M_1 \xrightarrow{f} M_2 \xrightarrow{g} M_3$ be U-exact. Then the sequence $S^{-1}M_1 \xrightarrow{S^{-1}(f)} S^{-1}M_2 \xrightarrow{S^{-1}(g)} S^{-1}M_3$ is $S^{-1}U$-exact, i.e., $ImS^{-1}(f) = (S^{-1}(g))^{-1}(S^{-1}U)$.*

Proof. Suppose that $[m, s] \in \mathrm{Im}S^{-1}(f)$ then there exists $m_1 \in M_1$ such that $[m, s] = [f(m_1), s]$. Since $f(m_1) \in \mathrm{Im}f = g^{-1}(U)$, there exists $u \in U$ such that $f(m_1) \in g^{-1}(u)$ and so $[f(m_1), s] \in \{ [x, s] \mid x \in g^{-1}(u) \}$ which implies $[f(m_1), s] \in (S^{-1}(g))^{-1}([u, s])$. Therefore $[f(m_1), s] \in (S^{-1}(g))^{-1}$ $(S^{-1}U)$.

Now, if $[u, t] \in (S^{-1}(g))^{-1}(S^{-1}U)$, then $[u, t] \in (S^{-1}(g))^{-1}(\{ [u, s] \mid u \in U,\ s \in S \})$ which implies $[u, t] \in \{ [x, s] \mid g(x) \in U,\ s \in S \}$. Therefore for some x where $g(x) \in U$ we have $[u, t] = [x, s]$. From $g(x) \in U$ we get $x \in \mathrm{Im}f$ and so $[x, s] \in \mathrm{Im}S^{-1}(f)$. Therefore $[u, t] \in \mathrm{Im}S^{-1}(f)$. □

3. The Fundamental Relations γ^* and ϵ^*

Consider the left H_v-module M over an H_v-ring R. The relation γ^* is the smallest equivalence relation on R such that the quotient R/γ^* is a ring. γ^* is called the fundamental equivalence relation on R and R/γ^* is called the fundamental ring, see [3], [4]. The fundamental relation ϵ^* on M over R is the smallest equivalence relation such that M/ϵ^* is a module over the ring R/γ^*, see [4].

According to [4], if $\mathcal{U}$ denotes the set of all expressions consisting of finite hyperoperations of either on R and M or the external hyperoperation applied on finite subsets of R and M. Then a relation ϵ can be defined on M whose transitive closure is the fundamental relation ϵ^*. The relation ϵ is defined as follows: for all $x, y \in M$,

$$x \epsilon y \Longleftrightarrow \{x, y\} \subseteq u, \quad \text{for some } u \in \mathcal{U}.$$

Suppose $\gamma^*(r)$ is the equivalence class containing $r \in R$ and $\epsilon^*(x)$ is the equivalence class containing $x \in M$. On M/ϵ^* the sum $\circ$ and the external product $\Box$ using the γ^* classes in R, are defined as follows: for all $x, y \in M$ and for all $r \in R$,

$$\epsilon^*(x) \circ \epsilon^*(y) = \epsilon^*(c), \quad \forall c \in \epsilon^*(x) + \epsilon^*(y),$$

$$\gamma^*(r) \Box \epsilon^*(x) = \epsilon^*(d), \quad \forall d \in \gamma^*(r).\epsilon^*(x).$$

Now, we will prove two theorems concerning the fundamental relations γ^* and ϵ^*.

Let ϵ_s^* be the fundamental equivalence relation on $S^{-1}M$ and $\mathcal{U}_s$ be the set of all expressions consisting of finite hyperoperations of either on $S^{-1}R$ and $S^{-1}M$ or of external hyperoperation. In this case $S^{-1}M/\epsilon_s^*$ is an $S^{-1}R/\gamma_s^*$-module.

THEOREM 3.1. *$S^{-1}M/\epsilon_s^*$ is an R/γ^*- module.*

Proof. We can define

$$\gamma^*(r) * \epsilon_s^*([m,s]) = \gamma_s^*([r,1]) \Box \epsilon_s^*([m,s]).$$

Then it is clear that $S^{-1}M/\epsilon_s^*$ is an R/γ^*-module. □

THEOREM 3.2. *There is an R/γ^*-homomorphism $h : M/\epsilon^* \longrightarrow S^{-1}M/\epsilon_s^*$.*

Proof. we define $h(\epsilon^*(m)) = \epsilon_s^*([m,1])$. First we prove that h is well defined. If $\epsilon^*(m_1) = \epsilon^*(m_2)$ then $m_1 \epsilon^* m_2$ which holds iff $\exists x_1, \dots, x_{m+1}$; $u_1, \dots, u_m \in \mathcal{U}$ with $x_1 = m_1$, $x_{m+1} = m_2$ such that $\{x_i, x_{i+1}\} \subseteq u_i, i = 1, \dots, m$ which implies $\{[x_i,1],[x_{i+1},1]\} \subseteq \langle\langle u_i, 1\rangle\rangle \in \mathcal{U}_s$. Therefore $[m_1,1]\epsilon_s^*[m_2,1]$ and so $\epsilon_s^*([m_1,1]) = \epsilon_s^*([m_2,1])$. Thus h is well-defined. h is a homomorphism because

$$h(\epsilon^*(a) \circ \epsilon^*(b)) = h(\epsilon^*(c)) = \epsilon_s^*([c,1]), \quad \forall c \in \epsilon^*(a) + \epsilon^*(b),$$

$$h(\epsilon^*(a)) \circ h(\epsilon^*(b)) = \epsilon_s^*([a,1]) \circ \epsilon_s^*([b,1]) = \epsilon_s^*([d,s]),$$

$$\forall [d,s] \in \epsilon_s^*([a,1]) \oplus \epsilon_s^*([b,1]),$$

setting $d = c \in a + b, s = 1$. So it is proved that $h(\epsilon^*(a) \circ \epsilon^*(b)) = h(\epsilon^*(a)) \circ h(\epsilon^*(b))$.
And also we have

$$h(\gamma^*(r) \Box \epsilon^*(m)) = h(\epsilon^*(a)) = \epsilon_s^*([a,1]), \quad \forall a \in \gamma^*(r).\epsilon^*(m),$$

$$\gamma^*(r) * h(\epsilon^*(m)) = \gamma^*(r) * \epsilon_s^*([m,1]) = \gamma_s^*([r,1]) \Box \epsilon_s^*([m,1]) = \epsilon_s^*([b,s]),$$

$$\forall [b,s] \in \gamma_s^*([r,1]) \odot \epsilon_s^*([m,1]).$$

Therefore, since we can take $b = a \in r \cdot m$ and $s = 1$, we get

$$h(\gamma^*(r) \Box \epsilon^*(m)) = \gamma^*(r) * h(\epsilon^*(m))$$

Hence h is a homomorphism of modules. □

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