

# P-e 곡선의 타원 특성을 이용한 전력계통 최대허용부하의 예측

論 文

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## Estimation of Maximum Loadability in Power Systems By Using Elliptic Properties of P-e Curve

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**Abstract** - This paper presents an efficient algorithm to estimate the maximum load level for heavily loaded power systems with the load-generation variation vector obtained by ELD (Economic Load Dispatch) and/or short term load forecasting while utilizing the elliptic pattern of the P-e curve. It is well known the power flow equation in the rectangular coordinate is fully quadratic. However, the coupling between e and f makes it difficult to take advantage of this quadratic characteristic. In this paper, the elliptic characteristics of P-e curve are illustrated and a simple technique is proposed to reflect the e-f coupling effects on the estimation of maximum loadability with theoretical analysis. An efficient estimation algorithm has been developed with the use of the elliptic properties of the P-e curve. The proposed algorithm is tested on IEEE 14 bus system, New England 39 bus system and IEEE 118 bus system, which shows that the maximum load level can be efficiently estimated with remarkable improvement in accuracy.

**Key Words** : Voltage stability, Maximum loadability, Ellipse approximation, P-e curve, e-f coupling effects

### I. Introduction

This paper presents a new approach to derive an energy integral reflecting transmission-line resistances and flux-decaying effects on the basis of an Equivalent Mechanical Model (EMM) for stability analysis of multimachine power systems.

The load limit concerned with voltage stability gives more precise information to the operators rather than the stability indices representing the proximity to the collapse point in some sense. In this respect, the CPFLOW (Continuation Power Flow) method has been introduced to calculate the maximum loadability in spite of its time-consuming iterative process [13-15]. This drawback of CPFLOW method can be overcome by significantly reducing the number of iteration with the aid of accurate estimation of the maximum loadability.

Chiang et al. presented an efficient method to calculate load and voltage margins, named look-ahead method. They have shown that on-line monitoring of voltage stability is possible to consider a number of contingency cases for the large power systems [16]. The feature of the look ahead

method is the fast estimation of maximum load level ( $\lambda_{max}$ ) by using the second order approximation of the power flow equation. Chiang's method is very useful for practical applications, while it has some room for improvement regarding the estimation accuracy. Since it adopted the polar coordinate with the state variables  $[\underline{V}, \underline{\theta}]$ , the truncation error in the second order approximation of the power flow equations degrades the estimation accuracy.

This paper presents an improved method to estimate the maximum load level by using the P-e curve rather than the P-V curve. The power flow equations can be represented by the pure quadratic form in terms of state variables [e, f] with respect to the rectangular coordinate. However, the P-e curve has the coupling between variable e and f, which makes the curve pattern somewhat unusual. Consequently, it is almost impossible to improve the accuracy in the maximum loadability estimation unless the e-f coupling is efficiently treated in the estimation procedures.

This paper shows first that the e-f coupling can be removed in a simple manner when a pair of power flow solutions are obtained, and next that it is possible to take into account the e-f coupling effects by using the distorted elliptic pattern for the high-voltage solution part of the P-e curve. This paper proposes a new algorithm to estimate  $\lambda_{max}$  by applying the curve fitting technique to the P-e curve with the use of the power flow solutions at

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three different load levels. Here, it is noted that the proposed algorithm does not require any unstable power flow solution.

The proposed algorithm has been tested for the IEEE 14-bus system, New England 39-bus system and the IEEE 118-bus system, which shows that the estimation accuracy can be remarkably improved and the iteration number in the max estimation can be considerably reduced.

## II. Elliptic Properties of P-e Curve

The elliptic characteristic of P-e curve with respect to the system load is illustrated with a simple 2-bus system and the e-f coupling is discussed. A simple technique is developed to remove the e-f couplings in multi-machine systems.

### The elliptic characteristics of P-e curve

A 2-bus power system is shown in Fig.1. Say that the generator internal voltage behind the reactance is constant with its angle being equal zero.

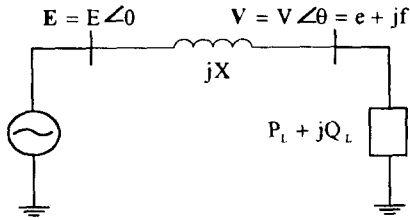


Fig. 1 2-bus power system

The real and reactive power equations for the above system are given by

$$P_L = -\frac{E}{X}V \sin \theta = -\frac{E}{X}f \quad (1)$$

$$Q_L = \frac{-V^2 + EV \cos \theta}{X} = \frac{-(e^2 + f^2) + E \cdot e}{X} \quad (2)$$

Equation (1) shows that  $f$ , which is the imaginary part of  $V$ , is directly proportional to real power load  $P_L$  and it can be represented by

$$f = KP_L \quad (3)$$

Provided that the system load increases with a constant power factor, one may assume the reactive power load to be  $Q_L = \alpha P_L$ . With the use of these relationships, (2) can be rewritten as:

$$\alpha P_L = \frac{-(e^2 + K^2 P_L^2) + E \cdot e}{X} \quad (4)$$

By rearranging the above equation, we can obtain the following elliptic equation for the P-e curve shown in Fig. 2:

$$e^2 - E \cdot e + K^2 P_L^2 + \alpha X P_L = 0 \quad (5)$$

Here we should emphasize that if the generator rotor angle is assumed not to be zero there exists the coupling of e-f, which makes the ellipse to be distorted. Moreover, the e-f coupling breaks the proportionality between  $P$  and  $f$ , producing another distorted elliptic curve. (The equations are not described here since it can be easily derived.)

### Removing the e-f coupling

The rectangular coordinate variables  $e$  and  $f$  can be represented in terms of  $V$ ,  $\theta$  as follows:

$$e_i = V_i \cos \theta_i \quad (6.a)$$

$$f_i = V_i \sin \theta_i \quad (6.b)$$

where  $i = 1, 2, \dots, n$

We assume that the e-f coupling for bus  $i$  results from the choice of the reference bus, and that there is a fictitious bus which can play the role of the generator bus in Fig.1 for bus  $i$ . If we let the phase angle of the fictitious bus be  $\theta_{is}$ , then we may remove the e-f coupling by rotating the phasor voltage  $V_i$  by  $-\theta_{is}$  as shown in Fig. 2. Let  $e'$  and  $f'$  denote the new rectangular components of phasor voltage  $V_i$  after rotating. Then, the relationships between  $(e, f)$  and  $(e', f')$  can be given as follows:

$$\begin{bmatrix} e_i \\ f_i \end{bmatrix} = \begin{bmatrix} \cos \theta_{is} & \sin \theta_{is} \\ -\sin \theta_{is} & \cos \theta_{is} \end{bmatrix} \begin{bmatrix} e_i \\ f_i \end{bmatrix} \quad (7)$$

In the case where a pair of load flow solutions (a SEP and an UEP) at bus  $i$  are given, the rotation angle  $\theta_{is}$  can be efficiently calculated as follows:

Let  $e_{i1} + jf_{i1}$  and  $e_{i2} + jf_{i2}$  denote a stable and an unstable solutions respectively at a specific load level, and assume that the e-f coupling be removed by the rotation of angle  $\theta_{is}$ . Then, variables  $f'_{i1}$  and  $f'_{i2}$ , both of which are directly proportional to  $P_{Li}$ , should be same by (1). Therefore, we can obtain the following equation from (7):

$$\begin{aligned} f_{i1}' &= -e_{i1} \sin \theta_{is} + f_{i1} \cos \theta_{is} \\ &= -e_{i2} \sin \theta_{is} + f_{i2} \cos \theta_{is} = f_{i2}' \end{aligned} \quad (8)$$

From the above equation (8), rotating angle  $\theta_{is}$  can be easily calculated as

$$\theta_{is} = \tan^{-1} \left( \frac{f_{i1} - f_{i2}}{e_{i1} - e_{i2}} \right) \quad \text{for bus } i \quad (9)$$

Here, it should be noted that the rotation angle  $\theta_{is}$  can be calculated for each bus  $i$ .

By using (9) and (7), the transformed rectangular voltages ( $e'$ ,  $f'$ ) can be calculated to remove the  $e$ - $f$  coupling. The transformed variable  $e'$  provides an undistorted ellipse for the P- $e$  curve.

As a result, it is possible to improve the accuracy in the estimation of the maximum loadability by utilizing the elliptic characteristics of the P- $e$  curve for the transformed variable  $e'$ . The removing of  $e$ - $f$  coupling is demonstrated in Fig. 3 for the New England 39 bus system. The Scenario 2) in section IV is used as a load-generation variation scheme. As shown in Fig. 3, the  $e$ - $f$  coupling can be efficiently removed by the proposed method and the transformed P- $e$  curve gives an almost untwisted ellipse around the collapse point.

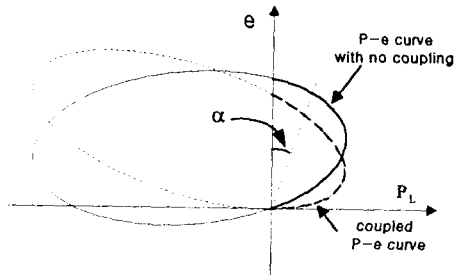


Fig. 2 P- $e$  curve with the consideration of  $e$ - $f$  coupling

Although this method is very interesting from the theoretical point of view, it is not practically useful since the calculation of an unstable solution usually requires long computation time, which is drastically increased with the system size. In this respect, we have shortly observed the theoretical significance of the maximum load level estimation using the transformed P- $e$  curve. In order to avoid the time-consuming calculation of unstable load flow solution, an alternative scheme has been developed to estimate the maximum loadability by using the distorted elliptic function for the P- $e$  curve as discussed in the next section.

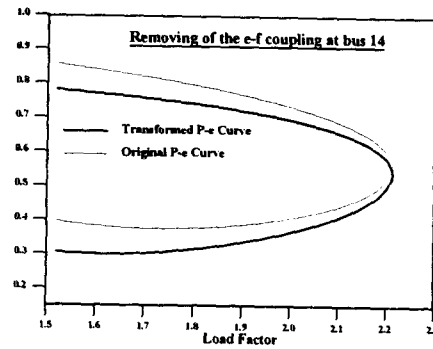


Fig. 3 Removing of the  $e$ - $f$  coupling in New England 39 bus system

### III. Rectangular-Coordinate Formulation of Maximum Loadability Estimation

Consider a static power flow equation with respect to rectangular coordinate variables as follows:

$$\mathbf{f}(\mathbf{x}) - \lambda \mathbf{b} = \mathbf{0} \quad (10)$$

where

$$\mathbf{x} = [\underline{e}^T, \underline{f}^T]^T \in \mathbb{R}^n$$

$\mathbf{f}(\mathbf{x}) \in \mathbb{R}^n$  : real quadratic functions

$\lambda \in \mathbb{R}^1$  : load level parameter

$\mathbf{b} \in \mathbb{R}^n$  : direction vector

The direction vector  $\mathbf{b}$  can be determined by the distribution of load-generation to each bus corresponding to the near-term bus wise forecasting of load demands and the ELD results. Here, it is noted that the load flow equation (1) has the quadratic form.

The point of this section is to find the maximum loadability associated with load-generation direction vector  $\mathbf{b}$  at the current operating point. With the use of  $\lambda_{\max}$ , we can compute the direct load margin (i.e. MW and/or MVAR) between the current operating point and the saddle - node bifurcation point. First, we can compute three load flow solutions, say  $x_1$ ,  $x_2$  and  $x_3$ , corresponding to different load levels  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  ( $\lambda_1 < \lambda_2 < \lambda_3$ ).

We can also compute the derivatives of the state variables with respect to the parameter  $\lambda$ , say  $\dot{\mathbf{x}}_1$ ,  $\dot{\mathbf{x}}_2$  and  $\dot{\mathbf{x}}_3$ . In order to calculate these derivatives, (10) can be linearized around the solution point  $(\mathbf{x}_0, \lambda_0)$  as follows:

$$\mathbf{f}_x \Delta \mathbf{x} - \mathbf{b} \Delta \lambda = \mathbf{0} \quad (11)$$

where  $\Delta \lambda$  and  $\Delta \mathbf{x}$  are the increments of variables, i.e.,  $\lambda - \lambda_0$  and  $\mathbf{x} - \mathbf{x}_0$ , respectively.

If the Jacobian matrix  $\mathbf{f}_x$  is non-singular, then the derivative vector  $\dot{\mathbf{x}}$  can be directly calculated from (11).

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{d\lambda} \cong \frac{\Delta \mathbf{x}}{\Delta \lambda} = \mathbf{f}_x^{-1} \mathbf{b} \quad (12)$$

In the above equation, the Jacobian is always nonsingular until the collapse point is reached by the solution algorithm.

Once the load flow solutions and their derivatives are calculated, then we can estimate the maximum load level  $\lambda_{\max}$  by utilizing the elliptic characteristics of the P-e curve at one of the weakest buses in the sense of voltage stability. The weakest load bus can be easily selected by observing the voltage sensitivity with respect to the change of  $\lambda$ .

The relative voltage sensitivity for each bus can be approximately calculated as follows:

$$\frac{1}{V_i} \frac{\partial V_i}{\partial \lambda} \approx \frac{(V_i|_{\lambda=\lambda_3} - V_i|_{\lambda=\lambda_2})}{V_i(\lambda_3 - \lambda_2)} \quad (13)$$

By using (13), we can easily select the weakest bus which has the largest relative voltage sensitivity. Let the selected bus be labeled bus  $i$ . At that bus selected, the maximum load level  $\lambda_{\max}$  can be estimated by applying the curve fitting technique for the  $\lambda$ -e curve at bus  $i$ .

The distorted elliptic equation for the  $\lambda$ -e curve can be described by the following general form:

$$\lambda^2 + \alpha \lambda e_i + \beta e_i^2 + \gamma \lambda + \zeta e_i + \psi = 0 \quad (14)$$

Differentiating with respect to  $\lambda$  the above equation gives the following equation.

$$2\lambda + \alpha(e_i + \lambda \dot{e}_i) + 2\beta e_i \dot{e}_i + \gamma + \zeta \dot{e}_i = 0 \quad (15)$$

The 5 coefficients in (14) can be easily computed as given in the Appendix if we know three upper load flow solutions and two derivatives of them at different load levels as shown in Fig.4. As mentioned earlier, the derivatives in (15) can be easily calculated by the load flow algorithm and (12). Arranging (14) with respect to

$e_i$  yields

$$\beta e_i^2 + (\alpha \lambda + \zeta) e_i + \lambda^2 + \gamma \lambda + \psi = 0 \quad (16)$$

Since the variable  $e_i$  must have a real solution we obtain the following inequality related to the quadratic determinant, which is given by

$$D = (\alpha^2 - 4\beta)\lambda^2 + 2(\alpha\zeta - 2\beta\gamma)\lambda + \zeta^2 - 4\beta\psi \geq 0 \quad (17)$$

From (17), we can derive a formula to estimate the maximum load level  $\hat{\lambda}_{\max}$  when  $D=0$  as follows:

$$\hat{\lambda}_{\max} = \frac{(2\beta\gamma - \alpha\zeta) - \sqrt{(2\beta\gamma - \alpha\zeta)^2 - (\alpha^2 - 4\beta)(\zeta^2 - 4\beta\psi)}}{\alpha^2 - 4\beta} \quad (18)$$

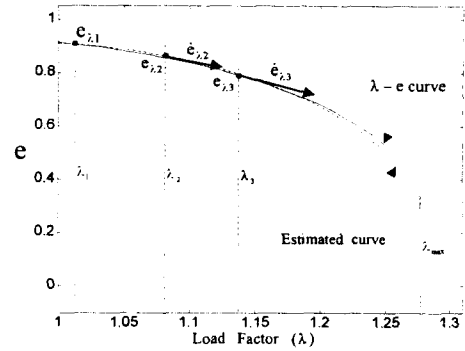


Fig. 4  $\lambda$ -e curve with upper solutions and ellipse curve fitting

The other root of  $D=0$  is dropped out since it is smaller than  $\hat{\lambda}_{\max}$  and is not relevant to the estimation of maximum load level. The above method improves the estimation accuracy of  $\lambda_{\max}$  remarkably since the power flow equation (10) is fully quadratic. In order to calculate the exact maximum load level  $\lambda_{\max}$ , it is required to repeat the estimation procedure by updating the load solution points. To avoid the divergence of load flow calculation in case of overestimated  $\hat{\lambda}_{\max}$ , we adopt the CPflow method where  $\lambda$  is regarded as a variable to be found with an initial value  $\hat{\lambda}_{\max}$  [13]. In the CPflow, the original set of equations is augmented by an additional equation which makes one of the state variables fixed to some specified value. In this process we specify  $e_i$  of the weakest bus  $i$  to  $\hat{e}_i^c$  as follows:

$$e_i - \hat{e}_i^c = 0 \quad (19)$$

where  $\hat{e}_i^c$  is a double root of (16) when  $\lambda = \hat{\lambda}_{max}$ .

The proposed estimation process is summarized as follows.

- i) Run conventional load flow on base case,  $\lambda_1$
- ii) Select appropriate load levels  $\lambda_2, \lambda_3$  and run the conventional load flow. Calculate  $de_i/d\lambda$  at  $\lambda_2$  and  $\lambda_3$ . Select the weakest bus according to (14).
- iii) Calculate five coefficients in (15) by using the information of  $e_i, \dot{e}_i$  and  $\lambda$ . Estimate the maximum load level  $\hat{\lambda}_{max}$  with (19)
- iv) Test  $|\hat{\lambda}_{max}^{k+1} - \hat{\lambda}_{max}^k| < \epsilon$ . If not, solve the augmented load flow equation including (20) by the CPflow method.
- v) Repeat iii) ~ iv) until the estimation process converges.

It should be noted that the proposed method weakly depends on the choice of the weakest bus, yielding almost same estimation efficiency since elliptic equation (14) can well-reflect the distortion caused by the e-f coupling.

#### IV. Numerical Tests

The proposed algorithm has been tested on three systems with two different schemes of load-generation variation. The first scenario of load-generation variation is to increase loads only with a constant power factor of the base case. The other is to consider both load and generation variation proportional to the base case load-generation. When  $\lambda = 1$ , the system load and generation is identical to the base case. The exact and the estimated maximum load levels are denoted by  $\lambda_{max}$  and  $\hat{\lambda}_{max}$  respectively.

The simulation results show that the proposed method provides an efficient algorithm to estimate the maximum load level with a remarkable improvement in the estimation accuracy compared with the look-ahead method.

##### Scenario 1) Load increase only

The Scenario 1 performs a simple test of the proposed algorithm by assuming the load increasingly only while the resulting power unbalance is covered by the slack bus. This scenario is a little far from the practical system operation, but frequently adopted to show the validity of the new algorithms since the simulation can be done in a

simple manner and its results can be precisely analyzed. The estimation results by the proposed method are mainly compared with those of the look-ahead method since the latter is now considered to be the most efficient method from the computational point of view. The test are performed with three sample systems and the results are listed in Table 1~3.

By observing the results, one can easily find that the proposed algorithm improves the estimation accuracy remarkably so that the second estimation guarantees a sufficient accuracy with the error within 1%. In Table 1, the first estimation by the proposed method produces an error greater than that by the look-ahead method. However, this comes from the fact that the proposed method uses  $\lambda_1$  which is far from the collapse point. If we use equally stepped load levels  $\lambda_1, \lambda_2$  and  $\lambda_3$ , the first estimation can also provide very accurate estimation, which can be directly confirmed by observing Table 2 and 3. Table 3 shows that the proposed algorithm remarkably reduces the iteration number to obtain the sufficient accuracy of the estimation compared with the look-ahead method.

##### Scenario 2) In case of both load and generation increase simultaneously

This scenario performs the simulation by increasing both the load and the corresponding generation reallocation. This simulation seems to be rather practical but may miss the worst case regarding the voltage stability. Consequently, it is desirable to adopt this scenario with Scenario 1 for the mutual supplement. The test results are listed in Table 4~6 for the same sample systems in Scenario 1. By observing the results, we can give almost the same discussions as in Scenario 1. It should be mentioned here that Table 4 shows that the look-ahead method provides more accurate result for the first estimation. When we consider the overall trend of both estimation methods, this is considered to be an unexpected result coming from a strange coincidence. In Table 6, the relatively large error in the first estimation by the proposed method also comes from the fact that using a solution point associated with a too low load level of  $\lambda_1 = 1$  compared with  $\lambda_2$  and  $\lambda_3$ .

By examining the results of both scenarios, we can conclude that the proposed algorithm remarkably improves the accuracy of the maximum loadability estimation, which enables us to reduce the computation time significantly. This can be a meaningful progress for the on-line application of voltage stability monitoring with numerous contingency cases.

Table 1. Test Results with IEEE 14-Bus System by Scenario 1

(Weak Bus : Bus 5,  $\lambda_{max} = 4.006692$ )

Iteration	Proposed Method					Look-ahead Method			
	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\hat{\lambda}_{max}$	Error %	$\lambda_1$	$\lambda_2$	$\hat{\lambda}_{max}$	Error %
1	1.00000	2.30000	2.40000	4.144082	3.42901	2.30000	2.40000	3.901424	2.62729
2	2.30000	2.40000	3.99738	4.007904	0.14458	2.40000	3.90142	3.960794	1.14552
3	2.40000	3.99738	4.00650	4.006692	0.00104	3.90142	3.96079	4.001073	0.14023
4	3.99738	4.00650	4.00668	4.006692	0.00007	3.96079	4.00107	4.006234	0.01142

Table 2. Test Results with New England 39-Bus System by Scenario 1

(Weak Bus : Bus 6,  $\lambda_{max} = 1.277155$ )

Iteration	Proposed Method					Look-ahead Method			
	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\hat{\lambda}_{max}$	Error %	$\lambda_1$	$\lambda_2$	$\hat{\lambda}_{max}$	Error %
1	1.00000	1.10000	1.20000	1.277694	0.04220	1.10000	1.20000	1.234728	3.32199
2	1.10000	1.20000	1.27312	1.277181	0.00203	1.20000	1.23472	1.262728	1.12962
3	1.20000	1.27312	1.27692	1.277156	0.00007	1.23472	1.26272	1.273684	0.27178
4	Converged					1.26272	1.27368	1.276683	0.03696

Table 3. Test Results with IEEE 118-Bus System by Scenario 1

(Weak Bus : Bus 47,  $\lambda_{max} = 1.878086$ )

Iteration	Proposed Method					Look-ahead Method			
	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\hat{\lambda}_{max}$	Error %	$\lambda_1$	$\lambda_2$	$\hat{\lambda}_{max}$	Error %
1	1.00000	1.10000	1.20000	1.900697	1.20393	1.10000	1.20000	1.651117	12.08511
2	1.10000	1.20000	1.87551	1.879003	0.04882	1.20000	1.65111	1.723931	8.20808
3	1.20000	1.87551	1.87804	1.878086	0.00000	1.65111	1.72393	1.820579	3.06120
4	1.87551	1.87804	1.87808	1.878086	0.00000	1.72393	1.82057	1.863916	0.75448

Table 4. Test Results with IEEE 14-Bus System by Scenario 2

(Weak Bus : Bus 5,  $\lambda_{max} = 4.062554$ )

Iteration	Proposed Method					Look-ahead Method			
	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\hat{\lambda}_{max}$	Error %	$\lambda_1$	$\lambda_2$	$\hat{\lambda}_{max}$	Error %
1	1.00000	2.60000	2.70000	4.167356	2.57971	2.60000	2.70000	3.996746	1.61989
2	2.60000	2.70000	4.04805	4.063918	0.03357	2.70000	3.99674	4.036672	0.63711
3	2.70000	4.04805	4.06213	4.062555	0.00002	3.99674	4.03667	4.060099	0.06045
4	4.04805	4.06213	4.06253	4.062555	0.00002	4.03667	4.06009	4.062408	0.00361

Table 5. Test Results with New England 39-Bus System by Scenario 2

(Weak Bus : Bus 7,  $\lambda_{max} = 2.211824$ )

Iteration	Proposed Method					Look-ahead Method			
	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\hat{\lambda}_{max}$	Error %	$\lambda_1$	$\lambda_2$	$\hat{\lambda}_{max}$	Error %
1	1.00000	1.10000	1.20000	2.189019	1.03105	1.10000	1.20000	1.844553	16.60489
2	1.10000	1.20000	2.16284	2.208698	0.14133	1.20000	1.84455	2.094614	5.29925
3	1.20000	2.16284	2.20846	2.211802	0.00099	1.84455	2.09461	2.189064	1.02901
4	2.16284	2.20846	2.21168	2.211824	0.00000	2.09461	2.18906	2.208137	0.16669

Table 6. Test Results with IEEE 118-Bus System by Scenario 2

(Weak Bus : Bus 38,  $\lambda_{max} = 3.239807$ )

Iteration	Proposed Method					Look-ahead Method			
	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\hat{\lambda}_{max}$	Error %	$\lambda_1$	$\lambda_2$	$\hat{\lambda}_{max}$	Error %
1	1.00000	2.60000	2.70000	3.409252	5.23009	2.60000	2.70000	3.157938	2.52697
2	2.60000	2.70000	3.14912	3.247151	0.22668	2.70000	3.15793	3.215159	0.76078
3	2.70000	3.14912	3.23489	3.239591	0.00667	3.15793	3.21515	3.237922	0.05818
4	3.14912	3.23489	3.23919	3.239808	0.00003	3.21515	3.23792	3.239774	0.00101

V. Conclusions

This paper has presented an efficient algorithm to estimate the maximum loadability index ( $\lambda_{max}$ ) by reflecting the near-term bus-wise forecasting of load demand and the corresponding generation reallocation. The elliptic characteristic of P-e curve has been illustrated with theoretical analysis. By utilizing the distorted elliptic characteristics of the P-e curve, an efficient algorithm has been developed to estimate the maximum loadability. The proposed algorithm improves the estimation accuracy remarkably compared with the look-ahead method which is now considered to be the most efficient from the computational point of view. This advantage comes from the fact that the proposed method is developed on the basis of curve fitting with the known pattern of the curve while the look-ahead method works with the curve of unknown pattern (It is noted that the P-V curve pattern is quite different from the quadratic). The proposed method reduces the computation time considerably by reducing the iteration number in the maximum loadability estimation, which is a meaningful progress for the on-line application of voltage stability monitoring with the consideration of numerous contingency cases. The proposed method has been tested on the IEEE 14 bus system, the New England 39 bus system, and the IEEE 118 bus system. The numerical results show that the proposed method can be well-applied to on-line power system voltage stability assessment.

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## VII. Appendix

### Calculation of Coefficients in Eq. (15)

Three load flow solutions  $(e_1, f_1)$ ,  $(e_2, f_2)$  and  $(e_3, f_3)$  are obtained corresponding to the three different load levels  $\lambda_1, \lambda_2$  and  $\lambda_3$ . By using these solutions, we have the following equations for the weakest bus  $i$  associated with (15).

$$\lambda_1^2 + \lambda_1 e_{1i} \alpha + e_{1i}^2 \beta + \lambda_1 \gamma + e_{1i} \zeta + \psi = 0 \quad (A.1)$$

$$\lambda_2^2 + \lambda_2 e_{2i} \alpha + e_{2i}^2 \beta + \lambda_2 \gamma + e_{2i} \zeta + \psi = 0 \quad (A.2)$$

$$\lambda_3^2 + \lambda_3 e_{3i} \alpha + e_{3i}^2 \beta + \lambda_3 \gamma + e_{3i} \zeta + \psi = 0 \quad (A.3)$$

In the above equations,  $e_{1i}, e_{2i}, e_{3i}$  denote the  $i$ th components of the solution vectors  $e_1, e_2, e_3$  respectively. The derivatives of  $e$  with respect to  $\lambda$  can be easily calculated by using (13). By substituting those derivatives and solution points, we can obtain the following equations for the weakest bus  $i$  associated with (16).

$$2\lambda_1 + (e_{1i} + \lambda_1 \dot{e}_{1i}) \alpha + 2e_{1i} \dot{e}_{1i} \beta + \gamma + \dot{e}_{1i} \zeta = 0 \quad (A.4)$$

$$2\lambda_2 + (e_{2i} + \lambda_2 \dot{e}_{2i}) \alpha + 2e_{2i} \dot{e}_{2i} \beta + \gamma + \dot{e}_{2i} \zeta = 0 \quad (A.5)$$

$$2\lambda_3 + (e_{3i} + \lambda_3 \dot{e}_{3i}) \alpha + 2e_{3i} \dot{e}_{3i} \beta + \gamma + \dot{e}_{3i} \zeta = 0 \quad (A.6)$$

The 5 unknowns of the coefficients can be solved by selecting 5 equations among the equations (A.1)-(A.6). Since the load flow solutions are more reliable than the derivatives of  $e$  and the higher load level is closer to the collapse point, it is natural to select the equations (A.1)-(A.3) and the equations (A.5) and (A.6). Consequently, we can calculate the coefficients by solving the other equations while disregarding (A.4).

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