고속충격하중을 받는 평면구조의 유한요소해석

Finite Element Simulation of High-Speed Impact in Plane Structure

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요 지

본 연구는 등방탄성체에 고속충격하중이 작용하는 경우에 대한 유한요소해석에 관한 것으로 대상구조는 여러가지 모양의 2차원 평면구조를 택하였다. Galerkin 방법을 이용하여 유한요소 정식화하였으며 직접시간적분법에 의해 수치해를 구하였다. 본 해석에서는 균열이 없는 평판으로 수치해와 이론해를 비교하여 수치해의 신뢰성을 확인하였으며, 0°, 30°, 45°경사 균열을 가진 평판에 적용한 3가지 예를 분석하였다. 수치해석 결과는 이론해의 결과와 상호 잘 일치하였다.

핵심용어 : 응력파, 초고속충격

Abstract

In this investigation the use of the finite element method in the analysis of impact-induced elastic stress waves in cracked plane is examined. The plane is assumed to be isotropic and elastic. The equation of motion is developed using Newton's second law of motion and the finite element formulation for the elastic stress wave propagation is developed using the Galerkin's method. Stress wave propagation include material-dependent wave speed and time-dependent stress fields. Numerical solution of elastic stress wave equation is iteratively obtained using a direct implicit scheme. The time-dependent part of the load is a step pulse. The solution obtained using the finite element method is compared with the solution obtained by using an analytic method. Numerical results show that there is a good agreement between the solution obtained by using an analytic method and the finite element solution in the analysis of the stress wave motion. Finally, the elastic stress wave intensity at the cracked area is discussed.

Keywords : *stress wave, hyper-velouity impact*

1. Introduction

In recent years there has been renewed interest in stress wave propagation in elastic media because of current developments in materials and structures. In the early 1970s, an analytic

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method of stress wave propagation had been developed for the one dimensional elasto-dynamic problem. The quantity of kinetic energy should be concerned about impact body in the case of an impact load.

Recently, a large number of investigators have studied the stress wave propagation problems dealing with boundary integral method[1),2)], direct time integration method of Kirchhoff equation[3),4)] and Dirichlet-to-Neumann(DtN) method[5)~10)]. The boundary integral method is required many boundary solutions to obtain an exact solutions at the boundary integral transform region, and in this method, the reliability of analysis is existed at a low velocity of stress wave. The direct time integration method of Kirchhoff equation requires a lot of memory device and the stress wave analysis is impossible when the analysis dimension is raised. DtN method is required an exact boundary conditions and the results of analysis have not obtained specific values in the space-time domain. Therefore, the general solutions of DtN method are obtained by the time convolutions.

For a dynamic problem, the Fourier transformation formulate the displacement field in the transformed domain by using the modal analysis and the inverse transformation can be evaluated by using the FFT[11),12)] and the displacement field in the space-time domain can be obtained. This method is accompanied by inverse transformation to obtain the space-time domain solutions.

The finite element method has been used for structural dynamic analysis. The convergence and accuracy of finite element analysis is dependent upon the discretization of space-time domain. The difficulty lies not in the formulation, but in the cumbersome calculation necessary to get convergence and accuracy.

This investigation propose a numerical method which combines the finite element method with the method of direct integration and stress wave propagation is studied through application of the numerical method. A finite element program for elastic stress wave propagation is developed in order to investigate the shape of stress field at time increment. The numerical solution of the propagation of hyper-velocity impact induced stress waves in indefective plane is obtained and compared with the solution obtained by the use of analytic method. The reliability and accuracy of the numerical analysis are compared with the analytic solution.

The results for stress fields of three cracked plane examples excited by an impact load in the space-time domain are studied. The shape of stress wave propagation and, in addition, the stress wave intensity are discussed on the cracked planes. It is the goal of this paper to provide a reliable method to predict these phenomena with a representative model.

2. Governing Equations

Stress wave equation in the isotropic elastic medium can be obtained by using force balance between the elastic and inertial forces acting on a small portion of the cube. In the tensor notation,

$$\frac{\partial T_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2} \tag{1}$$

where T_{ij} is the stress tensor and u_i is the displacement with Cartesian components and ρ and t are, respectively, the density of the medium and the time. For a two-dimensional problem, the time-dependent transient stress wave equations in the isotropic x-y plane are obtained by the following differential equation using the nabla operator :

$$\rho\frac{\partial^2 u}{\partial t^2} = (\lambda+2\mu)\nabla(\nabla\cdot u)$$

$$+(\lambda+\mu)\nabla\times\nabla\times u + f \quad (2)$$

where u is the displacement vector, f is the external force vector applied on the point and λ and μ are, respectively, the Lamé constant and the rigidity modulus of the medium. The longitudinal stress wave velocity V_l and the shear stress wave velocity V_s are given by

$$V_l = \sqrt{\frac{\lambda+2\mu}{\rho}}, \quad V_s = \sqrt{\frac{\mu}{\rho}} \quad (3)$$

The stress wave intensity or stress wave energy flux is obtained using the law of energy conservation (J.A. Stratton, 1941) by the following equations :

$$\frac{\partial w}{\partial t} + \triangle\cdot K = 0 \quad (4)$$

where $\frac{\partial w}{\partial t}$ represents the time rate change of energy per unit volume and K is the stress wave intensity. Using a Green's theorem, The time rate change of energy per unit volume is equal to zero. Thus

$$K_{t+\Delta t} = K_t \frac{S_t}{S_{t+\Delta t}} \quad (5)$$

where a subscript t stands for the time, $\triangle t$ is the discrete time increment and S is used to denote a stress wave propagated area.

The stress wave intensity is in inverse proportion to the ratio of propagated area with each other by changing propagated area at time increment. Physically, the above equation implies that the amplitude of stress wave front decreases with the radius from the impact load point.

3. Finite Element Approximation for Stress Wave Equations

For the two dimensional problem in the x-y plane, the functional $I_{(u)}$ of the equation (2) is defined using variational methods :

$$I_{(u)} = \int\int\Big(\frac{1}{2}(\lambda+2\mu)[(\nabla u_x)^2+(\nabla u_y)^2]$$

$$+\frac{1}{2}(\lambda+\mu)(\nabla\times u)^2 - \frac{1}{2}\rho\frac{\partial^2 u}{\partial t^2} + fu\Big)dx\ dy \quad (6)$$

where f is the applied external force vector, and the other symbols are as defined previously.

The finite element approximation to the governing equations given in the previous section is based on the 4-node quadrilateral isoparametric element. The finite element discretization given in equation (6) for the stress wave analysis is based on modelling in the variable, i.e., time and displacement, u_x and u_y can be approximated by the expressions :

$$u_x = \sum_{i=1}^{n} N_i(x,\ y)\, u_{xi}(t) \quad (7a)$$

$$u_y = \sum_{i=1}^{n} N_i(x,\ y)\, u_{yi}(t) \quad (7b)$$

where N_i is an interpolation function of x and y, u_{xi} and u_{yi} are nodal values of u_x and u_y, respectively. This shape function, N_i, satisfies the requirements of C^0 linear continuity condition.

Substituting the appropriate shape function for the elements into equation (6), and leads to

$$[K_{tt}]\{\ddot{u}\} + [K]\{u\} + \{F(t)\} = 0 \quad (8)$$

In this equation $[K_{tt}]$ and $[K]$ are, respectively, the inertia matrix and the stiffness matrix, $F(t)$ is the load vector, u is the unknown nodal displacement vector. Equation (8) is commonly found in the solution of dynamic problems and shows that the equation of stress wave propagation can be solved using the same approach as other dynamic problems.

In this investigation the transient response is discretized using a single-step scheme. A range of direct integration methods are at our disposal to solve this time differential problem numerically. For simplicity, let us choose implicit method[14),15)] :

$$u(t_n+\triangle t) = u(t_n)+\frac{\triangle t}{2}\{\dot{u}(t_n) + \dot{u}(t_n+\triangle t)\} \quad (9a)$$

$$\dot{u}(t_n+\triangle t) = \dot{u}(t_n)+\frac{\triangle t}{2}\{\ddot{u}(t_n) + \ddot{u}(t_n+\triangle t)\} \quad (9b)$$

where $\triangle t=t_{n+1} - t_n$ and $u(t_n)=u_n$ and the superposed "." signifies time differentiation.

Using equation (9), equation (8) becomes

$$[K^*]\{u\}_{n+1} = \{F^*\}_{n+1} \quad (10)$$

where $\{F^*\}_{n+1}=\{F(t+\triangle t)\}+[K_{tt}]\left(\frac{4}{\triangle t^2}\{u\}_n+\frac{4}{\triangle t}\{\dot{u}\}_n+\{\ddot{u}\}_n\right)$

and $[K^*] = [K] + \frac{4}{\triangle t^2}[K_{tt}]$.

The nodal displacements calculated from equation (10) are used to directly evaluate the node point stresses at time $t+\triangle t$

4. Finite Element Models and Material

The geometric profile of finite element analysis (FEA) model is shown in Fig. 1. Coordinate axes x and y are defined along the horizontal side and the vertical distance from the origin at the lower left corner of the plane. The plane is a regular square whose dimension is 200mm and the length of the defect of the plane is 60mm (a/W=0.3). Specifically the present paper deals with the following cases : (i) indefective plane; (ii) an angle of inclination of crack is 0°; (iii) an angle of inclination of crack is 30°; and (iv) an angle of inclination of crack is 45°. Parameters used to finite element models are listed in Table 1.

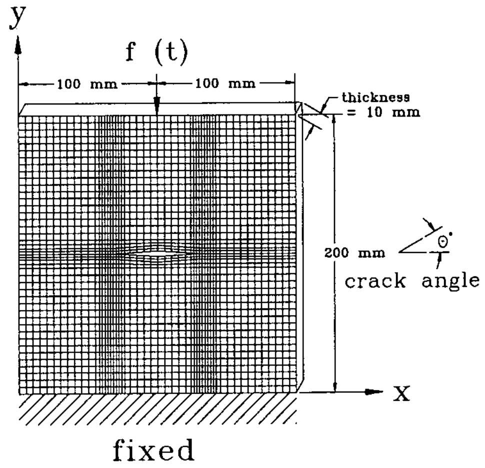


Fig. 1 The geometric profile and finite element model

For each phase we define an initial displacement, velocity and acceleration. The subscript 0 is taken to represent an initial condition :

Table 1 Material properties of FEA model

Parameter	Description	Value
E	Young's Modulus	62 GPa
G	Shear Modulus	25 GPa
ρ	Density	2300 kg/m³
ν	Poisson's Ratio	0.24
V_B	Stress Wave Speed at Bar	5191 m/sec
V_V	Stress Wave Speed at Bulk	5600 m/sec

$$u_0 = 0,\ \dot{u}_0 = 0,\ \ddot{u} = 0 \qquad (11)$$

The time integration of direct implicit scheme is limited by the size of the critical time step, which is 2μsec for these problems; in other words, for a stress wave traveling in FEA model, elements of 10mm per side give a good approximation. The general configurations of the finite element models used in the analysis are shown in Fig. 1 (0° crack angle).

Elements with dimensions of 2mm per side in the vicinity of crack are used. Outside of this region, elements with dimensions of 5mm per side are used to reduce calculation time and storage demand.

A total of 3600 elements and 3721 nodes is used in the finite element discretization for the indefective case, 2288 elements and 2400 nodes is used for the 0° inclined defective case, 1627 elements and 1766 nodes is used in the finite element discretization for the 30° inclined defective case and a total of 1565 elements and 1656 nodes is used for the 45° inclined defective case, respectively.

To study stress waves in the context of this model, we make the following assumptions: (i) the plane is subjected to a point unit step load, at the center of opposite site of fixed end; and (ii) stress wave may be reflected perfectly at the fixed boundary and may be transmitted perfectly at the infinite boundary. These boundary conditions are typical of what might be encountered in certain machineries.

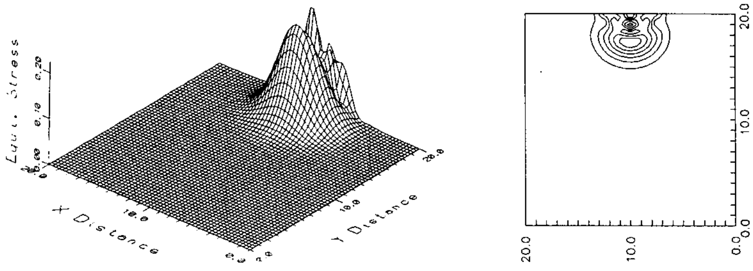
5. Results and Discussions

The finite element models proposed in the previous section are applied to the stress wave propagation of plane structure. The analytic results and numerical results are presented in Fig. 2~Fig. 6. All computations were carried on CRAYC-90 computer in double precision. Convergence and accuracy of the finite element solutions are obtained with indefective plane. This is demonstrated in Fig. 2~Fig. 3 by comparing the aspects of stress wave porpagation of analytic result with finite element results and a very good agreement between the analytical and the finite element solution was obtained.

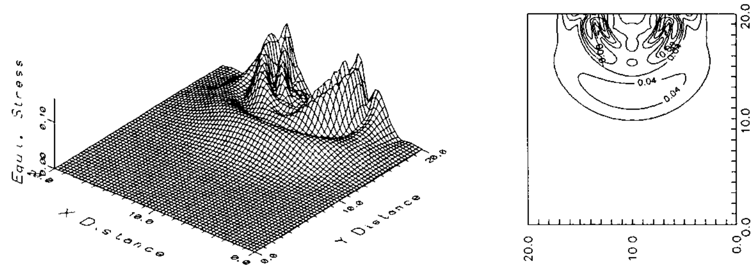
Fig. 2 shows typical patterns of stress wave propagation of an indefective plane. In the case of FEA model, the calculated stress wave speeds are 5191m/sec and 2705m/sec for longitudinal and shear stress wave component respectively. In Fig. 2(a) ~ (c) two different zones can be seen; the main lobe, and a small lobe at the right and left. Small lobes are shear stress wave together with the shear stress component of the longitudinal stress wave. This can be verified in Fig. 2(b) where some time passes and the two stress waves are separated due to their different velocities. The shear stress wave is propagated with a half speed as compared with the longitudinal stress wave. The direction of shear stress wave is get an odd angle of 45° from point load direction and the stress wave intensity of shear stress component is higher than that of longitudinal stress wave at any arbitrary time.

In Fig. 3 the analytical stress wave intensity and the numerical stress wave intensity are presented against the distance of propagated stress wave front. In these analyses, the stress wave intensity is assumed to decrease in terms of the ratio of propagated area. The difference of stress wave intensity between the analytical and the finite element solution was obtained to 4.37 percent.

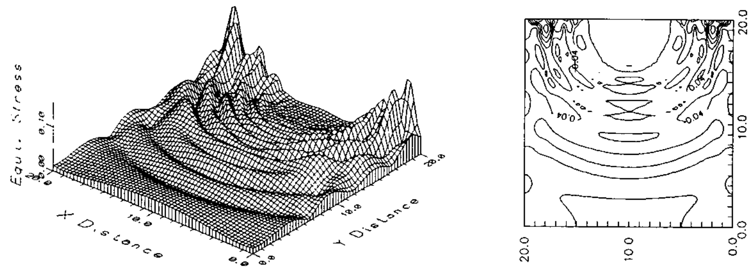
Since the main interest is to study the aspect



(a) time = 10 μsec



(b) time = 20 μsec



(c) time = 40 μsec

Fig. 2 The equivalent stress distribution of the plane without defect

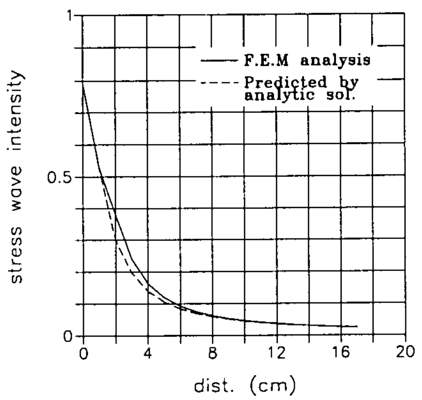
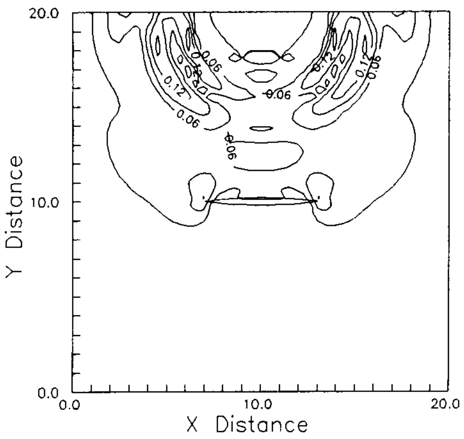
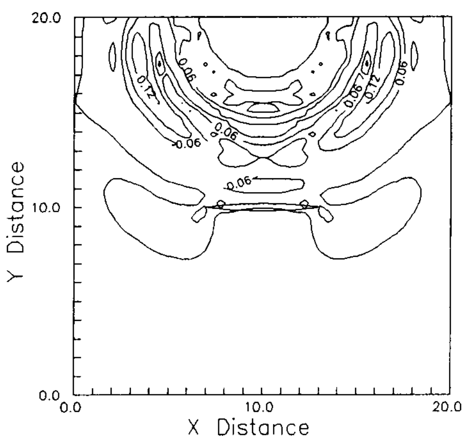


Fig. 3 Comparison of the time responses of stress wave intensity of the isotropic plane

of stress wave propagation on the fissure, the plane strain element is used. Fig. 4 shows the time responses of the equivalent stress components in the 0° angle of inclination of defective plane. Before arriving at the crack face, the stress wave is propagated with the same aspect as indefective plane. With respect to Fig. 4 the stress wave is bisected at the center of the crack face and the concentrated parts of stresses are moved toward the crack tip along the crack face.

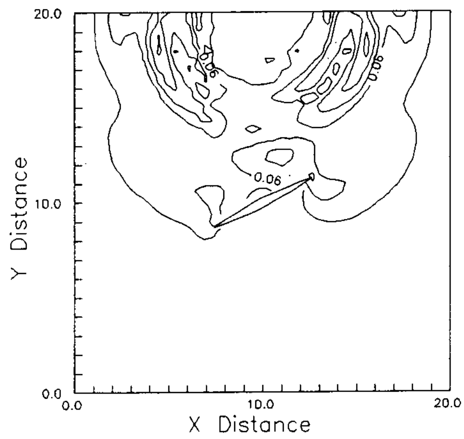


(a) time = 24 μsec

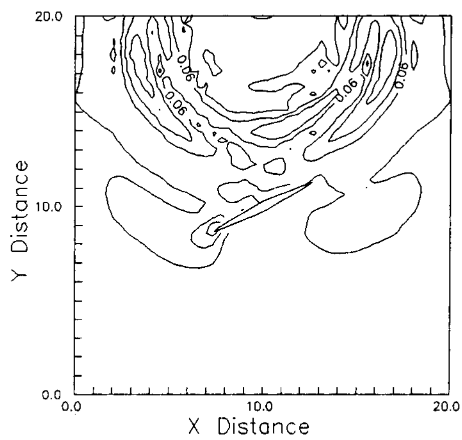


(b) time = 28 μsec

Fig. 4 Iso-stress distribution of the plane with an angle of inclination of crack is 0°



(a) time = 24 μsec



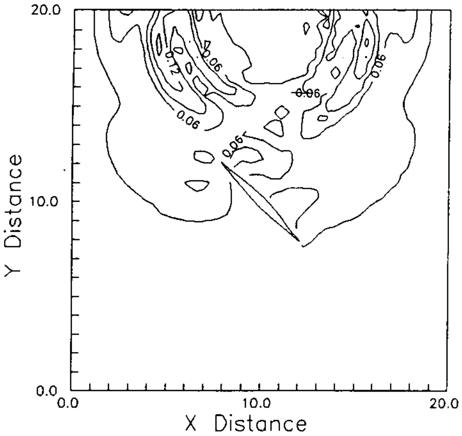
(b) time = 28 μsec

Fig. 5 Iso-stress distribution of the plane with an angle of inclination of crack is 30°

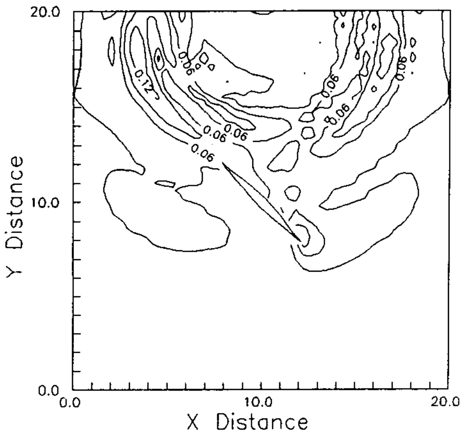
Fig. 5 shows the time responses of the equivalent stress components in the 30° angle of inclination of defective plane. With respect to Fig. 5 the stress wave is bisected at a quarter point of the crack face and the concentrated parts of stresses are moved toward the crack tip along the crack face.

Fig. 6 shows the time responses of the equivalent stress components in the 45° angle of inclination of defective plane. With respect to Fig. 6 the stress wave is bisected at one sixth point of the crack face and the concentrated parts of stresses are moved toward the crack tip along the crack face.

The stress wave is propagated with the same aspect as indefective plane before arriving at the crack face. Because of the crack angle of inclination, the arrival time of stress wave at the two crack tip sides are become different and the stress concentration is produced at the crack tip



(a) time = 24 μsec



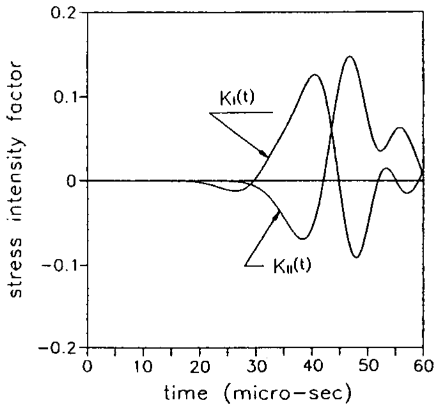
(b) time = 28 μsec

Fig. 6 Iso-stress distribution of the plane with an angle of inclination of crack is 45°

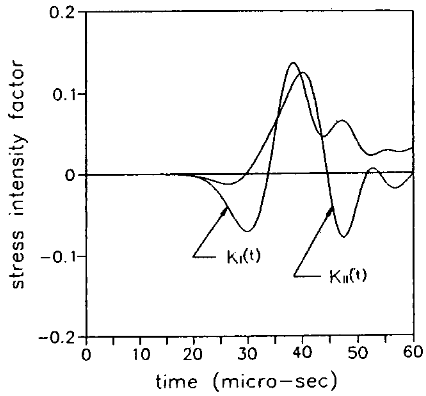
near the load point. The previously produced stress concentration part is moved to the counterpart of the crack along the crack face. The larger an angle of inclination of crack, the smaller a part of the stress wave reflected at the crack face. The crack tip stress distribution is changed very rapidly. The stress value at the crack tip is higher than that at the circumference of crack during stress wave propagation.

Next we take a vicinity of crack area excited by an impact load and obtain the response of stress field in the space-time domain. The stress at the crack can be converted to dynamic stress intensity factor by the displacement extrapolation of the crack tip elements. Fig. 7 shows the variations of stress intensity factor with respect to time for a 0° angle of inclination of crack and 45° angle of inclination of crack.

From the numerical results, the opening mode (mode I) and the in-plane shear mode (mode II) are appeared simultaneously when the stress wave pass by the crack tip. The stress wave field



(a) 0° crack angle



(b) 45° crack angle

Fig. 7 Dynamic stress intensity factor vs. time is separated into two parts at the defect and the stress wave intensity parts are moved toward the crack tip with a similar aspect at any defective case. Therefore, the difference of stress intensity factor is varied slightly in accordance with the three defective case.

The present author is thought the movement tendency of stress wave intensity parts to be the similar.

6. Conclusions

The formulation of the numerical method is presented to analyze the response of plane structure excited by impact loads. This method combines the finite element method with the direct integration method. This method is straightforward and easy to use. Moreover, this method can be used for the analysis of arbitrary planes with many types of cracks subjected to the point load or line load.

Finally, the stress wave intensity at the defective area is converted to dynamic stress intensity factor. Hence the present method can be easily used for two dimensional impact problem of defective plane structures and pre-estimate the stress distribution for any structural defects.

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