International Journal of Management Science
Vol 5, No 1, May 1999

RELATIVE PERFORMANCE COMPARISON OF GROUP CUSUM CHARTS

SungWoon Choi
Associate professor of the department of Industrial Engineering
Kyung Won University, Seongnam City, Kyungki Do, 461-701, Korea

SangHoon Lee
Associate professor of the department of Industrial Engineering
Kyung Won University, Seongnam City, Kyungki Do, 461-701, Korea

(Received December 1998 ; revision received April 1999)

ABSTRACT

Performance of the group cumulative sum (CUSUM) control scheme using multiple univariate CUSUM charts is more sensitive to the change of quality control (QC) characteristics than the control chart schemes based on the Hotelling statistic. We examine three group charts for multivariate normal data sets simulated with various correlation structures and shift directions in the mean vector. These group schemes apply the original measurement vectors, the scaled residual vectors from the regression of each variable on all others and the principal component vectors respectively to calculating the CUSUM statistics. They are also compared to the multivariate QC charts based on the Hotelling statistic by estimating average run lengths, coefficients of variation of run length and ranks in signaling order. On the basis of simulation results, we suggest a control chart scheme appropriate for specific quality control environment

1. INTRODUCTION

The evolution of modern technology is radically affecting data acquisition equipment and on-line computers used in industrial processes. Due to innovations in industrial quality control (QC), it is now common to monitor several correlated quality characteristic measurements together rather than a single measurement. For a QC process with multiple characteristics, either a multivariate control chart or a group control chart can be utilized to detect changes in the mean level of the process. The multivariate control chart is based on a multivariate statistic that involves information on the dependence between separate measurements. When the statistic exceeds a given threshold, the chart gives a signal for corrective action. Another approach for multiple measurements is the group control chart using the multiple univariate control charts that operate separate univariate charts simultaneously for each measurement being monitored. If any of the charts indicate a change in the mean level, corrective action is then required.

A control chart for the mean level of a sequence of observations can be considered as successive significance tests of the hypothesis for the unknown mean vector μ with the form

$$H_0 : \mu = \mu_0 \quad \text{vs.} \quad H_1 : \mu \neq \mu_0$$

where μ_0 represents the in-control mean vector of the desired condition. Denote the p component vector of quality characteristic measurements as $\mathrm{x}_n = (x_{n,1}, x_{n,2}, \cdots, x_{n,p})'$ where $x_{n,i}$ is the observation on the ith variable at time n. A typical assumption is that the observed measurements $\{\mathrm{x}_n, n = 1, 2, \cdots\}$ are independent and identically distributed multivariate normal random vectors whose covariance matrix Σ is known and constant. For simplicity, it can be also assumed without loss of generality that $\mu_0 = (0, 0, \cdots, 0)' = 0$ and Σ is normalized such that all diagonal elements are 1.

It is well known that the optimal affine invariant test statistic for a shift of the mean vector of the single observation x_n is the Hotelling statistic (Hotelling, 1947)

$$T^2 = (\mathrm{x}_n - \mu_0)' \Sigma^{-1} (\mathrm{x}_n - \mu_0) \tag{1}$$

which has a χ^2 distribution with p degrees of freedom. One of the control charts based on the Hotelling statistic is the Shewhart χ^2 chart. This chart gives an out-of-control signal as soon as T^2 in (1) exceeds a specified threshold h. The application of the Shewhart chart was discussed by Alt (1985). Affine equivalence makes the result of a test independent of the coordinate system in which x_n is measured and is a valuable property for a test in which there is no prior expectation that departures of the mean will be certain identifiable directions. This makes the Hotelling statistic particularly attractive for situations in which there is no particular direction to anticipated shifts in mean and suggests a multivariate approach of monitoring several correlated measurements at the same time to detect any shift in the mean level of sequential measurements away from the desired condition. When it is known that the shift in mean is likely to be in some particular directions however, the Hotelling statistic is inferior to other statistics (see for examples Hawkins, 1974).

Another control chart using T^2 was proposed by Alwan (1986). It is an univariate CUSUM procedure of T^2. Crosier (1988) suggested the CUSUM of T, rather than T^2. The positive square root of quadratic form of (1) puts the quantities on a more meaningful scale. Crosier (1988) also proposed another CUSUM scheme using the statistics

$$C_n = \sqrt{(\mathrm{s}_{n-1} + \mathrm{x}_n)' \Sigma^{-1} (\mathrm{s}_{n-1} + \mathrm{x}_n)}$$

where

$$\mathrm{s}_n = \begin{cases} \mathbf{0}, & \text{if } C_n \le k \\ (\mathrm{s}_{n-1} + \mathrm{x}_n - \mu_0)(1 - k/C_n), & \text{if } C_n > k \end{cases}$$

for $n = 1, 2, \cdots$ where $\mathrm{s}_0 = 0$ and $k > 0$. The signal of the scheme is based on the Hotelling statistic applied to the CUSUM vectors s_n instead of the T^2 using the observed measurement for the individual points, that is, it signals when $T_s^2 = \mathrm{s}_n' \Sigma^{-1} \mathrm{s}_n > h$ for a given threshold h. This control chart, using the acronym CCU in this paper, outperforms the CUSUM of T^2 or T in average run length (ARL) performance.

For multivariate normal processes, it is also common to use multiple univariate cumulative sum (CUSUM) control charts together to monitor the variables that jointly measure the quality of the processes. Woodall and Ncube (1985) proposed a group CUSUM chart for detecting a shift in the mean of a multivariate normal process. It consists of a set of individual univariate CUSUM charts of all the variables and its performance is evaluated by the collection of schemes. The interpretation of this group chart is simpler than that of the T^2- or T-based control charts, since the signal instantaneously identifies with the out-of-control variable. When the variables are highly correlated however, this approach is generally expected to have worse performance than the QC charts that take advantage of the correlation between variables. Nevertheless, quality control practice often applies the simultaneous univariate charts for correlated observations, but usually without analyzing the resulting performance. For the multivariate processes in which their characteristics are significantly correlated, Woodall and Ncube (1985) also recommended monitoring principal components with the group CUSUM chart. The original variables are linearly transformed to the new ones which are mutually independent via principal component analysis (Jackson, 1980), and the new variables are then applied to the simultaneous operation of independent univariate CUSUM charts. It may result in improving performance, but there is some loss of interpretability in terms of the original variables. An alternative approach using the transformation of the original variables for the group CUSUM chart was suggested by Hawkins (1991). It is based on the vector of scaled residuals from the regression of each variable on all others. He showed that this scheme can be quite effective for the case in which the shift directions of interest are specified.

Performance of the directionally-invariant control scheme for a multivariate normal processes depends on the mean vector μ and covariance matrix Σ only through the magnitude of noncentrality parameter

$$\eta_c = \sqrt{(\mu - \mu_0)'\Sigma^{-1}(\mu - \mu_0)} . \quad (2)$$

While the control charts based on the T^2 or T statistic are generally directionally invariant like the Shewhart and CCU charts, it is not true for the group CUSUM charts. Performance of the group schemes varies in the mean shift direction and the covariance structure for a constant value of noncentrality parameter. In this paper, three group CUSUM charts are extensively examined for multivariate normal processes with three aspects: performance measure, measurement and out-of-control characteristics. Two multivariate control schemes, the Shewhart and CCU charts, have also been compared to the group charts. For various quality control environments, this study analyzes performance of the control charts on the basis of 10,000 Monte Carlo independent simulation runs. In the next section, the approaches to examine the control charts for multivariate normal processes are discussed and the following sections present the numerical results and discussions for comparison of the group CUSUM charts and the multivariate control charts mentioned in this section. Finally, the last section contains summary and conclusions.

2. CONSIDERATIONS IN EXAMINING QC CHARTS

Performance of QC charts is usually evaluated by the ARL that is the average number of successive observations without an out-of-control signal. Monte Carlo simulation method is often used to compare the ARL performance of several control charts for monitoring multivariate processes(Crosier, 1988)(Hawkins, 1991) (Lowry, Woodall, Champ and Ridgon 1992). Different control charts are designed by simulation such that in-control ARL of each scheme is the same for the data with in-control characteristics, and the relative performance of various schemes can be then evaluated by comparing their ARLs for the data of out-of-control processes. Another performance measure is the coefficient of variation of run length (CVRL) (Yashchin, 1992). It is defined a function of ARL and standard deviation of the run length (SDRL), CVRL = SDRL /ARL and expresses a relative variation of the run length as a measure of dispersion of the run length. This study also suggests "ranks in signaling order" (RISO) as a measure to compare performance of different control charts. This statistic indicates the rank of the corresponding scheme in order of giving an out-of-control signal when several control charts are operated simultaneously for an identical process. It is considered as a relative measure for the run length for out-of-control signal, while the ARL is an absolute measure.

According to the environment of the process to be controlled, the observed measurements have different quality control characteristics. The essential characteristics for multivariate normal processes are determined by their mean vectors and covariance matrices . Assuming the zero in-control mean vector and the normalized covariance matrix, they depend on the correlation structure of the process. Doganaksoy, Faltin and Tucker (1991) used three correlation structures for identification of out-of-control quality characteristics in a multivariate manufacturing environment. These structures correspond respectively with the cases that all the variables are positively correlated, only some variables are correlated and the variables are correlated with mixed signs. This study uses six different types of correlation structure. The six structures are categorized into two classes: the positive type, in which all pairs of variables have an equal positive correlation, and the negative type, in which variables i and j for $i \neq j$ have a negative correlation if $i+j$ is odd and a positive correlation if $i+j$ is even. This study uses the structures in which absolute magnitudes of correlation between the variables are uniform. The positive types are denoted by P-2, P-5, P-8 with the absolute magnitudes of correlation of 0.2, 0.5, 0.8 respectively and the negatives types by N-2, N-5, N-8. For example of $p=4$,

$$\text{P-2} = \begin{bmatrix} 1 & 0.2 & 0.2 & 0.2 \\ 0.2 & 1 & 0.2 & 0.2 \\ 0.2 & 0.2 & 1 & 0.2 \\ 0.2 & 0.2 & 0.2 & 1 \end{bmatrix} \qquad \text{N-8} = \begin{bmatrix} 1 & -0.8 & 0.8 & -0.8 \\ -0.8 & 1 & -0.8 & 0.8 \\ 0.8 & -0.8 & 1 & -0.8 \\ -0.8 & 0.8 & -0.8 & 1 \end{bmatrix}$$

The second consideration for the environment to be controlled is out-of-control conditions from the desired situation. In QC for controlling the mean of multivariate process, the out-of-control process is usually characterized by the direction and distance of the shift in mean. To model the out-of-control mean process, three types of shift direction are considered:

Equal Shift, in which all components of μ are equal;

Symmetric Shift, which differs from Equal Shift in that the first half components of μ have different signs to the second half;

Only Shift, in which only a single component of μ is nonzero.

For Only Shift, we randomly choose one of all the variables for the out-of-control situation in each simulation run. It is not proper to use a measure independent on the correlation structure as a distance of the shift in mean for evaluating performance of the control charts related to different correlation structures and out-of-control conditions. The noncentrality parameter associated with the correlation structure, also called Mahalanobis distance, is chosen to represent the shift distance. This study assumes that the out-of-control process has a sudden and constant shift in mean.

3. GROUP CUSUM CHARTS

The technique frequently used for detection of a change in the mean of a normally distributed variable is a CUSUM chart scheme that is a set of sequential procedures based on likelihood ratios. A group CUSUM chart was suggested by extending the univariate CUSUM procedure to the multivariate normal process in Woodall and Ncube (1985). In this approach, p two-sided CUSUM charts are operated simultaneously to detect a shift in the mean vector of p-variate normal distribution and the out-of-control signal is given if any of the p univariate charts exceeds its control limit. The ith univariate CUSUM is operated for a given reference value $k_i > 0$ by forming the cumulative sums

$$U_{n,i} = \max(0, U_{n-1,i} + x_{n,i} - k_i)$$

and

$$L_{n,i} = \min(0, L_{n-1,i} + x_{n,i} + k_i)$$

for $n = 1, 2, \cdots$. Under the assumption of the zero in-control mean vector and the normalized covariance matrix, the reference values k_i are given equivalently for all the variables. This group chart, so called MCX, signals an out-of-control condition when

$$\text{MCX} = \max_i\left[\max(U_{n,i}, -L_{n,i})\right] > h \tag{3}$$

for a given threshold h. Hawkins (1993) discusses how to generalize these CUSUMs to others that are effective for detecting shifts in arbitrary but specific directions.

Departures from the in-control condition in multivariate processes may be expected to affect only a minority of the variables. With this idea, Hawkins (1991) proposed a measure for multivariate normal processes to test for shifts in mean. Realization x_n can be transformed to the regression-adjusted vector

$$\mathrm{z}_n = \left[\text{diagonal}(\Sigma^{-1})\right]^{-1/2}\Sigma^{-1}(\mathrm{x}_n - \mu_0)$$

that is the vector of scaled residuals from the regression of each variable on all the others. In case of a shift affecting a single variable, the control scheme using the vector z_n has the advantages of better separation of location from scale and immediate association to a signal with a particular variable. The group CUSUM chart that uses the regression-adjusted vector z_n for (3) is referred as to MCZ. The approach of using z_n is equivalent to the proposal of Healy (1987) when the "bad" mean vector differs from the in-control mean vector in only one component. Hawkins (1991) also suggested a measure using z_n to test for shifts in variability and a control chart using the Euclidean norm of the "resultant" vectors of the cumulative sums in the MCZ procedure.

Multivariate normal variables can be transformed to independent principal components by the spectral decomposition of the covariance matrix. The linear transformation for the successive measurements provides the possibility of separate control of the individual variables of a multivariate normal process (Woodall and Ncube, 1985; Jackson, 1985; Pignatiello and Runger, 1990). For $\mathrm{x}_n \sim N(\mu_0, \Sigma)$, the normalized principal component vector is

$$\mathrm{w}_n = \Lambda^{-1/2}\Psi'(\mathrm{w}_n - \mu_0)$$

where Ψ is the matrix of eigenvectors and Λ the diagonal matrix of eigenvalues associated with Ψ. Each component of w_n has the independent standard normal distribution, that is, $w_n \sim N(0,I)$. Thus, we directly applies the principal component vector w_n to the group CUSUM chart of (3) and denotes it as MCW.

The three group charts for 4-variate normal processes are examined with the six correlation types described in the previous section. Table 1 shows the transformation matrices of the regression-adjustment and the principal component analysis for the structures. The vectors for the CUSUM statistics in MCZ and MCW charts are calculated by premultiplying one of these matrices to the simulated data of multivariate normal distribution. This study uses the h values for the

Table 1. Transformation matrices for MCZ and MCW for 4-variate normal processes

$[\text{diagonal}(\Sigma^{-1})]^{-1/2}\Sigma^{-1}$					$\Lambda^{-1/2}\Psi'$			
				N-8				
1.955	0.602	-0.602	0.602		1.581	1.581	0.000	0.000
0.602	1.955	0.602	-0.602		0.913	-0.913	-1.826	0.000
-0.602	0.602	1.955	0.602		-0.271	0.271	-0.271	0.271
0.602	-0.602	0.602	1.955		0.645	-0.645	0.645	1.936
				N-5				
1.265	0.316	-0.316	0.316		1.000	1.000	0.000	0.000
0.316	1.265	0.316	-0.316		0.577	-0.577	-1.155	0.000
-0.316	0.316	1.265	0.316		-0.316	0.316	-0.316	0.316
0.316	-0.316	0.316	1.265		0.408	-0.408	0.408	1.225
				N-2				
1.046	0.149	-0.149	0.149		0.791	0.791	0.000	0.000
0.149	1.046	0.149	-0.149		0.456	-0.456	-0.913	0.000
-0.149	0.149	1.046	0.149		-0.395	0.395	-0.395	0.395
0.149	-0.149	0.149	1.046		0.323	-0.323	0.323	0.968
				P-2				
1.046	-0.149	-0.149	-0.149		0.791	-0.791	0.000	0.000
-0.149	1.046	-0.149	-0.149		-0.456	-0.456	0.913	0.000
-0.149	-0.149	1.046	-0.149		-0.395	-0.395	-0.395	-0.395
-0.149	-0.149	-0.149	1.046		-0.323	-0.323	-0.323	0.968
				P-5				
1.265	-0.316	-0.316	-0.316		1.000	-1.000	0.000	0.000
-0.316	1.265	-0.316	-0.316		-0.577	-0.577	1.155	0.000
-0.316	-0.316	1.265	-0.316		-0.316	-0.316	-0.316	-0.316
-0.316	-0.316	-0.316	1.265		-0.408	-0.408	-0.408	1.225
				P-8				
1.955	-0.602	-0.602	-0.602		1.581	-1.581	0.000	0.000
-0.602	1.955	-0.602	-0.602		-0.913	-0.913	1.826	0.000
-0.602	-0.602	1.955	-0.602		-0.271	-0.271	-0.271	-0.271
-0.602	-0.602	-0.602	1.955		-0.645	-0.645	-0.645	1.936

control limit, which result in ARL = 300 for independent 10,000 simulated processes with no mean shift. Table 2 contains the estimated values of h. As illustrated in this table, the estimates of h in all the three schemes are not uniform over the correlation types. It indicates that the ARL performance of the group charts depends on the correlation types, not just for the noncentral parameter values. The ARL performance of MCW is then expected to vary less than the other group charts in the correlation structures. Performance of the schemes is evaluated by comparing their ARLs. The reference values of all the group charts are chosen with a half of the standard deviation of individual variables for the cumulative sum statistics under our assumption of the normalized covariance matrix as in (Hawkins, 1991), that is, k_i = 0.5 for $\forall i$. The ARL performance is investigated in five distance levels of the mean shift for each chart respectively. Table 3 illustrates the absolute values of Euclidean shift distance in each direction for three shift types, which are associated with η_c = 0.2, 0.4, 0.8, 1.6 and 3.2 respectively.

Table 2. h values of in-control ARL=300 for three group CUSUM charts for different correlation types

QC	N-8	N-5	N-2	P-2	P-5	P-8
MCX	5.52	5.81	5.91	5.91	5.82	5.52
MCZ	5.89	5.90	5.92	5.92	5.89	5.88
MCW	5.92	5.90	5.91	5.91	5.90	5.92

Table 3. Values of Euclidean shift distance δ associated with magnitudes of η_c for 4-variate processes

Shift Type	η_c	N-8	N-5	N-2	P-2	P-5	P-8
	0.2	0.045	0.071	0.089	0.126	0.158	0.184
Equal	0.4	0.089	0.141	0.179	0.253	0.316	0.369
$\mu=(\delta,\delta,\delta,\delta)'$	0.8	0.179	0.283	0.358	0.506	0.632	0.738
	1.6	0.358	0.565	0.716	1.012	1.265	1.475
	3.2	0.716	1.131	1.431	2.204	2.530	2.95
	0.2	0.045	0.071	0.089	0.089	0.071	0.045
Symmetric	0.4	0.089	0.141	0.179	0.179	0.141	0.089
$\mu=(\delta,\delta,-\delta,-\delta)'$	0.8	0.179	0.283	0.358	0.358	0.283	0.179
	1.6	0.358	0.565	0.716	0.716	0.565	0.358
	3.2	0.716	1.131	1.431	1.431	1.131	0.716
	0.2	0.102	0.158	0.191	0.191	0.158	0.102
Only	0.4	0.205	0.316	0.382	0.382	0.316	0.205
$\mu=(\delta,0,0,0)'$	0.8	0.409	0.632	0.765	0.765	0.632	0.409
	1.6	0.818	1.265	1.530	1.530	1.265	0.818
	3.2	1.637	2.530	3.060	3.060	2.530	1.637

Table 4 shows the expected values of z_n and w_n according to the correlation types when the process mean shifts from the zero in-control level with a certain value of noncentrality parameter. Table 5 contains the results of ARLs by shifting the mean vector with for the three mean shift directions, which are obtained from independent 10,000 simulation runs. For the same value of noncentral parameter, the ARLs of MCX decrease from N-8 to P-8 continuously (N-8 > N-5 > N-2 > P-2 > P-5 > P-8) for Equal Shift, while they decrease as the absolute correlation is smaller in both the classes of correlation type for Symmetric and Only Shifts. For the same shift direction, MCX appears to have better performance as the Euclidean shift distance increases according to the correlation structure. The ARL performance of MCX, however, depends on the correlation structure for the constant Euclidean distance as shown in Table 6. The relation of the correlation structure to the change of the ARLs for a constant Euclidean distance is reverse to that for a constant value of noncentrality parameter. The negative class in Equal Shift and both the classes in Symmetric Shift have the effect of shifting the half of the variables in the direction opposite to the other half. Thus MCX will have the same ARLs for the same absolute corruption in the three cases. Contrary to MCX, the performance of MCZ appears to be poor for highly positive-correlated processes. It has shorter ARLs in the order of N-8 < N-5 < N-2 < P-2 < P-5 < P-8. In the MCZ scheme, the original variables are rescaled to unit variance by regressing a variable on all the other variables. The rescaled variables correspond to the residuals resulting from eliminating the effects of all the other variables by regression. When the means of two variables simultaneously change in the same direction, its effect is exaggerated in the process if they are positively correlated and reduced if they are negatively correlated. This fact is converse in the changes with opposite signs. Therefore, the regression adjustment gives rise to contraction in the original magnitude of mean shift with the positive correlation structure for Equal Shift, thereby resulting in greater ARLs. For Symmetric and Only Shifts, the ARLs of MCZ is not significantly different over the correlation structures for relatively larger shifts and its performance is best for Only Shift. Compared to MCX and MCZ, MCW appears to be less affected by the correlation structure and the shift direction for the noncentrality parameter of equal amount in the manner that its performance is consistent over the same class of correlation types. The ARLs for the positive class of correlation types are shorter than that for the negative class for the cases of shifting all the variables, while they slightly vary for Only Shift.

The properties of the performance of MCZ and MCW for different correlation structures will be strongly related to their transformation matrices in Table 1. For example, the best performance of MCZ for Only Shift results from using the transformation matrices with on-diagonal elements greater than 1 for the regression adjustment. As shown in the simulation results in Table 5, the ARL performance of MCZ and MCW is associated with the expectation of realizations of the regression-adjusted vector and the principal component vector (Table 4), since these values are the means of the data actually used for calculating the CUSUM statistics in the charts. The expectation of the modified vectors depends on the correlation structure and mean shift direction of the process. When shifting along the principal component axes in the process mean level, the expected values of the principal components are corresponding to the shift distances in the orthogonal directions and Table 3, 4, 5 can then provide the ARL performance information for this situation.

Table 4. Expected values of z_n and w_n for 4−variate processes of mean vector μ of η_c=0.4 with μ_0=0

	$E(z_n)=[\text{diagonal}(\Sigma^{-1})]^{-1/2}\Sigma^{-1}\mu$											
	$\mu=(\delta,\delta,\delta,\delta)'$				$\mu=(\delta,\delta,-\delta,-\delta)'$				$\mu=(\delta,0,0,0)'$			
N-8	0.23	0.23	0.23	0.23	0.23	0.23	-0.23	-0.23	0.40	0.12	-0.12	0.12
N-5	0.22	0.22	0.22	0.22	0.22	0.22	-0.22	-0.22	0.40	0.10	-0.10	0.10
N-2	0.21	0.21	0.21	0.21	0.21	0.21	-0.21	-0.21	0.40	0.06	-0.06	0.06
P-2	0.15	0.15	0.15	0.15	0.21	0.21	-0.21	-0.21	0.40	-0.06	-0.06	0.06
P-5	0.10	0.10	0.10	0.10	0.22	0.22	-0.22	-0.22	0.40	-0.10	-0.10	-0.10
P-8	0.06	0.06	0.06	0.06	0.23	0.23	-0.23	-0.23	0.40	-0.12	-0.12	-0.12
	$E(w_n)=\Lambda^{-1/2}\Psi'\mu$											
	$\mu=(\delta,\delta,\delta,\delta)'$				$\mu=(\delta,\delta,-\delta,-\delta)'$				$\mu=(\delta,0,0,0)'$			
N-8	0.28	-0.16	0.00	0.23	0.28	0.16	0.00	-0.23	0.32	0.19	-0.06	0.13
N-5	0.28	-0.16	0.00	0.23	0.28	0.16	0.00	-0.23	0.32	0.18	-0.10	0.13
N-2	0.28	-0.16	0.00	0.23	0.28	0.16	0.00	-0.23	0.30	0.17	-0.15	0.12
P-2	0.00	0.00	-0.40	0.00	0.00	-0.33	0.00	-0.23	0.30	-0.17	-0.15	0.12
P-5	0.00	0.00	-0.40	0.00	0.00	-0.33	0.00	-0.23	0.32	-0.18	-0.10	-0.13
P-8	0.00	0.00	-0.40	0.00	0.00	-0.33	0.00	-0.23	0.32	-0.19	-0.06	-0.13

Table 5. ARLs of three group CUSUM charts for 4−variate normal processes

Shift	QC	η_c	N-8	N-5	N-2	P-2	P-5	P-8
		0.2	269.6	239.9	217.8	169.7	145.4	136.4
		0.4	204.8	146.3	110.8	70.2	54.4	48.7
	MCX	0.8	104.8	52.9	33.9	20.4	16.1	14.3
		1.6	33.6	14.8	10.7	7.4	6.1	5.4
		3.2	10.1	5.9	4.7	3.4	2.9	2.5
		0.2	187.1	187.4	191.8	233.8	265.6	290.6
		0.4	81.7	82.9	87.7	134.4	195.4	261.6
Equal	MCZ	0.8	24.0	24.3	25.7	45.1	90.4	180.7
		1.6	8.3	8.5	8.8	13.2	24.7	74.6
		3.2	3.8	3.8	4.0	5.5	8.6	20.0
		0.2	197.7	197.4	194.5	193.1	190.7	190.5
		0.4	90.2	88.7	89.3	72.2	71.8	71.8
	MCW	0.8	24.7	24.7	24.7	17.9	17.9	18.0
		1.6	8.2	8.2	8.2	6.1	6.1	6.1
		3.2	3.7	3.6	3.7	2.8	2.7	2.7
		0.2	270.6	241.0	215.3	212.8	237.5	270.2
		0.4	206.8	146.7	110.5	109.5	144.8	205.7
	MCX	0.8	104.0	52.0	33.7	34.1	52.8	105.9
		1.6	33.5	14.7	10.6	10.7	14.7	33.5
		3.2	10.0	5.9	4.7	4.7	5.9	10.1
		0.2	187.5	187.7	194.2	195.1	189.3	187.7
		0.4	83.2	84.2	88.0	87.8	84.0	81.4
Sym-	MCZ	0.8	23.8	24.1	25.4	25.6	24.3	23.8
metric		1.6	8.2	8.4	8.7	8.7	8.4	8.2
		3.2	3.8	3.8	4.0	4.0	3.8	3.8
		0.2	199.4	196.8	198.5	197.7	197.3	199.3
		0.4	91.6	89.7	89.5	84.4	84.6	84.5
	MCW	0.8	24.6	24.6	24.8	21.9	21.8	22.0
		1.6	8.2	8.2	8.2	7.4	7.3	7.4
		3.2	3.6	3.6	3.6	3.3	3.3	3.3
		0.2	262.2	223.2	199.2	198.4	220.4	262.1
		0.4	176.8	108.7	79.0	79.1	108.6	176.1
	MCX	0.8	63.6	27.4	19.6	19.6	27.8	63.2
		1.6	15.9	8.3	6.5	6.5	8.3	16.0
		3.2	5.6	3.5	2.9	2.9	3.5	5.6
		0.2	183.9	186.1	189.5	188.7	183.8	181.1
		0.4	68.4	69.7	71.8	71.4	68.6	67.4
Only	MCZ	0.8	17.5	17.7	17.9	18.0	17.7	17.6
		1.6	6.1	6.1	6.1	6.1	6.1	6.1
		3.2	2.7	2.8	2.8	2.8	2.8	2.8
		0.2	196.1	193.4	195.7	197.8	197.0	195.1
		0.4	81.4	80.8	83.3	85.4	82.0	81.4
	MCW	0.8	21.1	21.7	22.9	22.6	21.6	21.1
		1.6	7.1	7.2	7.6	7.6	7.3	7.1
		3.2	3.1	3.2	3.3	3.4	3.2	3.1

Table 6. ARLs of MCX for constant Euclidean distance shifts

Shift	δ	N-8	N-5	N-2	P-2	P-5	P-8
	0.2	89.8	93.0	94.1	100.3	110.2	123.7
Equal	0.8	8.7	9.0	9.2	10.1	11.2	12.5
	3.2	2.0	2.1	2.1	2.2	2.3	2.4
	0.2	89.3	92.2	94.7	94.1	93.2	90.5
Symmetric	0.8	8.6	9.0	9.2	9.2	9.0	8.7
	3.2	2.0	2.1	2.1	2.1	2.1	2.0
	0.2	180.9	189.3	191.3	191.6	188.6	180.1
Only	0.8	16.5	17.5	18.1	18.0	17.7	16.7
	3.2	2.6	2.7	2.8	2.8	2.7	2.6

Theory would lead one to expect certain relationships in the performance of the three CUSUMs. First, MCZ is optimal for the Only Shift situation, and so would be expected to dominate the other two techniques. This is indeed seen to be the case. Next, for the correlation structures with equal shift elements, the leading principal component points in the direction $(1, 1, \cdots, 1)'$ so that by Healy's results (1987) MCW should be optimal for the detection of Equal Shift as shown in the simulation results. The simulations also show that MCZ and MCW are close competitors in ARL performance for the Symmetric Shift situation. While none of the situations simulated corresponds to that in which MCX is optimal, it is the best of the three for the Equal Shift direction with high positive correlations.

When comparing directionally variant charts, it is important to examine the directional sensitivity with aspect that concerns the interpretation of the signal from the schemes. Using Only Shift, Table 7 illustrates the results for the directional sensitivity of MCX and MCZ for constant η_c. Their performance is measured by the percentage of the 10,000 runs that cause a signal at the variable actually mean-shifted in the Only Shift direction. Both schemes show good performance of correctly detecting the out-of-control variable for relatively large shifts and have better results with the structures with smaller correlations. MCZ operates with higher degree of correctness than MCX for the highly correlated data. Although MCW robustly and consistently detects out-of-control situation in the mean level without great influence of the correlation structure for the equal noncentrality parameter, the principal component analysis is often objected in the QC process for multivariate processes, in which the interpretation is concerned rather than monitoring of the signal. It is difficult to interpret a physical meaning for the complicate linear transformation of the original variables. But, Hawkins (1991) mentioned that "in some problems, the principal components will be more interpretable than the original measurements — typically when the vector of measurements conforms at least approximately to the factor-analysis model."

Table 8 shows the ARL performance of the three group CUSUM charts with constant values of h over different correlation structures for p = 2, 4, 10. It can give some idea in designing the group charts according to the quality characteristics of correlation. The group charts are also compared in the next section for other numbers of variables with the multivariate QC charts.

Table 7. Percentages of MCX and MCZ runs giving out−of−control signal by the shifted variables for 4−variate processes

	MCX						MCZ					
η_c	N-8	N-5	N-2	P-2	P-5	P-8	N-8	N-5	N-2	P-2	P-5	P-8
0.2	37	46	52	51	45	37	49	50	52	52	51	49
0.4	62	76	82	82	76	62	76	78	81	82	79	76
0.8	89	95	97	97	96	90	93	95	96	97	95	93
1.6	98	99	100	100	99	98	98	99	100	99	99	99
3.2	100	100	100	100	100	100	100	100	100	100	100	100

Table 8. In−control ARLs of three group CUSUM charts for various h values for multivariate normal processes

QC	p	h	N-8	N-5	N-2	P-2	P-5	P-8
MCX	2	4.5	175	150	143	143	151	174
		5.5	464	406	391	390	408	469
		6.5	1229	1073	1053	1060	1088	1221
	4	5.0	185	140	124	123	137	185
		6.0	476	362	328	329	363	479
		7.0	1236	956	888	889	959	1236
	10	5.0	117	70	55	55	69	114
		6.5	444	274	227	228	271	439
		8.0	1766	1103	991	977	1112	1760
MCZ	2	4.5	175	151	143	144	152	176
		5.5	460	408	385	388	403	460
		6.5	1246	1101	1054	1055	1086	1224
	4	5.0	128	125	122	122	125	129
		6.0	335	332	326	326	334	341
		7.0	911	896	887	881	893	900
	10	5.0	53	53	53	53	53	53
		6.5	222	222	223	221	223	220
		8.0	968	985	991	986	978	974
MCW	2	4.5	144	144	143	143	141	140
		5.5	389	388	391	392	390	386
		6.5	1062	1053	1068	1060	1048	1061
	4	5.0	122	122	122	125	125	124
		6.0	325	331	330	329	327	327
		7.0	892	884	882	884	869	876
	10	5.0	52	52	52	52	52	52
		6.5	218	218	216	218	218	221
		8.0	975	974	954	985	978	971

4. COMPARATIVE PERFORMANCE OF QC CHART SCHEMES

The relative performances of the three group CUSUM charts at detecting a mean shift from in-control are compared to the two multivariate QC schemes, the Shewhart (the acronym SHW in this section) and CCU charts for p = 2, 4, 10 with three measures described in the second section. As in the analysis of the previous section, all the schemes are designed to give an out-of-control signal when the test statistic is greater than the threshold h of in-control ARL = 300, which are obtained by the simulation. The estimates of h used can be found in Table 2 and Table 10 for the group charts and in Table 9 for the multivariate charts. All the reference values of the group charts and CCU are set to 0.5. This analysis applies the two correlation types, N-5 and P-5 to independent 10,000 simulation runs with five different magnitudes of noncentrality parameter respectively . The results in this section are obtained by operating simultaneously the four charts for an identical multivariate normal process at each run for the two extreme situations of mean shift direction respectively: SHW, CCU, MCX, MCW for Equal Shift and SHW, CCU, MCX, MCZ for Only Shift.

Table 9. h values of in-control ARL=300 for multivariate QC charts for multivariate normal processes

p	Shewhart	CCU
2	11.43	35.73
4	15.75	76.76
10	26.29	252.70

Table 10. h values of in-control ARL=300 for three group CUSUM charts for multivariate normal processes

	MCX		MCZ		MCW	
p	N-5	P-5	N-5	P-5	N-5	P-5
2	5.20	5.19	5.21	5.22	5.23	5.24
10	6.60	6.62	6.81	6.82	6.84	6.82

Table 11 and Table 12 contain the results of three performance measures for the Equal and Only Shift directions respectively. The CVRL value represents a relative variation of the run length as a ratio of the average and sample standard deviation of independent run lengths, and the RISO value is the percentage of runs giving the earliest signal among the charts when simultaneously operating them for an identical out-of-control process. Better performance of the chart scheme is associated with smaller values of ARL and CVRL and larger values of RISO. As illustrated in these tables, the simulation results confirm that the multivariate QC charts based on T^{2}, SHW and CCU are directionally invariant. As long as the noncentrality parameter of the shift in mean is constant, they perform equivalently in ARL for both the mean shift directions and the different correlation types for the same number of variables. The ARL performance of CCU is best among these charts for small changes in process means. It is not true for large shifts, however. When shifting a large distance, MCX and MCW have shorter ARLs than CCU for P-5 and Equal Shift, in which the ARL performance of MCX is better than of that of CCU even for a small shift for p = 10. For the situation of shifting only in a particular variable, MCZ shows the best ARL performance for relatively larger values of noncentrality parameter. As shown in the estimated results of CVRL, the run lengths for signaling out-of-control are distributed from ARL with smaller scale as the shift is larger and CCU has the smallest values for our cases of measurement and out-of-control characteristics. For the process of higher variable dimension, CCU improves in its CVRL performance, but the performance of SHW becomes worse. The CVRL values of the group charts more or less change according to the quality control environment considered and the values of the directionally-invariant approaches depend only on the magnitude of noncentrality parameter for the equal variable dimension. The percentage of giving the quickest signal for the same out-of-control condition is closely related to the ARL value of the corresponding chart. This RISO performance of SHW is relatively better compared to the performance with the other measures, especially when shifting a very large distance in the mean vector of the process of a lower order. In quality control practice, the SHW chart is known to be quite effective in detecting a large shift. However, the estimated results of the CVRL measure for the SHW chart show that its run length distribution is widely spread even for the largest mean shift in which its ARL and RISO performances are best. For example, the SHW chart has the shortest ARL of 2.0 and the largest RISO of 0.88 for p = 2 and η_c =3.2, whereas its CVRL of 0.72 is much greater than the values of the other charts. The maximum run length of 10,000 simulations were estimated with 15, 6, 6, 5 for SHW, CCU, MCX and MCZ respectively when shifting only in the mean of a variable. It indicates that this chart usually responds most quickly on the shift of a very large amount in the process mean but it sometimes fails in signaling for some significant out-of-control condition. Table 13 gives a numerical example of an application of the control charts to a 2-variate normal process for a bad case of the SHW chart.

Table 11. Performance Comparison of QC charts for multivariate normal processes of Equal Shift

(ARL)		N-5					P-5				
p	QC	η_c =0.2	η_c =0.4	η_c =0.8	η_c =1.6	η_c =3.2	η_c =0.2	η_c =0.4	η_c =0.8	η_c =1.6	η_c =3.2
	SHW	265.9	199.4	87.6	16.9	2.0	267.7	199.3	87.8	17.1	2.0
2	CCU	143.1	50.4	15.1	5.8	2.7	144.2	50.7	15.2	5.8	2.7
	MCX	210.3	110	33.1	9.8	4.1	145.1	52.9	15.3	5.6	2.6
	MCW	167.8	60.5	15.8	5.5	2.5	165.4	60.1	15.9	5.5	2.5
	SHW	281.8	231.4	123.3	26.8	2.7	279.1	230.2	122.3	26.6	2.7
4	CCU	145.7	54.7	18.4	7.6	3.6	143.7	55.2	18.4	7.6	3.6
	MCX	239.5	146.2	52.8	14.8	5.9	145.4	54.4	16.1	6.1	2.9
	MCW	197.4	88.7	24.7	8.2	3.6	190.3	71.8	17.9	6.1	2.8
	SHW	290.1	261	173.0	52.2	4.8	289.5	261.4	173.9	52.5	4.9
10	CCU	158.7	65.9	26.6	12.0	5.9	159.8	66.2	26.5	12.0	5.9
	MCX	263.2	196.6	92.4	27.9	9.6	144.2	53.7	16.4	6.5	3.2
	MCW	239.5	137.8	44.1	13.4	5.8	222.4	91.3	20.8	6.9	3.1
(CVRL)		N-5					P-5				
p	QC	η_c =0.2	η_c =0.4	η_c =0.8	η_c =1.6	η_c =3.2	η_c =0.2	η_c =0.4	η_c =0.8	η_c =1.6	η_c =3.2
	SHW	1.00	1.01	1.00	0.97	0.71	1.01	1.00	1.01	0.97	0.72
2	CCU	0.94	0.82	0.56	0.34	0.23	0.94	0.80	0.55	0.34	0.24
	MCX	0.96	0.93	0.77	0.42	0.25	0.96	0.86	0.61	0.37	0.24
	MCW	0.95	0.86	0.61	0.37	0.24	0.95	0.86	0.61	0.36	0.24
	SHW	1.00	1.01	1.01	0.99	0.79	0.99	0.99	1.00	1.00	0.80
4	CCU	0.91	0.73	0.47	0.29	0.19	0.91	0.75	0.47	0.28	0.19
	MCX	0.98	0.94	0.84	0.48	0.27	0.94	0.86	0.59	0.35	0.23
	MCW	0.97	0.91	0.66	0.37	0.24	0.96	0.87	0.59	0.35	0.24
	SHW	1.01	1.00	0.99	0.98	0.89	1.00	1.00	0.99	1.00	0.91
10	CCU	0.80	0.59	0.35	0.22	0.15	0.82	0.58	0.35	0.22	0.15
	MCX	0.96	0.95	0.88	0.64	0.31	0.94	0.83	0.57	0.33	0.21
	MCW	0.97	0.92	0.74	0.40	0.24	0.94	0.87	0.58	0.33	0.22
(RISO)		N-5					P-5				
p	QC	η_c =0.2	η_c =0.4	η_c =0.8	η_c =1.6	η_c =3.2	η_c =0.2	η_c =0.4	η_c =0.8	η_c =1.6	η_c =3.2
	SHW	0.32	0.21	0.16	0.30	0.89	0.32	0.21	0.16	0.29	0.88
2	CCU	0.42	0.61	0.70	0.60	0.30	0.38	0.53	0.59	0.54	0.29
	MCX	0.28	0.20	0.12	0.08	0.03	0.43	0.51	0.60	0.65	0.35
	MCW	0.36	0.44	0.65	0.79	0.40	0.33	0.39	0.57	0.71	0.40
	SHW	0.29	0.19	0.14	0.26	0.85	0.28	0.17	0.12	0.21	0.77
4	CCU	0.36	0.57	0.70	0.65	0.27	0.29	0.37	0.24	0.09	0.09
	MCX	0.25	0.16	0.09	0.06	0.02	0.40	0.52	0.67	0.66	0.41
	MCW	0.27	0.26	0.31	0.45	0.27	0.25	0.27	0.47	0.68	0.49
	SHW	0.28	0.19	0.13	0.19	0.75	0.25	0.15	0.09	0.12	0.53
10	CCU	0.30	0.51	0.62	0.55	0.23	0.21	0.20	0.03	0.00	0.00
	MCX	0.24	0.16	0.11	0.08	0.03	0.42	0.62	0.82	0.80	0.57
	MCW	0.24	0.21	0.24	0.38	0.27	0.20	0.17	0.30	0.60	0.63

Table 12. Performance Comparison of QC charts for multivariate normal processes of Only Shift

	(ARL)	N-5					P-5				
p	QC	η_c=0.2	η_c=0.4	η_c=0.8	η_c=1.6	η_c=3.2	η_c=0.2	η_c=0.4	η_c=0.8	η_c=1.6	η_c=3.2
	SHW	268.6	198.3	87.7	17.0	2.0	271	201.9	88.5	16.8	2.0
2	CCU	144.4	50.6	15.2	5.7	2.7	144.3	51.3	15.2	5.8	2.7
	MCX	185.3	77.0	20.4	6.6	2.9	190.3	77.4	20.5	6.6	2.9
	MCZ	156.6	56.5	15.4	5.4	2.5	157.5	57.2	15.4	5.4	2.5
	SHW	279.1	230.9	121.2	26.6	2.8	281.4	229.5	121.2	26.8	2.8
4	CCU	144.6	55.0	18.4	7.6	3.6	145.3	54.9	18.5	7.6	3.6
	MCX	223.2	108.7	27.4	8.3	3.5	220.4	108.6	27.8	8.3	3.5
	MCZ	186	69.7	17.7	6.1	2.8	183.8	68.5	17.7	6.1	2.8
	SHW	291.2	261.6	173.0	52.1	4.9	290.8	260.2	173.9	52.2	4.9
10	CCU	157.4	66.0	26.6	12.0	5.9	157.5	65.8	26.7	12.0	5.9
	MCX	256.0	146.4	36.7	10.3	4.2	258.6	151.5	37.7	10.3	4.2
	MCZ	216.6	88.2	20.8	6.9	3.1	219.4	87.4	20.9	6.9	3.1
	(CVRL)	N-5					P-5				
p	QC	η_c=0.2	η_c=0.4	η_c=0.8	η_c=1.6	η_c=3.2	η_c=0.2	η_c=0.4	η_c=0.8	η_c=1.6	η_c=3.2
	SHW	1.01	1.01	1.00	0.98	0.72	1.02	1.02	0.98	0.98	0.72
2	CCU	0.94	0.81	0.56	0.34	0.24	0.95	0.82	0.57	0.34	0.24
	MCX	0.97	0.89	0.68	0.4	0.26	0.97	0.89	0.68	0.40	0.26
	MCZ	0.96	0.86	0.61	0.36	0.24	0.95	0.86	0.63	0.36	0.24
	SHW	1.00	1.01	0.99	0.99	0.79	1.00	1.00	0.99	1.00	0.79
4	CCU	0.91	0.74	0.47	0.28	0.19	0.91	0.73	0.46	0.28	0.19
	MCX	0.97	0.92	0.70	0.41	0.26	0.98	0.93	0.70	0.41	0.26
	MCZ	0.98	0.86	0.59	0.35	0.24	0.95	0.85	0.59	0.34	0.24
	SHW	1.01	1.01	1.00	0.98	0.90	1.00	1.01	0.99	0.98	0.89
10	CCU	0.81	0.58	0.35	0.22	0.15	0.81	0.58	0.35	0.22	0.15
	MCX	0.97	0.91	0.70	0.42	0.26	0.97	0.94	0.72	0.41	0.26
	MCZ	0.94	0.88	0.57	0.33	0.22	0.96	0.87	0.56	0.33	0.22
	(RISO)	N-5					P-5				
p	QC	η_c=0.2	η_c=0.4	η_c=0.8	η_c=1.6	η_c=3.2	η_c=0.2	η_c=0.4	η_c=0.8	η_c=1.6	η_c=3.2
	SHW	0.32	0.21	0.16	0.29	0.88	0.32	0.20	0.16	0.30	0.88
2	CCU	0.38	0.56	0.62	0.55	0.29	0.39	0.56	0.63	0.55	0.29
	MCX	0.31	0.29	0.31	0.37	0.25	0.31	0.30	0.31	0.37	0.24
	MCZ	0.40	0.49	0.67	0.77	0.42	0.39	0.49	0.67	0.77	0.41
	SHW	0.29	0.19	0.13	0.22	0.77	0.29	0.18	0.13	0.22	0.77
4	CCU	0.34	0.49	0.39	0.15	0.10	0.34	0.49	0.39	0.15	0.10
	MCX	0.25	0.21	0.20	0.23	0.19	0.25	0.21	0.21	0.23	0.20
	MCZ	0.30	0.36	0.64	0.82	0.51	0.30	0.37	0.65	0.82	0.51
	SHW	0.27	0.18	0.10	0.13	0.53	0.27	0.18	0.10	0.13	0.53
10	CCU	0.29	0.40	0.14	0.00	0.00	0.30	0.40	0.15	0.00	0.00
	MCX	0.24	0.18	0.17	0.16	0.18	0.24	0.18	0.17	0.16	0.17
	MCZ	0.27	0.35	0.73	0.87	0.67	0.27	0.35	0.73	0.87	0.67

Table 13. Example of QC control chart procedures for a 2−variate normal process of Only Shift with η_c =3.2

n	$X_{n,1}$	$X_{n,2}$	χ^2	$S_{n,1}$	$S_{n,2}$	T_S^2	MCX	$Z_{n,1}$	$Z_{n,2}$	MCZ
1	0.37	-2.00	6.49	0.30	-1.61	4.19	1.50	1.58	-2.52	2.02
2	1.20	-0.09	2.07	1.26	-1.43	7.25	1.09	1.44	-0.79	2.31
3	2.01	-0.08	5.62	2.94	-1.36	19.28	2.21	2.37	-1.26	3.89
4	2.04	-0.22	6.20	4.61	-1.46	40.20	3.75	2.48	-1.43	5.87
5	2.86	0.41	9.59				6.11			
6	1.79	-0.70	6.57							
7	1.80	0.18	3.91							
8	1.08	-1.09	4.72							
9	1.77	-0.88	7.27							
10	2.70	-0.25	10.66							
11	0.65	-0.47	3.19							
12	3.19	0.93	10.79							
13	3.25	0.69	11.77							

5. SUMMARY AND CONCLUSIONS

This study has presented extensive comparisons between three group CUSUM charts and two multivariate QC charts for multivariate normal processes using simulated data with various correlation structures and mean shift directions. The performance of the group CUSUM charts appears to be more or less affected by the correlation structure of the process, but MCW and MCZ work without significant variation when the process mean vector changes only in one variable. The CCU chart scheme is generally more effective in detecting a small departure from the in-control mean vector than the group CUSUM charts and has the smallest relative variation of the run length. When the process is positively correlated and experiences substantial shifts in the means of all variables, MCX and MCW have better ARL performance than CCU and the ARL performance of MCZ is superior for the shift of relatively larger amounts only in the mean of one variable. Although the Shewhart chart has an advantage in computation over the other complicate schemes as MCX does, it appears to be ineffective in detecting a small mean shift for multivariate processes. The Shewhart chart offers a good detection for a considerably large shift in the mean level, however, it is likely to fail in giving a signal on the out-of-control condition that the process changes without extreme outliers. For the out-of-control situation only in one variable, MCX and

MCZ operates with a high degree of correctness in detecting the right out-of-control variable, thereby being appropriate for the QC procedure to concern the interpretation in terms of original variables rather than monitoring the signal.

Overall, the multivariate QC chart CCU appears to provide good protection for the shifts in unanticipated directions in the mean level of multivariate processes. For some special QC environments, however, the group CUSUM charts offer better protection for the out-of-control situation. If one is interested in detecting a shift only in one specified direction, MCZ is the optimal proposal. When the process is positively correlated, it is susceptible to simultaneous shifts in the means of all variables. For this situation, MCX is quite effective for the protection, and the signal from MCX can correspond to an immediately identified variable. Another group chart MCW appears to be competitive to the other superior schemes for shifting relatively larger distances in the mean vector and is suggested for controlling the means of multivariate processes if the principal components are interpretable.

REFERENCES

[1] Alt, F.B., "Multivariate Quality Control," in *Encyclopedia of the Statistical Sciences* (Vol.6), eds. N.L. Johnson, S. Kotz and C.R. Read, New York: John Wiley(1985), 111-122.

[2] Alwan, L.C., "CUSUM Quality Control - Multivariate Approach," *Communications in Statistics - Theory and Methods*, 15(1986), 3531-3543.

[3] Crosier, R.B., "Multivariate Generalizations of Cumulative Sum Quality Control Schemes," *Technometrics*, 30(1988), 291-303.

[4] Doganaksoy, N., Faltin, F.W. and Tucker, W.T., "Identification of Out-of-Control Quality Characteristics in a Multivariate Manufacturing Environment," *Communications in Statistics — Theory and Methods*, 20(1991), 2775-2790.

[5] Jackson, J.E., "Multivariate Quality Control," *Communications in Statistics — Theory and Methods*, 14(1985), 2657-2688.

[6] Hawkins, D.M., "The Detection of Errors in Multivariate Data by Principal Component Analysis," *Journal of the American Statistical Association*, 22(1974), 275-286.

[7] Hawkins, D.M., "Multivariate Quality Control Based on Regression-Adjusted Variables," *Technometrics*, 33(1991), 61-75.

[8] Hawkins, D.M., "The Regression Adjustment for Variables in Multivariate Quality Control," *Journal of Quality Technology*, 25(1993), 170-180.

[9] Healy, J.D., "A Note on Multivariate CUSUM Procedures," *Technometrics*, 29(1987), 409-412.

[10] Hotelling, H., "Multivariate Quality Control — Illustrated by the Air Testing of Sample Bombsights," *Techniques of Statistical Analysis*, eds. C. Eisenhart, M.W. Hastay and W.A. Wallis, New York: McGraw-Hill(1947), 111-184.

[11] Lowry, C.A., Woodall, W.H., Champ, C.W. and Rigdon, S.E., "A Multivariate Exponentially Weighted Moving Average Control Chart," *Technometrics*, 34(1992), 46-53.

[12] Pignatiello, J.J., Jr. and Runger, G.C., "Comparisons of Multivariate CUSUM Charts," *Journal of Quality Technology*, 22(1990), 173-186.

[13] Woodall, W.H. and Ncube, M.M., "Multivariate CUSUM Quality Control Procedures," *Technometrics*, 27(1985), 285-292.

[14] Yashchin, E., "Analysis of CUSUM and Other Markov-type Control Schemes by Using Empirical Distributions," *Technometrics*, 34(1992), 54-63.