

## ON THE SEPARATING IDEALS OF SOME VECTOR-VALUED GROUP ALGEBRAS

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**ABSTRACT.** For a locally compact Abelian group  $G$ , and a commutative Banach algebra  $B$ , let  $L^1(G, B)$  be the Banach algebra of all Bochner integrable functions. We show that if  $G$  is noncompact and  $B$  is a semiprime Banach algebras in which every minimal prime ideal is contained in a regular maximal ideal, then  $L^1(G, B)$  contains no nontrivial separating ideal. As a consequence we deduce some automatic continuity results for  $L^1(G, B)$ .

### 1. Introduction

For any locally compact Abelian group  $G$ , and commutative Banach algebra  $B$ , let  $L^1(G, B)$  denote the convolution algebra of all integrable functions on  $G$  with values in  $B$ . As one might expect, there are some interesting similarities between  $B$  and  $L^1(G, B)$ . For instance,  $L^1(G, B)$  is semi-simple if and only if  $B$  is semi-simple, and the regular maximal ideals of  $L^1(G, B)$  are closely related in a natural way with the regular maximal ideals of both  $L^1(G, B)$  and  $B$ . Also,  $L^1(G, B)$  is Tauberian if and only if  $B$  is Tauberian. Refer to [8,9] for the proofs of the above results. Also it is easy to note that  $L^1(G, B)$  is semiprime when  $B$  is semiprime. The question whether the zero ideal is the only separating ideal in a semiprime Banach algebra still seems to be open. However, in this paper we prove that when  $G$  is a noncompact locally compact Abelian group, and  $B$  is a commutative semiprime Banach algebra (not necessarily *unital*) in which every minimal prime ideal is contained in a regular maximal ideal, then  $L^1(G, B)$  contains no non-trivial separating ideal. As a consequence we deduce some automatic continuity results

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for the algebra  $L^1(G, B)$ . Our results extend some of the results in [11] for *non unital* Banach algebras, and also extend some results in [7] for semiprime Banach algebras. For relevant information on  $L^1(G, B)$  and for related results in harmonic analysis on Abelian groups, see [5, 8, 9, 12].

## 2. Preliminaries

Let  $B$  be a commutative Banach algebra (not necessarily *unital*), and let  $G$  be a locally compact Abelian group with Haar measure  $m$ . Throughout the following, the dual group of  $G$  is denoted by  $\Gamma$  and the spectrum of  $B$  is denoted by  $\Delta(B)$ . Let  $L^1(G, B)$  denote the Banach algebra of all integrable function from  $G$  into  $B$ ,

$$(f * g)(t) := \int_G f(t-s)g(s)dm \quad \text{for all } f, g \in L^1(G, B) \text{ and } t \in G,$$

and let  $\|f\|_1 := \int_G \|f(t)\|dm(t)$  for all  $f \in L^1(G, B)$ . Recall that for any  $f \in L^1(G, B)$ , and  $\gamma$  in the dual group  $\Gamma$  of  $G$ ,  $\hat{f}(\gamma) = \int_G \overline{\gamma(t)}f(t)dm(t)$  is known as the vector-valued Fourier transform of  $f$  at  $\gamma$ . Furthermore for any  $\gamma \in \Gamma$ , let  $M_\gamma := \{f \in L^1(G, B) : \hat{f}(\gamma) = \theta\}$  where  $\theta$  is the zero vector of  $B$ . Clearly,  $M_\gamma$  is a closed ideal of  $L^1(G, B)$ . If  $B$  has no non-trivial zero divisors, then  $M_\gamma$  is a closed prime ideal of  $L^1(G, B)$ . Recall that an ideal  $I$  of a commutative Banach algebra is said to be prime if the product  $xy \in I$  only if either  $x \in I$  or  $y \in I$ . It is an easy consequence of the Hahn-Banach theorem that  $\bigcap_{\gamma \in \Gamma} M_\gamma$  is the zero ideal in  $L^1(G, B)$ . For any  $\gamma \in \Gamma$ ,  $\phi \in \Delta(B)$ , let

$$M_{\gamma, \phi} := \{f \in L^1(G, B) | \phi(\hat{f}(\gamma)) = 0\}.$$

The regular maximal ideals of  $L^1(G, B)$  are given by  $M_{\gamma, \phi}$  for some  $\gamma \in \Gamma$ , and  $\phi \in \Delta(B)$  ([8]).

For each  $f \in L^1(G)$ , and  $x \in B$ , we let

$$(f \otimes x)(s) = f(s)x \text{ for all } s \in G.$$

We recall some of the properties of the product  $f \otimes x$  in the following proposition.

**PROPOSITION 2.1.** *Let  $G$  be a locally compact Abelian group, and let  $B$  be a commutative Banach algebra. Let  $x, y \in B$ ;  $f, g \in L^1(G)$ ; and  $\gamma$  a non-trivial continuous character on  $G$ . Then,*

- (i)  $f \otimes x \in L^1(G, B)$ , and  $\|f \otimes x\|_1 = \|f\|_1 \|x\|$
- (ii)  $(f \pm g) \otimes x = f \otimes x \pm g \otimes x$
- (iii)  $\widehat{f \otimes x}(\gamma) = \hat{f}(\gamma)x$
- (iv)  $(f \otimes x) * (g \otimes x) = (f * g) \otimes xy$
- (v) If  $B$  has the multiplicative identity 1, then  $(f * g) \otimes x = (f \otimes x) * (g \otimes 1) = (f \otimes 1) * (g \otimes x)$
- (vi) If  $f_n \rightarrow f$  in  $L^1(G)$  and  $x_n \rightarrow x$  in  $B$ , then  $f_n \otimes x_n \rightarrow f \otimes x$  in  $L^1(G, B)$ .

### 3. Main Results

Before we get to the main results, we need the following lemmas.

**LEMMA 3.1.** *Let  $G$  be a noncompact locally compact Abelian group,  $B$  a commutative Banach algebra, and  $f$  a non-zero function in  $L^1(G, B)$ . For a given  $\gamma$  in the dual group  $\Gamma$  of  $G$  and a positive number  $\epsilon$ , there exist  $f_1, f_2, \dots, f_n$  in  $L^1(G)$  with compactly supported Fourier transforms and  $x_1, x_2, \dots, x_n$  in  $B$  such that*

$$\|f - \sum_{i=1}^n f_i \otimes x_i\| < \epsilon + \|\hat{f}(\gamma)\|,$$

where  $\hat{f}_i(\gamma) = 0$  for  $1 \leq i \leq n$ .

*Proof.* Since finite linear combinations of the elements of the form  $h \otimes x$  where  $h \in L^1(G)$ , and  $x \in B$  are dense in  $L^1(G, B)$ , and the functions in  $L^1(G)$  with compactly supported Fourier transforms are dense in  $L^1(G)$ , there exist  $h_1, h_2, \dots, h_n$  in  $L^1(G)$  with compactly supported Fourier transforms and  $x_1, x_2, \dots, x_n$  in  $B$  such that

$$\|f - \sum_{i=1}^n h_i \otimes x_i\| < \frac{\epsilon}{2}. \text{ For } 1 \leq i \leq n, \text{ let } \text{Supp } \hat{h}_i = \{\alpha \in \Gamma : \hat{h}_i(\alpha) \neq 0\}.$$

For each  $1 \leq i \leq n$ , we define

$$g(t) = \frac{\chi_{(\cup_{j=1}^n \text{Supp } \hat{h}_j)}}{m(\cup_{j=1}^n \text{Supp } \hat{h}_j)} \gamma(t),$$

where  $\chi_{(\cup_{j=1}^n \text{Supp } \hat{h}_j)}$  is the characteristic function of  $(\cup_{j=1}^n \text{Supp } \hat{h}_j)$ , and  $f_i = h_i - \hat{h}_i(\gamma)g$ . Clearly  $g$  and the  $f_i$ 's belong to  $L^1(G)$ . It is easy to see that  $\hat{g}(\gamma) = 1$ ,  $\hat{f}_i(\gamma) = 0$  for each  $i$ , and  $\|g\|_1 = 1$ . We have

$$\begin{aligned} & \|f - \sum_{i=1}^n (f_i \otimes x_i) - g \otimes \hat{f}(\gamma)\| \\ &= \|f - \sum_{i=1}^n (h_i \otimes x_i) + \sum_{i=1}^n (h_i \otimes x_i) - \sum_{i=1}^n (f_i \otimes x_i) - g \otimes \hat{f}(\gamma)\| \\ &\leq \|f - \sum_{i=1}^n (h_i \otimes x_i)\| + \|\sum_{i=1}^n (h_i - f_i) \otimes x_i - g \otimes \hat{f}(\gamma)\| \quad \dots (A) \end{aligned}$$

Furthermore,

$$\begin{aligned} & \|\sum_{i=1}^n (h_i - f_i) \otimes x_i - g \otimes \hat{f}(\gamma)\| = \|\sum_{i=1}^n \hat{h}_i(\gamma)g \otimes x_i - g \otimes \hat{f}(\gamma)\| \\ &= \int_G \|\sum_{i=1}^n \hat{h}_i(\gamma)g(t)x_i - g(t)\hat{f}(\gamma)\| dm(t) \\ &= \frac{1}{m(\cup_{j=1}^n \text{Supp } \hat{h}_j)} \\ & \quad \int_G \|\sum_{i=1}^n \hat{h}_i(\gamma)\chi_{(\cup_{j=1}^n \text{Supp } \hat{h}_j)}\gamma(t)x_i - \chi_{(\cup_{j=1}^n \text{Supp } \hat{h}_j)}\gamma(t)\hat{f}(\gamma)\| dm(t) \\ &= \frac{1}{m(\cup_{j=1}^n \text{Supp } \hat{h}_j)} \int_{(\cup_{j=1}^n \text{Supp } h_j)} \|\sum_{i=1}^n \hat{h}_i(\gamma)x_i - \hat{f}(\gamma)\| dm(t) \\ &= \|\sum_{i=1}^n \hat{h}_i(\gamma)x_i - \hat{f}(\gamma)\| \leq \|f - \sum_{i=1}^n h_i \otimes x_i\| < \frac{\epsilon}{2} \quad \dots (B) \end{aligned}$$

From (A) and (B) it follows that  $\|F - \sum_{i=1}^n f_i \otimes x_i - g \otimes \hat{f}(\gamma)\| < \epsilon$ . Hence

$\|f - \sum_{i=1}^n f_i \otimes x_i\| < \epsilon + \|\hat{f}(\gamma)\|$ . This completes the proof of the Lemma.  $\square$

LEMMA 3.2. Let  $G$  be a noncompact locally compact Abelian group,  $B$  a commutative Banach algebra, and  $f$  a non-zero function in  $L^1(G, B)$ . For a given  $\gamma$  in the dual group  $\Gamma$  of  $G$  and a given positive number  $\epsilon > 0$ , there exist  $g_1, g_2, \dots, g_n$  in  $L^1(G)$ , a neighborhood  $V$  of  $\gamma$ , and  $x_1, x_2, \dots, x_n$  in  $B$  such that

$$\|f - \sum_{i=1}^n g_i \otimes x_i\| < \epsilon + \|\hat{f}(\gamma)\|$$

where  $\hat{g}_i = 0$  on  $V$  for  $1 \leq i \leq n$ .

*Proof.* By Lemma 3.1, there exist  $f_1, f_2, \dots, f_n$  in  $L^1(G)$  with compactly supported Fourier transforms, and  $x_1, x_2, \dots, x_n$  in  $B$  such that

$$\|f - \sum_{i=1}^n f_i \otimes x_i\| < \frac{\epsilon}{2} + \|\hat{f}(\gamma)\|$$

where  $\hat{f}_i(\gamma) = 0$ . Since  $L^1(G)$  satisfies the Ditkin's condition ([12]), there exist  $g_1, g_2, \dots, g_n$  in  $L^1(G)$ , and a neighborhood  $V$  of  $\gamma$  such that  $\hat{g}_i = 0$  on  $V$ , and

$$\|f_i - g_i\|_1 < \frac{\epsilon}{2 \left(1 + \sum_{i=1}^n \|x_i\|\right)}$$

for  $1 \leq i \leq n$ . Now

$$\begin{aligned} \|f - \sum_{i=1}^n g_i \otimes x_i\|_1 &\leq \|f - \sum_{i=1}^n f_i \otimes x_i\|_1 + \left\| \sum_{i=1}^n (f_i - g_i) \otimes x_i \right\|_1 \\ &\leq \frac{\epsilon}{2} + \|\hat{f}(\gamma)\| + \sum_{i=1}^n \|f_i - g_i\|_1 \|x_i\| \\ &< \frac{\epsilon}{2} + \|\hat{f}(\gamma)\| + \frac{\epsilon}{2 \left(1 + \sum_{i=1}^n \|x_i\|\right)} \left( \sum_{i=1}^n \|x_i\| \right) \\ &= \epsilon + \|\hat{f}(\gamma)\|. \end{aligned}$$

□

**COROLLARY 3.3.** *Let  $f \in L^1(G, B)$ , and  $\gamma \in \Gamma$  such that  $\hat{f}(\gamma) = \theta$ . Given  $\epsilon > 0$ , there exist  $g_1, g_2, \dots, g_n$  in  $L^1(G)$  with a vanishing Fourier transform in a neighborhood  $V$  of  $\gamma$ , and  $x_1, x_2, \dots, x_n$  in  $B$  such that*

$$\|f - \sum_{i=1}^n g_i \otimes x_i\| < \epsilon.$$

*Proof.* Obviously follows from the Lemma 3.2. □

Now we are ready for the main results of the section.

**THEOREM 3.4.** *Let  $G$  be a locally compact Abelian group,  $\gamma$  a continuous character on  $G$ , and  $\mathcal{P}$  a prime ideal contained in  $M_\gamma$ . Then  $\mathcal{P}$  is dense in  $M_\gamma$ .*

*Proof.* Let  $\mathcal{P}$  be a prime ideal of  $L^1(G, B)$  contained in  $M_\gamma$ . Let  $f$  be a function with  $\hat{f}$  identically equal to the zero vector in a neighborhood  $V$  of  $\gamma$ . We claim that  $f$  belongs to  $\mathcal{P}$ . For, if  $g$  belongs to  $L^1(G)$  with  $\hat{g}(\gamma) \neq 0$ ,  $\hat{g} = 0$  on  $\Gamma - V$ , and  $x$  a non-zero vector in  $B$ , then  $(g \otimes x) * f = \theta$  (the zero vector of  $L^1(G, B)$ ). Since  $\mathcal{P}$  is a prime ideal of  $L^1(G, B)$ , either  $g \otimes x \in \mathcal{P}$  or  $f \in \mathcal{P}$ . But  $g \hat{\otimes} x(\gamma) = \hat{g}(\gamma)x \neq \theta$ . Hence  $f \in \mathcal{P}$ . Thus all the functions  $f$  in  $L^1(G, B)$  with vanishing Fourier transforms in a neighborhood of  $\gamma$  belong to  $\mathcal{P}$ . Hence by Lemma 3.2, it follows that  $\mathcal{P}$  is dense in  $M_\gamma$ . This completes the proof of the theorem. □

**THEOREM 3.5.** *Let  $G$  be a noncompact locally compact Abelian group, and  $B$  be a commutative Banach algebra. If  $\mathcal{P}$  is a closed prime ideal of  $L^1(G, B)$  contained in  $M_{\gamma, \phi}$  for some  $\gamma \in \Gamma$ , and  $\phi \in \Delta(B)$ , then  $\mathcal{P}$  contains  $M_\gamma$ . Furthermore  $\mathcal{P}$  does not contain  $M_\sigma$  for any  $\sigma \neq \gamma$ .*

*Proof.* Let  $f \in M_\gamma$ . By Corollary 3.3,  $f$  can be approximated by a function  $g$  in  $L^1(G, B)$  with vanishing Fourier transform in a neighborhood  $V$  of  $\gamma$ . By an argument similar to the one given in Theorem 3.4, we can show  $g \in \mathcal{P}$ . Since  $\mathcal{P}$  is a closed ideal, it follows that  $f \in \mathcal{P}$ . Thus  $M_\gamma$  is contained in  $\mathcal{P}$ . Let  $\sigma \in \Gamma$  such that  $\sigma \neq \gamma$ . Suppose  $V_\sigma$  and  $V_\gamma$  are compact neighborhoods of  $\sigma$  and  $\gamma$  respectively such that  $V_\sigma \cap V_\gamma = \emptyset$ . Then there exist functions  $f_\sigma$  and  $f_\gamma$  from  $G$  into the complex plane with the support of  $\hat{f}_\sigma$  contained in  $V_\sigma$  and the support

of  $\hat{f}_\gamma$  contained in  $V_\gamma$  such that  $\hat{f}_\sigma(\sigma) = 1$  and  $\hat{f}_\gamma(\gamma) = 1$ . Let  $x, y \in B$  such that  $\phi(x)\phi(y) \neq 0$ . Then  $f_\sigma \otimes x, f_\gamma \otimes y \in L^1(G, B)$  such that  $(f_\sigma \otimes x) * (f_\sigma \otimes y) = \Theta$ . Since  $\mathcal{P}$  is a prime ideal contained in  $M_{\gamma, \sigma}$ , we get  $f_\sigma \otimes x \in \mathcal{P}$ . Obviously  $f_\gamma \otimes y \notin \mathcal{P}$ . However  $f_\gamma \otimes y \in M_\sigma$ . Therefore  $M_\sigma$  is not contained in  $\mathcal{P}$ .  $\square$

#### 4. Applications

Recall that a closed ideal  $S$  of a commutative Banach algebra  $A$  is called a separating ideal ([3]) if it satisfies the following condition: For each sequence  $\{a_k\}_{k \geq 1}$  in  $A$  there is a positive integer  $n$  such that  $\overline{a_1 a_2 \cdots a_n S} = \overline{a_1 a_2 \cdots a_k S}$  ( $k \geq n$ ). For any derivation  $D$  on  $A$ , let  $\mathfrak{S}(D) =: \{a \in A \mid \text{there is a sequence } \{a_n\} \text{ in } A \text{ with } a_n \rightarrow 0 \text{ and } Da_n \rightarrow a\}$ . For any epimorphism  $h$  from a commutative Banach algebra  $X$  onto  $A$ , let  $\mathfrak{S}(h) =: \{a \in A \mid \text{there is a sequence } \{x_n\} \text{ in } X \text{ with } x_n \rightarrow 0 \text{ and } h(x_n) \rightarrow a\}$ . It is easy to show that  $\mathfrak{S}(D)$  and  $\mathfrak{S}(h)$  are closed ideals of  $A$ . By the closed graph theorem  $D$  is continuous if and only if  $\mathfrak{S}(D)$  is zero. Similarly  $h$  is continuous if and only if  $\mathfrak{S}(h)$  is zero. It is well known that  $\mathfrak{S}(D)$  and  $\mathfrak{S}(h)$  are separating ideals of  $A$  ([13]). For further information on separating ideals, their relation to the prime ideals of the Banach algebra, and for related results on automatic continuity theory, see [1, 2, 3, 4, 6, 10].

Now we are ready to state one of the main results of the section.

**THEOREM 4.1.** *Let  $G$  be a noncompact locally compact Abelian group  $G$ , and  $B$  a commutative semiprime Banach algebra in which every minimal prime ideal is contained in a regular maximal ideal. Then  $L^1(G, B)$  contains no nontrivial separating ideal.*

**LEMMA 4.2.** *Let  $G$  be a noncompact locally compact Abelian group  $G$ , and  $B$  a commutative semiprime Banach algebra. For any  $\gamma \in \Gamma$ ,  $M_\gamma = \bigcap_{\mathcal{P} \in \mathcal{I}_\gamma} \mathcal{P}$  where  $\mathcal{I}_\gamma$  is the set of all minimal prime ideals of  $L^1(G, B)$  containing  $M_\gamma$ .*

*Proof.* Let  $f \in \bigcap_{\mathcal{P} \in \mathcal{I}_\gamma} \mathcal{P}$ . Since there is a one-to-one correspondence between the prime ideals of the quotient algebra  $L^1(G, B)/M_\gamma$  and

the prime ideals of the algebra  $L^1(G, B)$  containing  $M_\gamma$ , there exists a positive integer  $n$  such that  $\underbrace{f * f * \cdots * f}_{n \text{ times}} \in M_\gamma$ . This implies  $(\hat{f}(\gamma))^n = \theta$ . Since  $B$  is semiprime,  $\hat{f}(\gamma) = \theta$ . Hence  $f \in M_\gamma$ .  $\square$

*Proof of Theorem 4.1.* If possible assume that  $\mathfrak{S}$  is a nontrivial separating ideal in  $L^1(G, B)$ .

CLAIM.  $\mathfrak{S}$  is contained in all but finitely many  $M_\gamma$  for  $\gamma \in \Gamma$ .

*Proof of the Claim.* Let  $\mathcal{M}$  be the set of all minimal prime ideals of  $L^1(G, B)$  not containing  $\mathfrak{S}$ . By [3]  $\mathcal{M}$  is a finite set. Let

$$\mathcal{M}_\Delta = \{\mathcal{P} \in \mathcal{M} | \mathcal{P} \subseteq M_{\gamma, \phi} \text{ for some } (\gamma, \phi) \in \Gamma \times \Delta(B)\}$$

and  $\mathcal{M}_{\Delta'} = \mathcal{M} - \mathcal{M}_\Delta$ . By Theorem 3.5, each member of  $\mathcal{M}_\Delta$  contains a unique  $M_\gamma$  for some  $\gamma \in \Gamma$ . Let  $\Gamma_{\mathcal{M}_\Delta} = \{\gamma \in \Gamma | M_\gamma \subseteq \mathcal{P} \text{ for some } \mathcal{P} \in \mathcal{M}_\Delta\}$ . Obviously  $\Gamma_{\mathcal{M}_\Delta}$  is a finite set. Since  $\mathfrak{S}$  is contained in all but finitely many closed prime ideals of  $L^1(G, B)$  ([3]), and since any prime ideal contains a minimal prime ideal, it follows that  $\Gamma_{\mathcal{M}_\Delta}$  is not empty. Let  $\gamma \in \Gamma - \Gamma_{\mathcal{M}_\Delta}$ . By Lemma 4.2,  $M_\gamma = \bigcap_{\mathcal{P} \in \mathcal{I}_\gamma} \mathcal{P}$  where  $\mathcal{I}_\gamma$  is the set consisting of all minimal prime ideals of  $L^1(G, B)$  containing  $M_\gamma$ . Write  $\mathcal{I}_\gamma = \mathcal{I}_\Delta \cup \mathcal{I}_{\Delta'} \cup \mathcal{I}_{\Delta''}$  where

$$\mathcal{I}_\Delta = \{\mathcal{P} \in \mathcal{I}_\gamma | \mathcal{P} \subseteq M_{\gamma, \phi} \text{ for some } \phi \in \Delta(B)\},$$

$$\mathcal{I}_{\Delta'} = \{\mathcal{P} \in \mathcal{I}_\gamma | \mathcal{P} \text{ contains } \mathfrak{S}, \text{ and } \mathcal{P} \not\subseteq M_{\gamma, \phi} \text{ for each } \phi \in \Delta(B)\}$$

and

$$\mathcal{I}_{\Delta''} = \{\mathcal{P} \in \mathcal{I}_\gamma | \mathcal{P} \text{ does not contain } \mathfrak{S} \text{ and } \mathcal{P} \not\subseteq M_{\gamma, \phi} \text{ for each } \phi \in \Delta(B)\}.$$

Notice that  $\mathcal{I}_{\Delta''}$  is at most a finite set, and each  $\mathcal{P}$  in  $\mathcal{I}_\Delta$  contains  $\mathfrak{S}$ . Obviously

$$\mathcal{M} = \left( \bigcap_{\mathcal{P} \in \mathcal{I}_\Delta \cup \mathcal{I}_{\Delta'}} \mathcal{P} \right) \cap \left( \bigcap_{\mathcal{P} \in \mathcal{I}_{\Delta''}} \mathcal{P} \right).$$

In the above, if  $\mathcal{I}_{\Delta''}$  is empty then  $\bigcap_{\mathcal{P} \in \mathcal{I}_{\Delta''}} \mathcal{P}$  is taken to be  $L^1(G, B)$ . Since  $\mathcal{I}_{\Delta''}$  is at most a finite set, and  $M_{\gamma, \phi}$  is a prime ideal for each  $\phi \in$



$\Delta(B), \bigcap_{\mathcal{P} \in \mathcal{I}_{\Delta''}} \mathcal{P} \not\subseteq M_{\gamma, \phi}$ . Let  $f \in \bigcap_{\mathcal{P} \in \mathcal{I}_{\Delta} \cup \mathcal{I}_{\Delta'}} \mathcal{P}$ . Choose  $g \in (\bigcap_{\mathcal{P} \in \mathcal{I}_{\Delta''}} \mathcal{P} \setminus M_{\gamma, \phi})$ . Then  $fg \in M_{\gamma}$ . Since  $\phi(\hat{g}(\gamma)) \neq 0$  for each  $\phi \in \Delta(B)$ , by the assumption on  $B$ ,  $\hat{f}(\gamma)$  belongs to every minimal prime ideal of  $B$ . Since  $B$  is semiprime,  $\hat{f}(\gamma) = \theta$ . Thus  $M_{\gamma} = \bigcap_{\mathcal{P} \in \mathcal{I}_{\Delta} \cup \mathcal{I}_{\Delta'}} \mathcal{P}$ . This implies  $\mathfrak{S} \subset M_{\gamma}$ . This completes the proof of the claim.

For the remainder of the proof, the argument is similar to Theorem 3.3 of [7]. Let  $\Gamma_{\mathcal{M}_{\Delta}} = \{\gamma_1, \gamma_2, \dots, \gamma_n\}$ . Let  $h \in (G \cap (\bigcap_{i=2}^n M_{\gamma_i})) \setminus M_{\gamma_1}$ . Since there exists a minimal prime ideal  $\mathcal{P} \in \mathcal{M}$  contains  $M_{\gamma_1}$  but not any of the  $M_{\gamma_i}$ 's for  $2 \leq i \leq n$ , such a function  $h$  exists. Since  $\hat{h}(\gamma_1) \neq \theta$ , there exists a continuous linear functional  $\lambda$  on  $B$  such that  $\lambda(\hat{f}(\gamma_1)) \neq 0$ . Consider the basic open set

$$N = \{\gamma \in \Gamma : |\lambda(\hat{h}(\gamma)) - \lambda(\hat{h}(\gamma_1))| < |\lambda(\hat{h}(\gamma_1))|\}$$

of  $\Gamma$  containing  $\gamma_1$ . Since  $G$  is a noncompact Abelian group,  $\gamma_1$  is not an isolated point in  $\Gamma$ . By the choice of  $h$ , the characters  $\gamma_2, \gamma_3, \dots, \gamma_n$  do not belong to  $N$ . Hence there exists a character  $\gamma_0 \in \Gamma \setminus \{\gamma_1, \gamma_2, \dots, \gamma_n\}$  such that  $\gamma_0 \in N$ . Since  $\mathfrak{S}$  is contained in  $M_{\gamma_0}$ ,  $\hat{h}(\gamma_0) = \theta$ . Hence  $|\lambda(\hat{h}(\gamma_1))| = |\lambda(\hat{h}(\gamma_1)) - \lambda(\hat{h}(\gamma_0))| < |\lambda(\hat{h}(\gamma_1))|$ . This is a contradiction. Therefore  $L^1(G, B)$  does not contain a non-trivial separating ideal.  $\square$

The following result extends Theorem 3.3 of [7] (which in turn extends Theorem 5 of [11]) to some semiprime Banach algebras which do not possess the multiplicative identity.

**THEOREM 4.3.** *Let  $G$  be a noncompact locally compact Abelian group, and  $B$  be a commutative semiprime Banach algebra in which every minimal prime ideal is contained in a regular maximal ideal. Then every derivation on  $L^1(G, B)$  is continuous. Also every epimorphism from a commutative Banach algebra onto  $L^1(G, B)$  is continuous.*

*Proof.* Obviously follows from Theorem 4.1 and the closed graph theorem.  $\square$

REMARK. If  $B$  has the multiplicative identity then every proper prime ideal is contained in a maximal ideal of  $B$ . Even if  $B$  does not have the multiplicative identity, in most of the algebras every minimal prime ideal is contained in a regular maximal ideal. Therefore the assumption in the above theorem that every minimal prime ideal contained in a regular maximal ideal of the algebra is not too restrictive.

## References

- [1] W. G. Bade and P. C. Curtis, Jr., *Prime ideals and automatic continuity for Banach algebras*, J. Funct. Anal. **29** (1978), 88–103.
- [2] P. C. Curtis Jr., *Derivations on commutative Banach algebras*, in Proceedings, Long Beach, 1981, Lecture Notes in Math. (Springer-Verlag, Berlin, Heidelberg, New York) **975** (1983), 328–333.
- [3] J. Cusack, *Automatic continuity and topologically simple radical Banach algebras*, J. London Math. **16** (1977), 493–500.
- [4] H. G. Dales, *Automatic continuity: A survey*, Bull. London Math. Soc. **10** (1978), 129–183.
- [5] J. Diestel and J. J. Uhl, *Vector measures*, vol. 15, Math surveys, Amer. Math. Soc., Providence, RI, 1977.
- [6] R. Garimella, *On nilpotency of the separating ideal of a derivation*, Proc. Amer. Math. Soc. **117** (1993), 167–174.
- [7] ———, *On continuity of derivations and epimorphisms on some vector-valued group algebras*, Bull. Austral. Math. Soc. **56** (1997), 209–215.
- [8] A. Hausner, *The Tauberian Theorem for Group algebras of vector-valued functions*, Pacific J. Math. **7** (1957), 1603–1610.
- [9] G. P. Johnson, *Space of function with values in a Banach algebra*, Trans. Amer. Math. Soc. **92** (1959), 411–429.
- [10] M. M. Neumann, *Automatic continuity of linear operations in functional analysis*, vol. 38, Surveys and recent results II, North-Holland Math. Studies (North-Holland, Amsterdam, New York), 1980, pp. 269–296.
- [11] M. M. Neumann and M. V. Velasco, *Continuity of epimorphisms and derivations on vector-valued group algebras*, vol. 68, Arch. Math. (Basel), 1997, No. 2, pp. 151–158.
- [12] W. Rudin, *Fourier analysis on groups*, Interscience, New York, London, 1962.
- [13] A. M. Sinclair, *Automatic continuity of linear operators*, vol. 21, London Mathematical Society, Lecture Notes Series, Cambridge Univ. Press, London/New York, 1976.

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