

Algorithm for Generating Traffic Distributions in ATM Networks using 2-D LHCA

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ABSTRACT

Using Asynchronous Transfer Mode(ATM) which is a high-bandwidth, low-delay, cell switching and multiplexing technology, Broadband-Integrated Services Digital Network (B-ISDN) can support communication services of all kinds. To evaluate the performance of ATM networks, traffic source models to meet the requirements are demanded. We can obtain random traffic distribution for ATM networks by using the Cellular Automata (CA) which have effective random pattern generation capability. In this paper we propose an algorithm using 2-D LHCA to generate more effective random patterns with good random characteristics. And we show that the randomness by 2-D LHCA is better than that of the randomness by 1-D LHCA.

2-D LHCA를 이용한 ATM 망에서의 트래픽 분포 발생 알고리즘

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요 약

비동기식 전송모드(ATM)는 대역폭이 높고 전송지연이 적은 셀 교환 및 셀 다중화 기술이며, 이러한 ATM을 이용하여 광대역 종합 정보통신망(B-ISDN)은 다양한 통신 서비스를 제공할 수 있다. ATM 망 성능을 평가하기 위하여 실제상황에 맞는 트래픽 소스 모델이 필요하다. 효과적으로 랜덤패턴을 생성하는 셀룰라 오토마타를 이용하여 ATM 망에서의 트래픽 분포를 얻을 수 있다. 본 논문에서는 좀더 효과적인 랜덤패턴을 생성하기 위하여 2-D LHCA를 이용하는 알고리즘을 제안한다. 제안된 2-D LHCA에 의한 랜덤성이 1-D LHCA에 의한 랜덤성 보다 더 좋다는 것을 보인다.

1. Introduction

ATM can accomodate variable bit-rate services because slots are allocated to services on demand. It should provide the ability to transport connection

oriented and connectionless traffic and negotiated Quality of Services (QoS) to the end user. Also, to a network provider, it enables the transport of different traffic types through the same network. To evaluate the performance of ATM networks, traffic source models to meet the requirements are demanded. Therefore, it is necessary to generate cell arrivals closely resembling the traffic existed in real networks. In approaching for modeling traffic sources, traditional stochastic processes such as Poisson process, exponential distribution, Bernoulli distribution and Markov process are used. But these

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modeling techniques are not sufficient to simulate actual traffic[1].

Cellular Automata have been introduced by Von Neumann and Ulam as models of self-organizing and self-reproducing behaviors[2,3]. A CA is a necessity in many application areas such as test pattern generation, pseudo-random number generation, cryptography, error correcting codes and signature analysis[4-9]. Since a CA has a regular uniform array of nearest neighbor interconnection with combinational logic, it can effectively generate random patterns which have good randomness characteristics. It is classified into One-Dimensional CA(1-D CA) and Two-Dimensional CA(2-D CA) by its neighborhood connection and transition rule. In recent years, One-Dimensional Linear Hybrid CA(1-D LHCA) has been proposed as an alternative to Linear Feedback Shift Register(LFSR)[10].

In this paper, we consider 2-D LHCA as extensions of 1-D LHCA which can display much better random patterns than those generated by 1-D LHCA or LFSR. We propose boundary conditions of 2-D LHCA to find 2-D LHCA rules whose characteristic polynomials are primitive polynomials of degree 32 and propose an algorithm for generating traffic distribution using 2-D LHCA. It is well-known that a CA has a maximum length if and only if the characteristic polynomial of the transition matrix is primitive. But it is very difficult to find 2-D LHCA rules whose characteristic polynomials are primitive polynomials of degree 32. We give three 2-D LHCA rules whose characteristic polynomials are primitive polynomials of degree 32 respectively. And we show that the randomness of our result is better than the randomness by 1-D LHCA by the comparison of 1-D LHCA and 2-D LHCA.

2. Preliminaries

2.1 Definition[12]

A primitive polynomial $p(x)$ of degree n is an irreducible polynomial such that the minimum

value of m for which $p(x)$ divides $x^m + 1$ is $2^n - 1$.

It is well-known that a CA has a maximum length if and only if the characteristic polynomial of the transition matrix is primitive[13].

2.2 Example

a) The polynomial $x^4 + x^3 + x^2 + x + 1$ is irreducible but not primitive.

b) The polynomial

$$x^{32} + x^{29} + x^{28} + x^{27} + x^{25} + x^{24} + x^{21} + x^{20} + x^{18} + x^{16} + x^{15} + x^{14} + x^{12} + x^6 + x^5 + x^4 + x^2 + x + 1$$

is primitive.

2.3 Definition[5]

A 2-D CA is a generalization of 1-D CA, where the cells are arranged in a two-dimensional grid with connections among the neighboring cells. A 2-D CA is composed of mn cells organized as an $m \times n$ array with m rows and n columns. The state transition of the 2-D CA can be represented by an $mn \times mn$ binary matrix. The next state of a cell depends on its four neighbors(top, left, bottom, right) and itself(five-neighborhood dependency). Thus, the next state q of the (i, j) -th cell of a 2-D CA is given by

$$q_{ij}^{t+1} = f(q_{ij}^t, q_{i-1,j}^t, q_{i,j-1}^t, q_{i+1,j}^t, q_{i,j+1}^t)$$

From now we consider only 2-D CA with a linear neighborhood relationship(XOR function). For the five neighborhood dependency, rule can be expressed as a 5-bit number(self, top, left, bottom, right), where each bit signifies the presence of the corresponding dependency. The process of partitioning the state transition matrix of a 2-D CA is reported in [6].

3. Main Results

In this section, we propose boundary conditions of 2-D LHCA and propose an algorithm for genera-

ting traffic distributions using 2-D LHCA. And we give three 2-D LHCA rules whose characteristic polynomials are primitive polynomials of degree 32 respectively.

3.1 Definitions

3.1.1 Boundary conditions

First, boundary cells in $m \times n$ 2-D CA are given by

- a_{i1} : left-most cell in each row
- a_{in} : right-most cell in each row
- a_{1j} : top-most cell in each column
- a_{mj} : bottom-most cell in each column

where $i = 1, \dots, m$, $j = 1, \dots, n$

Then boundary conditions can be classified as following.

- a) Class 1 : Null Boundary 2-D CA(See Fig. 1)
- The boundary values are all zero.

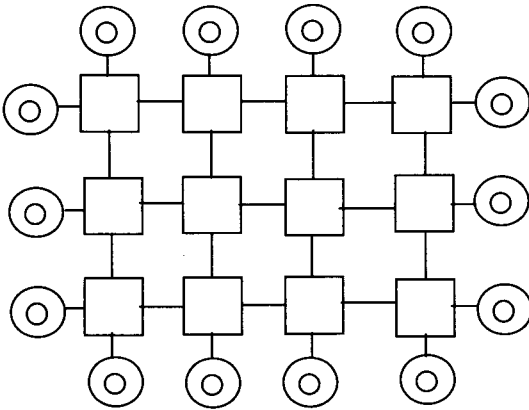


Fig. 1. Null Boundary 2-D CA

- b) Class 2 : Intermediate Boundary 2-D CA

- Boundary conditions are divided into three types by different boundary value of left (right)-most cell in each row. And boundary values of top and bottom are zero.

① Type 1 : Pure 2-D IBCA(See Fig. 2)

- Intermediate boundary condition is applied to only a_{11} and a_{mn} .

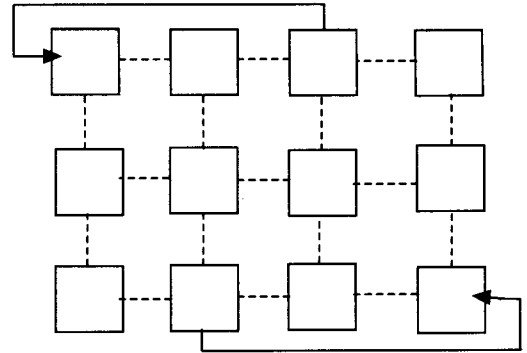


Fig. 2. Type 1 : Pure 2-D IBCA

The rest of boundary cells have null boundary condition.

And a_{13} be the left neighbor of a_{11} and $a_{m,n-2}$ are the right neighbor of a_{mn}

② Type 2 : Inner 2-D IBCA(See Fig. 3)

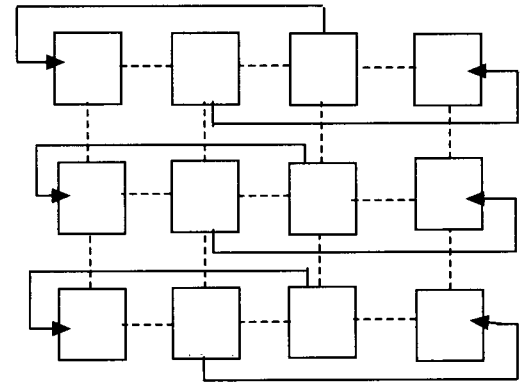


Fig. 3. Type 2 : Inner 2-D IBCA

- Intermediate boundary condition is applied to only a_{i1} and a_{in} where $i = 1, \dots, m$. That is, left neighbor of a_{i1} is a_{i3} and right neighbor of a_{in} is $a_{i,n-2}$.

③ Type 3 : Outer 2-D IBCA(See Fig. 4)

- a_{11} , a_{mn} cells obey Type 1. Left neighbor of a_{i1} is $a_{i-1,n}$ and right neighbor of a_{in} is $a_{i+1,1}$.

We use a video traffic source. Now, a single video source is approximated by an autoregressive

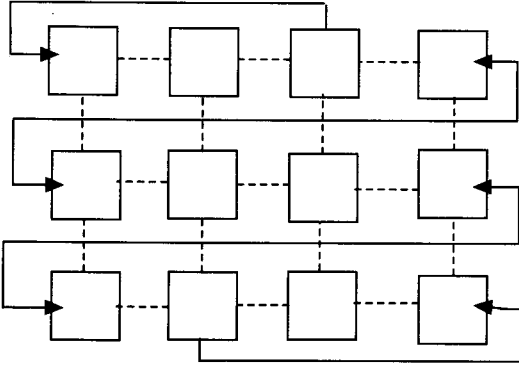


Fig. 4. Type 3 : Outer 2-D IBCA

process. The definition of an autoregressive process is as follows:

3.1.2 AR(1) Model[1,2]

An autoregressive process $\{X(n)\}$ is given by

$$X(n) = \sum_{m=1}^K \lambda_m X(n-m) + \mu w(n),$$

where $X(n)$ represents the source bit rate during the n -th frame; K is the model order; $w(n)$ is a Gaussian random process; and $\lambda_m (m=1, 2, \dots, K)$ and μ are coefficients.

The steady-state distribution of X is Gaussian. It is shown that the first order autoregressive Markov model $X(n) = \lambda_1 X(n-1) + \mu w(n)$ is sufficient for engineering purposes. This model provides the rather accurate approximation of the bit rate of a single video source without scene changes. This model is suitable for use in simulations[4].

3.2 Algorithm

The well-known Rejection Method can be used to simulate a random variable having density function $g(x)$. We can use this as the basis for simulating from the continuous distribution having density $f(x)$ by simulating Y from g and then accepting this simulated value with a probability proportional to $\frac{f(Y)}{g(Y)}$.

Let c be a constant such that

$$\frac{f(y)}{g(y)} \leq c$$

for all y .

Since each iteration during executing the algorithm will, independently, result in an accepted value with probability $P\left\{U \leq \frac{f(Y)}{cg(Y)}\right\} = \frac{1}{c}$, it follows that the number of iterations is geometric with mean c [14].

To simulate a unit Gaussian random variable Z (that is, one with mean 0 and variance 1), note that the absolute value of Z has density function

$$f(x) = \frac{2}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad 0 < x < \infty$$

We will start by simulating from the above density by using the rejection method with

$$g(x) = e^{-x}, \quad 0 < x < \infty$$

Now, note that

$$\frac{f(x)}{g(x)} = \sqrt{\frac{2e}{\pi}} \exp\left\{-\frac{(x-1)^2}{2}\right\} \leq \sqrt{\frac{2e}{\pi}}$$

$$\text{Let } c = \sqrt{\frac{2e}{\pi}}. \text{ Then } \frac{f(x)}{g(x)} \leq c.$$

The algorithm is as follows :

3.2.1 The algorithm for generating a Gaussian distribution using 2-D LHCA

step 1 : Generate R_1, R_2 ;

step 2 : Compute

$$Y_1 = -\log(R_1), Y_2 = -\log(R_2);$$

step 3 : If $Y_2 - \frac{(Y_1-1)^2}{2} > 0$,

$$\text{set } Y = Y_2 - \frac{(Y_1-1)^2}{2}$$

and go to step 4 ;

Otherwise go to step 1 ;

step 4 : Generate a random number U and set

$$Z = \begin{cases} Y_1, & \text{if } U \leq \frac{1}{2} \\ -Y_1, & \text{if } U > \frac{1}{2} \end{cases}$$

The random variables Z and Y generated by the above are independent with Z being normal with mean 0 and variance 1 and Y being exponential with rate 1.

And Z can be replaced by $\mu + \sigma z$ to have a Gaussian random variable with mean μ and variance σ^2 . The other variables Y_1 , Y_2 and Y are all exponential variables with mean 1. If U is a uniform variable on $(0,1)$, $-\log U$ is a exponential variable with mean 1 [14].

3.2.2 2-D LHCA Rules

Three 2-D LHCA's are used to generate three independent uniform variables (R_1 , R_2 and U).

Here, R_2 is generated by the rule

$$\begin{bmatrix} 23 & 30 & 23 & 30 & 23 & 30 & 23 & 30 \\ 30 & 23 & 30 & 23 & 30 & 23 & 30 & 23 \\ 23 & 30 & 23 & 30 & 23 & 30 & 23 & 30 \\ 30 & 23 & 30 & 23 & 30 & 23 & 30 & 23 \end{bmatrix} \text{ with Type 1,}$$

R_1 is generated by the rule

$$\begin{bmatrix} 7 & 14 & 7 & 14 & 7 & 14 & 7 & 14 \\ 14 & 7 & 14 & 7 & 14 & 7 & 14 & 7 \\ 7 & 14 & 7 & 14 & 7 & 14 & 7 & 14 \\ 14 & 7 & 14 & 7 & 14 & 7 & 14 & 7 \end{bmatrix} \text{ with Type 1}$$

and

U is generated by the rule

$$\begin{bmatrix} 21 & 5 & 21 & 21 & 21 & 21 & 5 & 21 \\ 21 & 21 & 21 & 5 & 21 & 21 & 21 & 21 \\ 21 & 5 & 5 & 21 & 21 & 21 & 21 & 5 \\ 5 & 21 & 5 & 5 & 5 & 21 & 21 & 5 \end{bmatrix} \text{ with Type 3.}$$

These three rules have the primitive polynomials:

$$x^{32} + x^{29} + x^{27} + x^{26} + x^{25} + x^{24} + x^{19} + x^{16} + x^{15} + x^{12} + x^{10} + x^7 + x^6 + x + 1,$$

$$x^{32} + x^{29} + x^{28} + x^{27} + x^{25} + x^{24} + x^{21} + x^{20} + x^{18} + x^{16} + x^{15} + x^{14} + x^{12} + x^6 + x^5 + x^4 + x^2 + x + 1$$

and $x^{32} + x^{31} + x^{30} + x^{10} + 1$ respectively.

We found the above three 2-D CA rules by many simulations.

Fig. 5 shows the simulating result that is the Probability Density Function curve of the generated

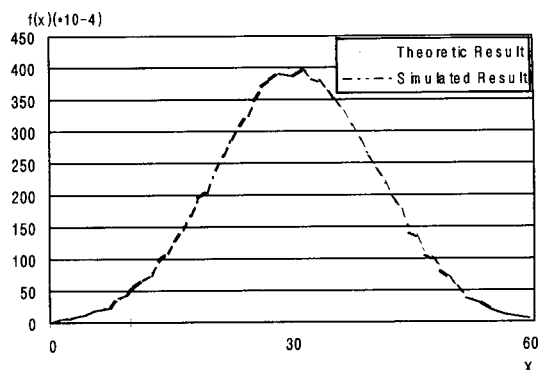


Fig. 5. Probability Density Function curve of a generated Gaussian random sequence

Gaussian random sequence using 2-D LHCA.

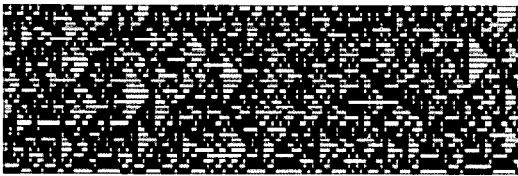
3.3 Statistical testing

The statistical testing results are satisfactory as following:

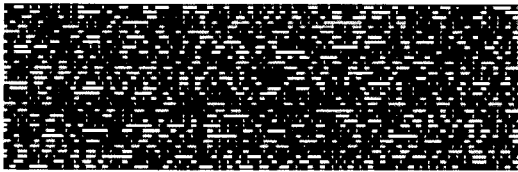
N	2015	Sum Wgts	2015
Mean	29.82471	Sum	60096.79
Std Dev	9.995745	Variance	99.91491
Skewness	-0.06845	Kurtosis	-0.28458
USS	1993598	CSS	201228.6
CV	33.51498	Std Mean	0.222678
T:Mean=0	133.9364	Pr> T	0.0001
Sgn Rank	1015551	Pr>= S	0.0001
Num ^=0	2015		
M(Sign)	1005.5	Pr>= M	0.0001
D:Normal	0.016822	Pr>D	>0.15

3.4 Comparison of 1-D LHCA and 2-D LHCA

In this section we compare 1-D LHCA and 2-D LHCA. The test criteria considered for this comparative study include bit frequency test, bit auto-/cross-correlation and visual inspection of state-time diagram. In fact, in Figure 5 it is not possible to compare the pdf curves of generated Gaussian random sequences by 1-D LHCA and 2-D LHCA. So we give a figure (Fig.6) and a table (Table 1)



(a)



(b)

Fig. 6. (a) Random non-zero initial sites with 1-D LHCA.
(b) Random non-zero initial sites with 2-D LHCA

comparing the randomness of 1-D LHCA and 2-D LHCA. Fig.6 and Table 1 [6] show that the randomness of 2-D LHCA is better than that of 1-D LHCA.

4. Conclusion

In this paper, we proposed new boundary conditions of 2-D LHCA and proposed an algorithm for generating traffic distributions using 2-D LHCA in ATM networks.

By the comparison of 1-D LHCA and 2-D LHCA we showed that the randomness of our result is better than that of the randomness by 1-D LHCA. Also, we found three 2-D LHCA rules whose characteristic polynomials are primitive polynomials of degree 32 respectively.

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Table 1. < Comparison of 1-D LHCA & 2-D LHCA Randomness >

		32-cell 1-D LHCA			4×8 2-D LHCA		
		First 8 cells	First 16 cells	All 32 cells	One row 8-cell	Two rows 16-cell	Four rows 32-cell
Bit frequency	n=100 n=500 n=1000	[43,55]% n=225	[42,57]% [47,53]% [48,53]%	[44,60]% [46,53]% [46,54]%	[48,50]% n=225	[48,51]% 50% 50%	[45,54]% [48,50]% 50%
Bit auto- correlation	t=1 t=20 t=200	[.46,.55] [.43,.56]	[.46,.52] [.48,.54] [.47,.52]	[.48,.53] [.48,.53] [.45,.53]	[.49,.50] [.48,.52]	.50 .50 [.49,.50]	[.49,.50] .50 .50
Cross- correlation	Between any two columns	[.46,.53]	[.49,.51]	[.48,.51]	.50	.50	[.49,.50]
Visual test	Visual inspection	Triangles visible	Triangles visible	No regularity	No regularity	No regularity	No regularity

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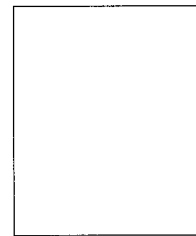
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