Comparison of Genetic Algorithms and Simulated Annealing for Multiprocessor Task Allocation

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ABSTRACT

We present two heuristic algorithms for the task allocation problem (NP-complete problem) in parallel computing. The problem is to find an optimal mapping of multiple communicating tasks of a parallel program onto the multiple processing nodes of a distributed memory multiprocessor. The purpose of mapping these tasks onto the nodes of the target architecture is the minimization of the parallel execution time without sacrificing solution quality. Many heuristic approaches have been employed to obtain satisfactory mapping. Our heuristics are based on genetic algorithms and simulated annealing. We formulate an objective function as a total computational cost for a mapping configuration, and evaluate the performance of our heuristic algorithms. We compare the quality of solutions and times derived by the random, greedy, genetic, and annealing algorithms. Our experimental findings from a simulation study of the allocation algorithms are presented.
by Bokhari[1]. The purpose of mapping multiple
tasks into the multiprocessor nodes of the parallel
architecture is the minimization of the parallel
execution time without sacrificing solution quality.
To minimize run time, tasks should be evenly
distributed across the nodes, while the communica-
tion cost in message passing among processors
should be minimized. The mapping objective is An
objective function is defined to formalize the map-
ning goal as a minimization process. In general,
obtaining an optimal solution of the mapping prob-
lem is computationally intractable: the mapping problem
is known to be NP-complete[1,6]. Therefore, heu-
ristic approaches are commonly employed to obtain a
satisfactory near-optimal solution in a reasonable
time. Task allocation in distributed-memory multi-
processor systems means a mapping of a given
problem to the target system. Task allocation con-
stitutes of partitioning the problem into a set of disjoint
subproblems (tasks) and allocating these tasks
to the processors of the parallel architecture in such
a way the total computational cost is minimized.

In this paper, we present two allocation algo-
rithms. Our algorithms are based on the two types
of stochastic search and optimization techniques:
Genetic Algorithm (GA's)[7,8] and Simulated
Annealing (SA)[10]. These two techniques are mod-
eled on processes found in thermodynamics, genetics,
and natural evolution, and are being used in artificial
intelligence systems. They have been applied suc-
cessfully to difficult NP-complete problems[2,3,4,8,
9,11,12,13,14,15,16]. This study is motivated by the
fact that there is a lack of comparative studies of
GAs and SA. We explore the connections between
these heuristics. We also compare the mapping qual-
ities and times derived by the genetic and annealing
algorithms against those derived by a random and a
greedy allocation algorithms. A simulation is de-
veloped to evaluate the performance of the allocation
algorithms.

We review previous task allocation strategies. The
task allocation problem and some of its derivatives
are NP-complete. Therefore, heuristic algorithms that
approximate optimal solutions have been developed.
Some of these approaches[1,2,5,9,12,15,17,18] dealt
in a similar manner, with the mapping of the problem
graph (a set of communicating processes) onto a target
architecture with a fixed interconnection topology.

Bokhari[1] proposes a mapping scheme of distrib-
uted processors that uses two input adjacency ma-
trices to represent the problem graph (the job mod-
ules and the intercommunications) onto the system
graph (the processors and the interconnections), and
then applies an exhaustive pairwise exchange of two
job modules. The objective function used is to
maximize graph cardinality— the number of matched
edges in the problem that fall on the links on the
system graph. The basic assumption in the scheme
is that all the problem edges are considered iden-
tical, i.e., they have the same weight. However,
more general problem graph may have different
weights on edges.

Lee and Aggarwal[11] extend the Bokhari’s
approach by incorporating a set of objective func-
tions that accurately quantify communication over-
head into the problem. The optimality of mapping
the problem graph onto the system graph is eval-
uated by the objective functions with a more repre-
sentative communication measure. They
has developed a mapping algorithm based on the
objective functions, where they first makes an initial
assignment and then iteratively apply a pairwise
exchange scheme to the initial assignment. The
approach is still restrictive; it utilizes a fixed path
routing scheme for the network traffic.

Bollinger and Mickiff[2] formulate a two-phase
mapping strategy to map a logical system onto a
physical architecture using the simulated annealing
algorithm, where the first phase, process annealing,
assigns parallel processes onto processing nodes and
the connection annealing phase schedules traffic con-
sections on network data links so as to minimize interprocess communication conflicts. Objective functions that accurately quantify communication cost are derived to evaluate the quality of generated mapping. This effort improved upon [11] in which it utilizes the information concerning the actual routing rules.

Du and Maryanski[5] attack a variation of the mapping problem. This variation concerns the allocation of data in a dynamically reconfigurable environment. The allocation algorithm employs a set of "benefit" functions and a greedy search algorithm. The underlying execution architecture is based on a client/server model, a heterogeneous system. Although their problem closely resembles our data allocation problem, as the underlying architectural model significantly differs from our parallel execution environment, their assumptions are not relevant to our problem.

Driesche and Piessens[4] have studied the genetic algorithm for static load balancing. They present that combining genetic algorithms with simple heuristics can drastically improve the efficiency. In their paper, the usefulness of genetic algorithms in the context of dynamic load balancing is not assessed.

Mansour and Fox[12] have proposed sequential genetic algorithm for the task allocation problem in parallel computing. The cost function used is a quadratic objective function. Their problem more closely resembles our problem but our efforts are based on genetic and simulated annealing algorithms.

Hong and McMillin[9] have applied the cost measurement and error tolerance scheme based on the hill climbing nature to the composite stock cutting problem in an Intel iPSC/2 multicomputer. The asynchronous simulated annealing algorithm they proposed is based on the spatial decomposition method. Their experimental results show that the parallel algorithm results in packing densities as almost same as the sequential algorithm does with almost linear speedup.

Woodside and Montorton [18] generalize a heuristic solution based on bin-packing for finding load balanced allocations of independent tasks to multiprocessors. They introduce communicating tasks into the algorithm which are to be allocated onto a bus-connected processors and present a static allocator which could be incorporated into an automated compiler for distributed systems.

The remainder of this paper is organized as follows. Section 2 formulates the task allocation problem studied in this work. Sections 3 and 4 describe our genetic and annealing algorithm approaches to the given problem. In Section 5, we compare the solution quality derived by our heuristic algorithms against those derived by a random and a greedy allocation algorithms, and the simulation results of the algorithms are presented. Finally, the conclusions and future work are given in Section 6.

2. The Task Allocation Problem

The mapping problem we study in this work is formulated. We first define terms and notations to be used. A parallel program can be modeled by a weighted task graph, \( G(V_c, E_c) \), in which vertices, \( V_i = \{T_1, T_2, \ldots, T_d\} \), denote the tasks of a parallel program and undirected edges, \( E_c = \{(T_i, T_j) \mid 1 \leq i, j \leq d\} \), represent interaction between tasks. Each vertex of \( G \) is assigned a weight \( w_i \) which denotes the computation cost of the task \( T_i \). Each edge is assigned a weight \( w_{ij}(T_i, T_j) \) denoting the amount of interaction between tasks \( T_i \) and \( T_j \) for \( 1 \leq i, j \leq d \).

A parallel architecture is represented by an undirected processor graph \( G_p(V_p, E_p) \) where \( V_p = \{P_1, P_2, \ldots, P_p\} \) and \( E_p = \{(P_m, P_n) \mid 1 \leq m, n \leq p, m+n\} \). The vertices \( V_p \) represent the processors of the target multicomputer, and the edges \( E_p \) indicate the bidirectional communication links. In a parallel system, mapping leads to partitioning the task graph into subgraphs allocated to the processors. Tasks are partitioned into as many equally weighted clus-
ters as the number of processors. Each cluster is then assigned by a one-to-one mapping to a processor of the multiprocessor, and so the interprocessor communication cost is minimized.

Given a task graph $G_t(V_t,E_t)$ and a processor graph $G_p(V_p,E_p)$, the allocation problem consists of finding a mapping scheme $F: V_t \rightarrow V_p$, which maps the $t$ tasks of the graph $G_t$ to the $p$ processors of $G_p$, and minimizes the computation and communication cost. Let the set of vertices assigned to a cluster be $R(h)$, i.e., $R(h) = \{ T_i \in V_t : F(T_i) = h, 1 \leq i \leq t \}$. The computation cost (for weight $w_i$) of every cluster can be expressed as

$$W(h) = \sum_{T_i \in R(h)} w_i$$  \hspace{1cm} (1)$$

The communication cost of all the edges from a cluster is given by

$$C(h) = \sum_{T_i \in R(h)} \sum_{T_j \in R(h)} w_e(T_i, T_j)$$  \hspace{1cm} (2)$$

An objective function which estimates the total parallel execution time including the computation and communication cost for a mapping configuration, is defined as

$$OF = \sum_{n} \sum_{h} W(P_n^h) + \beta \sum_{n} \sum_{m} C(P_n^h)$$  \hspace{1cm} (3)$$

$W(P_n^h)$ is the computation workload of node $P_n$ that is, $W(P_n^h) = \text{Max}_n(P_n^h)$ for all $n, 1 \leq n \leq p$ and $h \leq c$, where $P_n^h$ is the number of tasks of cluster $C_n$ allocated to node $P_n$. $C(P_n^h)$ is the interprocessor communication cost (matrix) between node $n$ and node $m$, specified as $C(P_n^h) = $ Max$(P_n^h M_{P_n^h}(F(V_t))))$, $V_t, V_j \subseteq C_n F(V_t)$ is the processor number in the range 0 to $|V_t| - 1$, onto which the task $i$ is mapped. $\beta$ is a constant representing the relative importance of communication with respect to computation and the cost of unit computation/cost of unit communication in a machine.

3. A Genetic Algorithm Approach

Genetic algorithms simulate the survival of the fittest among individuals in nature over generations for solving a problem. Each generation consists of a population of individuals, a set of character strings. Each individual represents a point in the search space and a possible solution. We use a genetic algorithm based approach with a distributed population model [11, 12] which has the advantage of implicit parallelism and reduces the possibility of premature convergence. The population model involves partitioning the population into individual subpopulations. Isolated evolution with interaction among subpopulations takes place over successive generations. Our genetic mapping algorithm consists of four phases—initialization, reproduction, crossover, and mutation.

3.1 Representation

In the coding scheme, the set of tasks (a finite-length string) is represented by a task vector which is a sequence of integers ranging from 0 to $d - 1$. A permutation of the sequence defines an assignment of the tasks onto the nodes. A task entry $D_i$ found at position $i$ of a vector represents an assignment of task $D_i$ onto node $X_m$, where $m = i$ modulus $n$, and $n$ is the number of nodes. As an example, if we have a vector (1, 10, 2, 5, 8, 4, 6, 9, 7, 3, 0), assuming that we have four processors, the vector leads to the following assignment: Node 0 contains tasks: $D_1$, $D_5$, and $D_9$; Node 1 contains tasks: $D_0$, $D_4$, and $D_7$; Node 2 contains tasks: $D_2$, $D_8$, and $D_6$; Node 3 contains tasks: $D_3$ and $D_6$.

3.2 Initialization

The first step of the algorithm is to initialize the population of individuals. In the initialization phase, a set of random permutations of the task vector is uniformly generated. Each permutation represents a possible allocation of the tasks onto the nodes.

A near-optimal allocation is generated by repetitively...
modifying the permutations. A permutation matrix, $P_0$ ($0 \leq i \leq n - 1, 0 \leq j \leq d - 1$) is created. Every row of $P$, $P_0$ ($0 \leq i \leq n - 1$) is a complete permutation of all tasks $D_k$ ($0 \leq j \leq d - 1$). We define the mapping function, $f_i : D \mapsto X$ for any given row of $P$, $P_0$ ($0 \leq i \leq n - 1$) as $f(D_k) = j \mod n$, where $j$ is the index in row $P_i$ of task $D_k$, ($0 \leq k \leq d - 1$).

(Table 1) The permutation matrix $P_{0}$

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>10</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>4</td>
<td>6</td>
<td>9</td>
<td>7</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>2</td>
<td>4</td>
<td>10</td>
<td>6</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>0</td>
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<td>1</td>
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<tr>
<td>2</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>9</td>
<td>6</td>
<td>1</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>8</td>
<td>7</td>
<td>10</td>
<td>6</td>
<td>0</td>
<td>9</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

If $n = 4$, row $P_0$ implies that tasks 0 through 10 are mapped to nodes 2, 0, 2, 1, 1, 3, 2, 0, 0, 3, 1, respectively.

3.3 Reproduction

The reproduction phase selects a new set of task allocations for use in the next generation using the OF. The selection process is based on the goodness/fitness value of the current permutations. The allocation with a higher value of goodness has a higher probability of producing one or more offspring in the next generation. Upon the completion of each reproduction phase, the old, poor allocations are replaced by the birth of the new, good permutations.

3.4 Crossover

The crossover phase represents the cross fertilization of permutations similar to the composition of genes from both parents in a birth. It consists of a position-wise exchange of values between each randomly paired permutations. Two point crossover is performed on a pair of individuals by swapping contiguous segments of genes. The segment boundaries are randomly selected and are the same in both parents. Two random numbers are chosen and serve as the bounds for the position-wise exchange. Each task of the first permutation which falls within the determined bounds is swapped with the corresponding task of the second permutation, and likewise the second permutation with the first.

3.5 Mutation

The mutation phase is incorporated into the algorithm to prevent premature local convergence in the population. The mutation rate is designated by the probability of mutation. During this phase, a permutation is randomly modified with a low probability: a pair of tasks in an allocation is position-wise swapped.

3.6 Termination Condition

The termination/convergence condition is reached if all permutations are identical or if the number of generations is greater than a predetermined maximum generation limit. In our experimentation, the number of generations is obtained for five different runs and the maximum limit was set at 1500. The number of generations is implementation-dependent and must be specified carefully to obtain the best solution quality.

We now combine all the processes above to form the complete genetic algorithm for mapping.

Algorithm 1.

1. Initialization Randomly generate initial population of individuals.
2. Repeat steps 3-7 until the algorithm terminates
3. Evaluate goodness of individuals in population.
4. Reproduction - Select the string with the highest goodness value
   a. Crossover - Pick two strings and position-wise swap with a probability of crossover
   b. Mutation - Randomly modify the string with a probability of mutation
5. Preserve the best solution so far.

4. A Simulated Annealing Approach

Our simulated annealing solution to the task allocation problem is based on the SA algorithm [13] that consists of annealing steps for producing the
Algorithm 2.
1. Set an initial temperature $T_{\text{ini}} = T_{\text{ini}}$. 
2. Set an initial configuration $S = S_0$. 
3. Calculate the cost value $C = \text{calculate}(S)$. 
4. While $\nabla C < 0$ (frozen termination condition) do 
5. Determine the vertices $V_{\text{rej}}$ to be moved. 
6. While (not yet in equilibrium) do 
7. Generate new configuration $S' = \text{perturb}(S)$. 
8. Calculate new cost value $C' = \text{calculate}(S')$. 
9. Calculate the cost difference $\Delta C = C' - C$. 
10. If $\Delta C < 0$ 
11. then $S = S'$ 
12. else $S = S'$ with $(e^{-\Delta C/T_{\text{new}}}) > \text{random}(0,1)$; 
13. End while (with step 6). 
14. Reduce temperature $T_{\text{new}}$. 
15. End while (with step 4).

In the algorithm, a move (perturbation) is accomplished by a random remapping of a randomly chosen configuration. A remapping that leads to a lower or identical cost is always accepted, whereas increase in the cost is only allowed with the probability $e^{-\Delta C/T_{\text{new}}}$ known as the Metropolis criterion. Acceptance probabilities of moves are controlled by a temperature $T_{\text{new}}$. The algorithm uses Eq. (3) as the cost function.

In the SA implementation, the cooling schedule policy must be specified carefully. The initial configurations are obtained by a random allocation of tasks among processors. The initial temperature is then determined such that the acceptance probability of uphill moves in the cost function is initially 0.9. Equilibrium is detected by sampling cost dynamically as the assignment is perturbed. The equilibrium at a temperature means the probability distribution of configurations has reached a steady state. Temperature is decreased by the cooling schedule in Eq. (4), making small changes in temperature. An exponential cooling schedule is used because the use of logarithmic cooling schedules requires too much computation time.

\[ T_{\text{new}}(i+1) = C \cdot T_{\text{new}}(i) \]  

where $C < 1$ and commonly very close to 1. The constant was set to 0.98 in our implementation. $T_{\text{new}}(i)$ is the current temperature that will be decreased. Eq. (4) determines the next temperature as a fraction $C$ of the present one. SA is considered converged if one of the following two conditions is satisfied: (1) if the number of accepted moves is zero, or (2) if no further progress in the mapping quality is made for a given number of annealing steps.

5. Experimental Results

We implemented and tested Algorithms 1 and 2 discussed in Sections 3 and 4 on random task graphs, respectively. The results of our mapping algorithms are favorably compared against those of a random and a greedy algorithms. The random algorithm assigns each task cluster on a randomly selected processor with no regard to computation times or communication requirements. The greedy algorithm is based on a best fit mapping strategy, where the clusters are listed in order of increasing cost, $c_0 \leq c_1 \leq \cdots \leq c_n$ and then the algorithm finds the smallest $i$ such that $c_i \leq c$, where $c$ is the cost of the cluster set to be assigned. All these algorithms were implemented in C on a Sparc workstation. Our experimental findings from a simulation study of the algorithms are reported. The total value of OF of Eq. (3) is used to assess the quality of the solutions produced by the algorithms. The performance measures are the solution quality and the CPU time for algorithm execution.

Vertices $V_i$ of task graphs are randomly generated and mapped into 16-node hypercubes. The computation and communication costs are generated randomly. Each vertex ($T_i$, $1 \leq i \leq t$) has an integer weight ranging from 1 to 10. $T_i$ is connected to randomly selected vertices that will be in the range 1 to 16. Edges $E_i$ are weighted randomly with
integer values between 1 and 10. Throughout the simulations, the following GA parameter values were used: A total population size = 50, the crossover probability = 0.85, the mutation probability = 0.02, and the maximum generation limit = 1500. The parameter setting for SA was specified in the previous section.

(Table 2) Comparison of four algorithms with different task graphs

<table>
<thead>
<tr>
<th>(V, E)</th>
<th>Random</th>
<th>Greedy</th>
<th>Genetic</th>
<th>Annealing</th>
</tr>
</thead>
<tbody>
<tr>
<td>(200,1000)</td>
<td>7617</td>
<td>7455</td>
<td>6785</td>
<td>6811</td>
</tr>
<tr>
<td>(300,2000)</td>
<td>14450</td>
<td>14450</td>
<td>12847</td>
<td>12309</td>
</tr>
<tr>
<td>(400,4000)</td>
<td>38052</td>
<td>36714</td>
<td>31754</td>
<td>31487</td>
</tr>
</tbody>
</table>

(Table 3) Comparison of four algorithms with different task graphs

<table>
<thead>
<tr>
<th>(V, E)</th>
<th>Random</th>
<th>Greedy</th>
<th>Genetic</th>
<th>Annealing</th>
</tr>
</thead>
<tbody>
<tr>
<td>(400,1000)</td>
<td>9408</td>
<td>9055</td>
<td>8516</td>
<td>8394</td>
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<tr>
<td>(400,2000)</td>
<td>17308</td>
<td>17549</td>
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<td>(400,4000)</td>
<td>38052</td>
<td>36714</td>
<td>31754</td>
<td>31487</td>
</tr>
</tbody>
</table>

(Table 2) compares the cost of the solutions derived by the random, greedy, genetic, and annealing algorithms for several different problem instances. The figures here are the average values of five different runs. The comparison results in (Table 2) represents the best solutions obtained by the four algorithms in these multiple runs. (Table 3) also compares the cost of the solutions derived by the four algorithms for different problem instances. However, the same value (400) of V, was used for each problem with different E.

(Table 4) Time (in sec.) of four algorithms with different task graphs

<table>
<thead>
<tr>
<th>(V, E)</th>
<th>Random</th>
<th>Greedy</th>
<th>Genetic</th>
<th>Annealing</th>
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<tbody>
<tr>
<td>(400,1000)</td>
<td>42</td>
<td>96</td>
<td>144</td>
<td>187</td>
</tr>
<tr>
<td>(400,2000)</td>
<td>74</td>
<td>166</td>
<td>298</td>
<td>314</td>
</tr>
<tr>
<td>(400,4000)</td>
<td>82</td>
<td>180</td>
<td>453</td>
<td>566</td>
</tr>
</tbody>
</table>

(Fig. 1) demonstrates the relatively negligible effects of mutation on the allocation derived for (400, 4000). The cost for different generations. The values of mutation rates used were in the range 0.005-0.05. The results indicate that lower cost of the GA is achieved with the larger number of generations and the impact of varying the mutation rate is minimal. For additional experimental results, see [14].

The performance obtained using GA and SA approaches is comparable. From Table 2, 3, and 4, we can see that the solution quality by the genetic method are slightly better in smaller problems but the annealing solution is better in a larger problem. Genetic algorithms search from a population of
points, while only a single point in simulated annealing is perturbed. The time consuming evaluation of the objection function can be done in parallel for a whole population, as can the reproduction of offspring individual solutions. The implicit parallelism of genetic search tends to evolve good solutions in a shorter time. The genetic search time can be reduced by parallelizing the sequential GA on multiprocessor systems. It has been shown that, in prior studies[11] near linear speedups can be observed by an asynchronous parallel GA. In distributed-memory multicomputers, the scalability of SA is lower than that of GA due to inherent global synchronization involved.

(Fig. 2)

(Figure 2) illustrates the behavior of the genetic and annealing algorithms. The cost of the solutions is plotted against the number of OF evaluations. Each of the algorithms approaches a suboptimal solution when the fluctuating cost is stabilized. The genetic approach gives greater initial improvements than the annealing one. The annealing runs tend to converge late in comparison with the genetic. The results show that the genetic approach makes initial improvements larger and the progress of annealing is consistent with the larger number of evaluations.

6. Conclusions

We have studied the task allocation problem (an NP-complete problem) in parallel computing. We have presented two heuristic solutions based on genetic algorithms and simulated annealing. We evaluated the performance of our allocation algorithms by orders of magnitude, and compared the quality of solutions derived by the random, greedy, genetic, and annealing algorithms. Experimental results from a simulation study of the algorithms were obtained. The solution quality derived by the genetic or annealing algorithm was found to be superior to those of either the random or the greedy. The performance obtained using the genetic or annealing approaches is comparable. The genetic approach makes initial improvements greater and the annealing process is consistent with the larger number of iterations. In our ongoing research, we are developing a hybrid algorithm which combines the benefit of genetic and annealing algorithms. Related research works along with this direction can be found in [15, 16].

References


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