■ 論 文 ■

A Numerical Algorithm for Evaluating Progression Efficiency along Coordinated Arterials Using Shock Wave Theory

충격파이론을 응용한 간선도로 신호연동화의 효율 평가를 위한 알고리즘의 개발

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목 차

요 약

신호화 간선도로에서 연동신호계획의 효율성이 간선도로 교통운영의 핵심사안이다. 본 논문에서는 연동화 신호시간 계획의 우수성을 평가하는 컴퓨터 알고리즘을 제시한다. 링크내에서 이동하는 차량군의 동적 특성과 옵셋값의 변화에 민감한 지체도의 산정이 수학모형의 중요부분이다. 개발되는 알고리즘의 이론적인 배경은 Michalopoulos의 해석적 모형에 기반을 두고 있는데, 이 모형을 다양한 교통조건이나 신호조건에 적용되도록 하는 실용화 과정이 본 논문에 기술되어 있다. 제시된 알고리즘에 대하여 옵셋값의 변화에 따른 지체도의 민감도 평가를 실시하였다. 평가를 통하여 본 알고리즘은 교통수요, 링크길이, 회전교통량비율과 같은 교통조건의 변화에 따라 적절한 민감도와 추정력을 보이는 것으로 판단되었다.

Ⅰ. Introduction

The simulation of traffic stream flow has been the subject of extensive research during the past few decades. Although numerous traffic models have been proposed and tested against field data for uninterrupted flow, relatively few research works have been done about the behavior of traffic when it is interrupted. In urban arterials, traffic interruptions occur most frequently at signalized intersections. In signalized arterials, coordination of signals is one of the most complex problems in traffic engineering practice. The efficiency of a coordination scheme can be evaluated using the traffic simulation model describing the movements of platoons in signalized links. This paper aims at proposing an algorithm to evaluate the performance of coordinated signal timing plans for the signalized arterials.

TRANSYT (Robertson 1969) is the most widely used tool for the simulation and optimization of the signal timing for signalized networks. It simulates traffic behavior using the well-known traffic model for describing platoon behavior engaged in TRANSYT, such as Robertson's platoon dispersion model. TRANSYT has some shortcomings in modeling traffic, however. One is that TRANSYT describes the platoon dispersion behavior reasonably well, but does not consider the platoon compression. The phenomenon of the platoon compression happens in real-world traffic. Another is that TRANSYT provides disutility measures like vehicular delay occurring only at stop lines of intersections during red time, but does not provide the disutility measures along links. Actually, vehicular delay occurs at the road section between adjacent intersections as well as at intersection itself.

The hydrodynamic theory, which was developed by Lighthill and Whitham (1955), has been recognized as one of the outstanding models in analyzing traffic phenomena. Stephanopoulos and Michalopoulos (1979) suggested the dynamics of formation and dissipation of queues at isolated signalized intersections by applying the theory of shock wave derived from the hydrodynamic theory. Michalopoulos *et al.* (1980a, 1980b) developed a mathematical modeling of the traffic dynamics in links between signalized intersections. Applying the shock wave theory, they derived analytical expressions for describing platoon propagation along the road. This model reflects not only traffic platoon dispersion but also platoon compression. Since the model describes the evolution of traffic waves and queues in both time and space, rather than time alone, it provides traffic performance at not only intersection stop-line but also the road section between adjacent intersections.

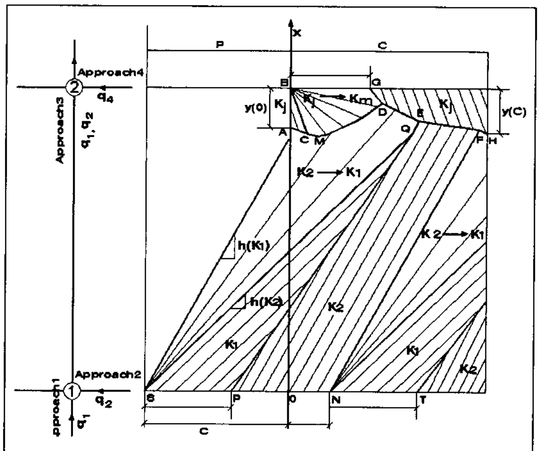
Authors felt the necessity of more works in applying the Michalopoulos's model to various traffic, roadway and signalized conditions due to the limitation of its analytical solution approach. Analytical derivation of the large number of possible queue length developments is tedious and almost impossible in real problem application. Computerized algorithm can handle such an huge computational work efficiently. In this paper, an algorithm for numerical solution based on the Michalopoulos's analytical model is presented, which is applicable for personal computers with ease. Michalopoulos's model provides density values in time-space regions as final solution. The measures of effectiveness adopted in general for describing the operating performance of signal arterials are delay and travel speed. Thus, a conversion model of the density values into the delay was developed in this paper. The algorithm can be extended to modeling multi-phase signal operations with ease.

Ⅱ. Michalopoulos' Analytical Model

Before describing a computer algorithm, we briefly introduce the Michalopoulos's analytical model. For more detailed descriptions, refer to Michalopoulos et al. (1980a).

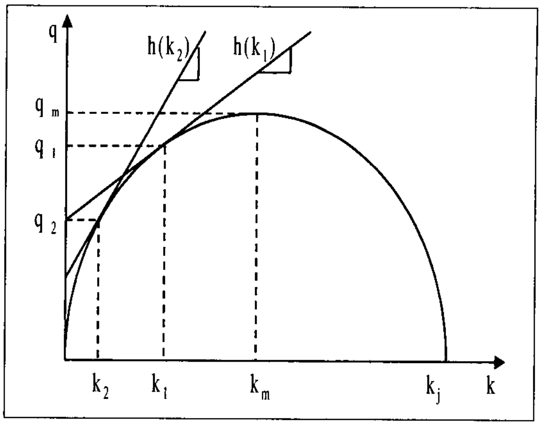
Consider the system of two signalized intersections shown in the left-hand-side of ⟨Figure 1⟩. Assuming two-phase signal operation of both intersections, the right-hand-side of the figure shows an example of shock wave developments between two signalized intersections. ⟨Figure 2⟩ shows a flow-density (q-k) curve. Two platoons with traffic flows, q_1, q_2 and the corresponding densities k_1, k_2 respectively, depart from upstream intersection. Tangents at points 1 and 2 ($h(k_1)$, $h(k_2)$) represent the wave speed for these two platoons. The intersection of these two sets of waves has a slope equal to the chord connecting the two points on the q-k curve, and this intersection represents the path of the shock wave. It should be noted that the waves on the time-space diagram in this analysis are not trajectories of vehicles but lines of constant flow and thus lines of constant speed. (Gerlough, 1975)

As shown in ⟨Figure 1⟩, the combinations of two intersecting waves generate various types of shock waves. Every possible shock waves falls into 13 patterns as shown in ⟨Figure 3⟩. The path of each shock wave pattern is listed in ⟨Table 1⟩. In this table, t_0 represents the starting time of the shock wave generation; t_3, the termination time of green interval of downstream intersection ; and dist, link length.

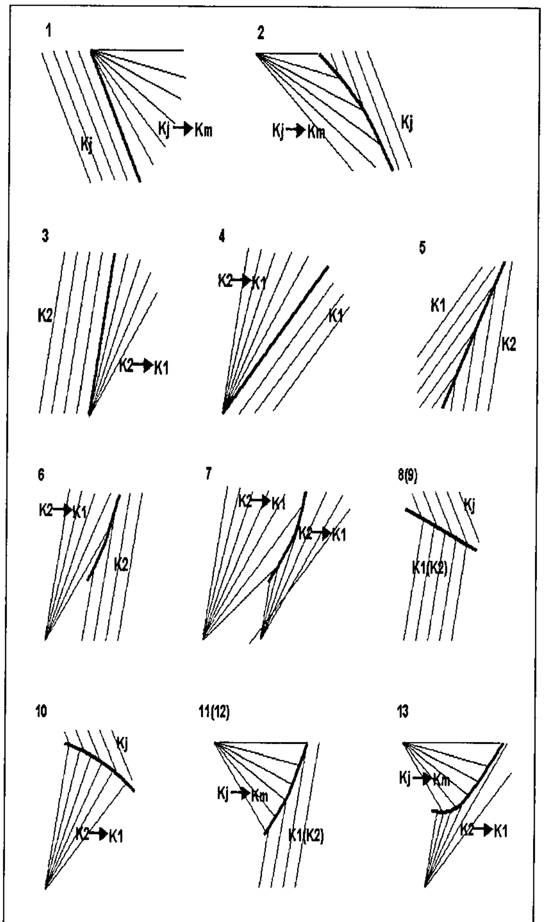


Source : Michalopoulos, P.G., et al (1980a)

⟨Figure 1⟩ Shock Wave Developments between Two Signalized Intersections



⟨Figure 2⟩ The Flow-Density Curve



Source : Michalopoulos, P.G., et al (1980a)

⟨Figure 3⟩ Shock Wave Patterns

For instance, shock-wave line PQ in ⟨Figure 1⟩ is generated when wave $h(k_1)$ intersects wave $h(k_2)$.

which belongs to Pattern No 5 in ⟨Figure 3⟩. The generated shock wave moves with speed u_{12} given by

$$u_{12} = \frac{q_2 - q_1}{k_2 - k_1}$$

If the q-k curve is assumed linear, the above equation becomes:

$$u_{12} = \frac{u_f}{k_j}(k_j - k_2 - k_1)$$

where u_f represents free flow speed and k_j, jam density. The path of line PQ is given by

$$X = \frac{u_f}{k_j}(k_j - k_1 - k_2)(t - g_1) \qquad (1)$$

where g_1 represents green interval for flow q_1. Equation 1 corresponds to Equation 5 in ⟨Table 1⟩. The path of the remaining shock-wave lines can be derived in similar manner.

When two shock waves collide, new shock wave is generated. The pattern of the new shock wave is determined based

⟨Table 1⟩ Equations of Shock Wave Pattern

NO	EQUATION
1	$dist - u_f(t - t_3)$
2	$dist - u_f(t - t_0) + (t_0 - t_3)^{1/2}(t - t_3)^{1/2}$
3	$h(K_1)(t - t_1)$
4	$h(K_2)(t - t_1)$
5	$\frac{u_f(K_j - K_1 - K_2)}{K_j}(t - t_0)$
6	$h(K_2)(t - t_1) - ((h(K_2) - h(K_1))(t_0 - t_1)^{1/2}(t - t_1)^{1/2}$
7	$\frac{x_0}{(t_0 - t_1)^{1/2}(t_0 - c - t_1)^{1/2}}(t - t_1)^{1/2}(t - c - t_1)^{1/2}$
8	$x_0 - \frac{u_f K_1(t - t_0)}{K_j}$
9	$x_0 - \frac{u_f K_2(t - t_0)}{K_j}$
10	$\left(u_f + \frac{x_0}{(t_0 - t_1)}\right)(t_0 - t_1)^{1/2}(t - t_1)^{1/2} - u_f(t - t_1)$
11	$dist + h(K_1)(t - t_3) - \left(h(K_1) + \frac{(dist - x_0)}{(t_0 - t_3)}\right)(t_0 - t_3)^{1/2}(t - t_3)^{1/2}$
12	$dist + h(K_2)(t - t_3) + \left(h(K_2) + \frac{(dist - x_0)}{(t_0 - t_3)}\right)(t_0 - t_3)^{1/2}(t - t_3)^{1/2}$
13	$\frac{\left(x_0 - \frac{dist(t_0 - t_1)}{(t_3 - t_1)}\right)}{(t_0 - t_1)^{1/2}(t_0 - t_3)^{1/2}}(t - t_1)^{1/2}(t - t_3)^{1/2} + \frac{dist(t - t_1)}{(t_3 - t_1)}$

Source : Michalopoulos, P.G., et al (1980a)

on the types of colliding shock waves. For example, when shock-wave SQ meets shock-wave PQ at point Q, shock-wave QE is newly generated, as shown in ⟨Figure 1⟩. The coordinates of the meet points (C, D, E, F and Q) which are generated due to the intersection of two shock-wave lines or curves can be solved from the geometric relationship.

The platoon dynamics occurring signalized arterials can be described through calculating the shock-wave paths and colliding points. Because the full description of all equations is quite lengthy, we do not describe any further in this paper. Readers interested in further details should consult Michalopoulos (1980a).

III. Numerical Algorithm

The shock-wave developments within the link are directly affected by the traffic arrivals, the link length, the signal control policy and the initial condition. Due to the numerous combinations of these parameters, a large number of downstream queue formations are possible. Because analytical derivation of the large number of possible queue length developments was tedious and almost impossible, a numerical algorithm is suggested in this paper. The conversion process of density values into delay should be added for practical applications. The following seven steps describes the proposed algorithm for calculating delay and queue length occurring due to the effect of progression quality.

Step 1. Input data.

Input data required are: major flow q_1 and minor flow q_2; signal timing data such as common cycle length, green times for two intersections, and offset value; link length; and the q-k curve. Even though the linear q-k relationship was used in this section for simplicity, the general q-k curve using a differentiable function could be applicable with minor efforts.

Step 2. Calculate density values and wave speeds.

When the linear q-k curve is used, the density k_l and wave speed $h(k_l)$ corresponding to flow q_l can be calculated using the following equations:

$$k_1 = \frac{k_j}{2} - \sqrt{\left(\frac{k_j}{2}\right)^2 - \frac{k_j}{u_f} q_1}$$

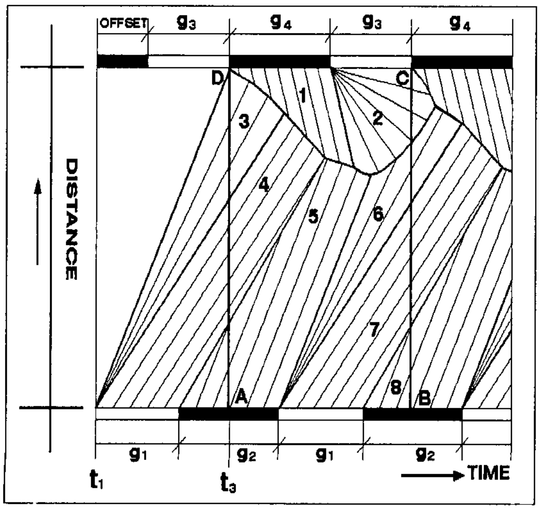
$$h(k_1) = u_f\left(1 - \frac{2k_1}{k_j}\right)$$

Step 3. Prepare shock wave table.

A shock wave is generated when two waves intersect. The shock wave types fall into 13 patterns as shown in ⟨Figure 3⟩. The path of each shock wave pattern is listed in ⟨Table 1⟩.

Step 4. Calculate shock wave location.

The procedure uses the time-marching technique; that is, the solution is progressively obtained by marching in steps of time. The calculation initiates at the starting time t_l of main phase g_l of upstream intersection, as shown in ⟨Figure 4⟩. At time t, the distance coordinates of shock waves are calculated using the equations listed in ⟨Table 1⟩.



⟨Figure 4⟩ Shock Wave Computation Using Time-Marching Approach

Step 5. Check the collision of two shock waves.

When two shock waves collide, new shock wave is generated.

Thus, the checking procedure is conducted in this step. If the collision occurs at current time step, the colliding shock waves are discarded, and the path of the new shock wave is calculated for next time step. Otherwise, go to next step.

Step 6. Calculate the area of homogeneous density region.

At time step t, several different density regions occur along link. The distance of the individual density region should be accumulated during entire simulation period, and then the area of each region is determined after processing last time step. Even if the procedure starts at t_1, actual simulation period is one cycle from t_3, as show in ⟨Figure 4⟩. Thus, the observation region is a rectangular surrounded by Points A, B, C, and D.

Step 7. Calculate delay.

The area S_i for individual density region i is directly related to the delay occurred at the link between two intersections. Two types of delays are obtained: intersection delay and link delay. From ⟨Figure 4⟩, regions 1 and 2 are related to the intersection delay; and regions 3, 4, 5, 6, 7 and 8, the link delay. The intersection delay d_u and the link delay d_l are calculated using the following equations.

$$d_u = \frac{\sum_{i=1}^{2} k_i S_i}{C(q_1/\lambda_1 + q_2/\lambda_2)}$$

$$d_l = \frac{\sum_{i=3}^{8} k_i S_i}{C(q_1/\lambda_1 + q_2/\lambda_2)} - \frac{l}{u_f}$$

where C denotes cycle length; λ_i, green ratio g_i/C. The variables used in the equations above have the following units: d_u and d_l in sec/veh; k in veh/m; S in m-sec; C in sec; q in veh/sec;

and u_f in m/sec.

General delay equations consist of uniform delay term and random delay term. Since random fluctuation of traffic arrival was not considered in the model development, the intersection delay d_u calculated in this procedure is the uniform delay. The random delay term dr should be added to the delay equation. Because the development of the random delay is not a concern of this paper, one might adopt one of the approved delay equations like the 1985 HCM delay equation (1985). Total link delay per vehicle is expressed as:

$$d_t = d_u + d_r + d_l$$

The algorithm described above is focused on the calculation of delay. Queue lengths and the number of stops could be calculated using similar manners. For details, consult Baek (1998).

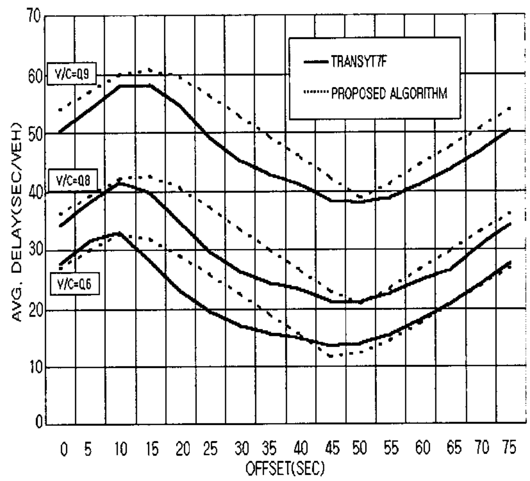
Ⅳ. Performance Test

To test the ability of the proposed algorithm to predict delays for progressed movements, the TRANSYT-7F simulation program (1983) was used to estimate delays for a simplified arterial system. The arterial chosen as base case was assumed to have the following characteristics:

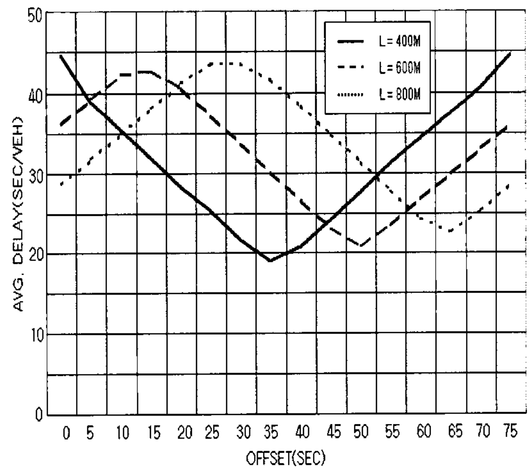
C = 75 seconds
g_1 = 35 seconds
g_3 = 30 seconds
v/c = 0.8
q_1/q_2 = 4
and, d = 600 meter

To explore the effect of demand level, the following v/c ratios were tested: 0.6(low demand), 0.8(medium demand), and 0.9(heavy demand). ⟨Figure 5⟩ shows the comparison of estimated uniform delay du between TRANSYT-7F and the proposed algorithm. In general, delays predicted by the proposed algorithm appear to be close to those calculated by TRANSYT-7F. The proposed algorithm has a tendency to slightly overestimate delay at the offset range less than the minimum delay offset values.

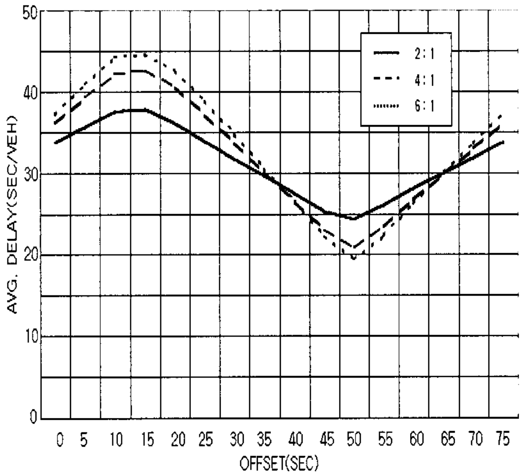
To test the reasonability of the proposed algorithm, two scenarios were evaluated over the entire range of offset intervals. One used three link lengths d : 400, 600, and 800 meter; and the other employed three turning volume ratio q_1/q_2 from upstream signal: 2, 4, and 6.



⟨Figure 5⟩ Delay-Offset Analysis Comparing the Proposed Algorithm With TRANSYT-7F



⟨Figure 6⟩ The Delay Predicted According to different Link Lengths



〈Figure 7〉 The Delay Predicted by Changing Turning Volume Ratios

〈Figure 6〉 shows the delay predicted by the proposed algorithm according to different link lengths. From this figure, it is found that the optimal offsetincreases as link lengths become longer, as expected. Minimum delay value for each link length increases slightly, as link length becomes longer. In long link, progression effect in reducing delay is diminished owing to the platoon dispersion phenomena.

The delay predicted by changing turning volume ratios is shown in 〈Figure 7〉. As side-street flow becomes closer to main-street flow, the delay-offset relationship becomes flatter. This result was expected.

Ⅴ. Conclusions

A numerical algorithm is presented that evaluate the performance of coordinated signal timing plans for the signalized arterials. From performance test, it was observed that the algorithm performed reasonably well. The delay estimated using the proposed algorithm are shown to be sensitive to the quality of progression, as well as to traffic demand, link length, and turning flow ratio from upstream signal. The delay-offset relationship predicted from the algorithm produced consistent results with the delays generated by TRANSYT-7F. The advantage of the proposed algorithm over the TRANSYT model lies in a) modeling the phenomenon of platoon dispersion as well as compression, engaged in the shock wave theory (Michalopoulos *et al.* 1980); b) computing link delay.

The second phase of this research will be to collect field data to calibrate the q-k curve, and to conduct performance test of the algorithm using actual data. The algorithm could be developed to be a simulation model for signalized arterials by improving the algorithm and adding auxiliary features. This research is under way by authors.

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