학술기사

SACS를 이용한 Fatigue 해석의 이론적 배경

Fatigue Analysis of Theoretical Background Using SACS

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Structures subjected to large numbers of cycles of oscillating loads may fracture even at very low nominal stress levels. This mode of failure, called fatigue, has been recognized for many years and a grate deal of research has been carried out to develop analysis and design procedures to minimize the likelihood of such failures. It is particularly important to have such procedures available for expensive structures in hostile environments with the presence of human life. Offshore structures involve all three of these requisites. The most common approach to estimating the ability of a structure to survive repetitive loading is based on the Palmgren-Miner accumulation of damage hypothesis. The theory rests on several assumptions. Within the framework of the Palmgren-Miner theory two basic approaches Spectral (or Statistical) fatigue analysis and Deterministic fatigue analysis to fatigue analysis are currently in use.

Fatigue is a post-processing program of the SACS suite that evaluates the performance of the structure with respect to fatigue failure. It uses the common solution file as its fundamental database supplemented by several input lines where the user selects from among the various analysis and design options available. Some of the major theory and features of this Fatigue module are:

1. S-N Curves

Numerous S-N curves are intrinsic to the Fatigue program. The API-RP2A X and X' curves are represented mathematically as:

$$N(s) = 2\times 10^{6}\left(\frac{\Delta\sigma}{\Delta\sigma_{ref}}\right)^{-m}$$

Where N(s) is the allowable number of cycles for stress range $\Delta\sigma$ and m is the inverse log-log slope of the S-N curve. The scale effect correction factor applied to N(s) when incorporating API

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thickness dependent effects is taken as:

$$N'(s) = N(s)[\frac{t}{t_0}]^{-0.25m}$$

Where N'(s) is the allowable number of cycles for stress range $\Delta\sigma$, t_0 is the limiting branch thickness and t is the member thickness. For the S-N curve from Norwegian Standards (NS3472E), the following equations allowing for material thickness variations are used:

In Air:

$N(s) \le 10^7$

$$\log N(s) = 12.16 - 0.75 \times \log \frac{t}{32} - 3.0 \times \log \Delta\sigma$$

$N(s) > 10^7$

$$\log N(s) = 15.62 - 1.25 \times \log \frac{t}{32} - 5.0 \times \log \Delta\sigma$$

In Water:

$N(s) \le 10^8$

$$\log N(s) = 12.16 - 0.75 \times \log \frac{t}{32} - 3.0 \times \log \Delta\sigma$$

2. Load Path Dependant SCF

Load path dependant SCF's are determined using the following procedure: For any tubular connection, all braces that lie in a plane with the chord or within 15 degrees of that plane are considered in the calculation of load paths and SCF's. The chord member(s) are selected based on the following hierarchy:

1) Largest diameter
2) Largest wall thickness
3) Highest yield stress
4) Member that are in-line with a 5 degree tolerance

The connection is first checked for K-Joint consideration. Only multiple braces on the same side of the chord are considered as part of the K-joint. For any brace, the axial load component normal to the chord is balances by the axial load component normal to the chord in other braces on the same side of the chord. The brace with the smallest normal axial force is considered first with the brace containing the largest opposing normal axial force. The balanced load is subtracted from the opposing brace and the process is repeated until all K-Joints are identified.

Any X or cross joint path are considered next. Only braces on opposites sides of the chord are considered as part of the X-Joint. The remaining unbalanced K-Joint axial load component normal to the chord is balanced by the axial load component normal to the chord in an opposing brace on the opposite side of the chord. The brace with the largest opposing normal axial force is considered first. The balanced load is subtracted from opposing brace and the process is repeated until all X-Joints are identified.

T/Y load paths are identified last. Braces with the remaining unbalanced axial load component normal to the chord is classified T/Y-Joints. The Load Path SCF is calculated as a weighted average of the K, X and T/Y load path percentages.

3. Spectral Fatigue

The spectral approach to fatigue is an attempt to account for the random nature of a confused sea in a rational manner. The method assumes that there is a definable relation between wave height and stress ranges at the connections, and that at any point the elevation of the sea above its mean value is a stationary Gaussian random process. These assumptions are most applicable for low to moderate seastates. Since these are the seastates of interest in fatigue studies, this assumption can reasonably be accepted.

The following section gives a more detailed background on the theory of spectral fatigue. For non-tubular members and plates, stress points at the extreme outer fibers are evaluated. For tubular connections, Fatigue investigates eight points on the chord and brace sides of the connection. For the purpose of this discussion, it is assumed that we are interested in calculating the total fatigue damage at one specific point of the connection.

3.1 Linear Systems

It is shown in standard reference that linear systems whose properties do not change with time can be characterized in the frequency domain by an expression of the form:

$$Y(f) = H(f)X(f) \tag{1}$$

Where:

F = Frequency.

X(f) = Fourier transform of the excitation

Y(f) = Fourier transform of the response.

H(f) = Transfer function.

The transfer function (also called the frequency response function) can be thought of as the amplitude of the sinusoidal response when the excitation is a sinusoid of unit amplitude. Equation (1) can be extended to the case of much response function to a given excitation by interpreting the terms in a matrix sense. In subscripted notation it is written as:

$$Y_i(f) = H_i(f)X(f) \tag{2}$$

In equation (2) Y and H are N×1 matrices(or N component vectors) and X is a scalar(or 1×1 matrix). Taking the outer product of eq.(2) with itself results in the following:

$$Y_i(f)Y_j(f) = H_i(f)H_j(f)X^2(f) \tag{3}$$

If the excitation, x(t), is a random function of time, then its Fourier transform, X(f), is also a random function, as are those of the responses. $Y_I(f)$. In this case equation (3) is a relation between random functions (note, however, that the transfer functions, $H_I(f)$, are well defined and not random).

We represent the average value of a random variable, Z, by the notation Z. The average of both sides of equation (3) gives:

$$Y_i Y_j = H_i H_j X^2 \tag{4}$$

For our purpose we will be interested only in the diagonal terms of this matrix equation:

$$Y_i^2 = H_i^2 X^2 \tag{5}$$

For any random function defined in the frequency domain, Z(f), the function $Z^2(f)$ is called the power spectral density (or sometimes the mean-square spectral density) of the process and is designated by:

$$S_z(f) = Z^2(f) \tag{6}$$

It is shown in standard references that the mean-square value of a stationary random function of time, y(t), (a stationary process is one whose statistics do not change with time) is give by:

$$y^2(t) = \int_0^\infty S(f)df \tag{7}$$

The square root of this is called the root-mean-square (RMS) value. Combining this definition with equations (5), (6) and (7) we get the RMS value of the response of our system:

$$Y_{RMS} = \sqrt{\int_0^\infty H_i^2(f)S_h(h)df} \tag{8}$$

For fatigue analysis of offshore structures, the excitation is the elevation of the sea's surface at a point as a function of time, h(t), and the responses of interest or stationary random function are the hot spot stress ranges at the connections. Here stress range is defined as the difference between successive maximum and minimum peaks in the plot of stress vs. time.

Thus if the spectral density of a particular seastate $S_h(f)$, is known, and the transfer function $H_i(f)$ for the point can be calculated, then the statistical cyclic stress range(RMS cyclic stress range) at that point for this particular seastate is:

$$\sigma_{RMS\ i}=\sqrt{\int_0^{\infty} H_i^2(f) S_h(f) df} \qquad (9)$$

3.2 Transfer Function

3.2.1 Cyclic Wave Loading

A transfer function defines the ratio of the range of cyclic stress to wave height as a function of frequency (usually for one direction of wave). If, for each frequency, the input to the system is a unit amplitude sinusoid of that frequency, then the steady state amplitude of the response is the transfer function at that frequency. In our case the input is the elevation of the sea at a point above its undisturbed position (wave height) and the responses are the brace stresses at the connections. In reality our system is not truly linear so the fundamental relationship of equation (1) is only approximately true, but the approximation is a very good one if the waves characterizing the fatigue environment are not too large. The Airy linear wave theory results in wave profiles that are pure sinusoids, however for waves of small amplitude (as are typical in fatigue studies) the profiles are nearly sinusoidal and thus these waves can reasonably serve as transfer function generators.

To generate a transfer function for a particular fatigue load case (wave direction), several waves of various heights but constant steepness are used to load the structure. These waves need not necessarily be the waves from the fatigue environment, but waves chosen based on the dynamics of the structure. The stress is calculated at various wave positions (per the user). The difference between the maximum and minimum stress, called the stress range, is determined for each wave (see figure below).

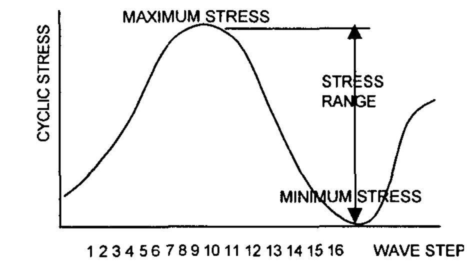

Fig. 1 Stress range

Diving these stress ranges by one-half of the corresponding wave height produces stress ranges for waves of unit amplitude (for sinusoidal waves, wave height equals twice the wave amplitude). The relationship between the stress ranges of unit amplitude and the corresponding wave frequency for all waves considered is the transfer function (see figure below).

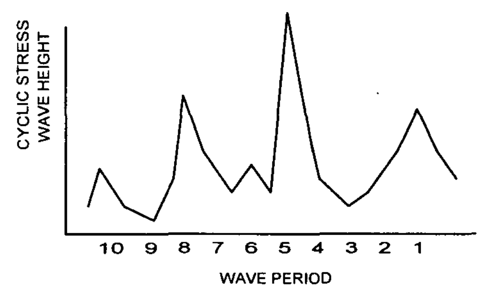

Fig 2. The relationship between the stress ranges of unit amplitude and the corresponding wave frequency

3.2.2 Cyclic Wind Loading

For wind spectral analysis, a mechanical transfer function for each mode is used by the program. The mechanical transfer function for any mode, H(f), defines the ratio of the range of cyclic stress as a function of the wind frequency and the mode natural frequency as follows:

$$H(f)=\frac{1}{K_i}[[1-\left(\frac{f}{f_n}\right)^2]^2+[2c\left(\frac{f}{f_n}\right)^2]^2]^{-½}$$

where K_i is the generalized stiffness, f_n is the natural frequency and c is the percent damping.

3.3 Spectral Density

3.3.1 Wave Height Spectra

Wave height spectra are used to characterize the random behavior of waves statistically. From a wave spectrum, a wave height spectral density relating the probability distribution at various frequencies can be developed. Three forms of wave height spectral density function are commonly used in the offshore industry, all of which are incorporated into Fatigue; they are:

Pierson-Moskowitz Spectrum(Bretschneider's Form)

$$S_{PM}(F^*)=\frac{5h_s^2T_0}{16}\frac{1}{(F^*)^5}\exp[-\frac{5}{4}(F^*)^{-4}]$$

JONSWAP(Joint North Sea Wave project) Spectrum

$$S_J(F^*)=\frac{S_{PM}(F^*)}{C}\exp\left\{1n\gamma\exp[-\frac{(F^*-1)}{2\sigma^2}]\right\}$$

The terms in these expressions are:

h_3 = Significant wave height, defined as the average height of the 1/3 highest waves.

T_0 = Dominant wave period, the period for which S(f) is a maximum.

F^* = Dimensionless frequency, f/f_0, what f_0 is the frequency corresponding to T_0.

γ, σ and C are parameters characterizing the JONSWAP spectrum. The following defaults and built in to the program:

γ = 3.3
ρ = 0.07 for $F^*<1$ 0.09 for M1>1
C = 1.525

Onchi-hubble Double Peak spectrum

$$S_{OH}(f)=\frac{\pi}{2}\sum_{j=1}^{2}\frac{[4(4\lambda_j+1)\pi^4f^4_{pf}]^{\lambda j}}{\Gamma(\lambda_j)}\frac{h_{sf}^2}{(2\pi f)^{4\lambda_j+1}}\exp[\frac{4\lambda_j+1}{4}(\frac{f_{pf}}{f})^4]$$

Where f is the wave frequency, λ is the peakedness, h_s is significant wave height and f_p is the spectral peak frequency.

3.3.2 Wind Spectra

The generalized force spectrum used for wind spectral analysis is a Harris wind spectrum with gust effects spatial correlation and mean wind velocity variation. For any mode, the generalized force spectrum $S_i(f)$ is taken as:

$$S_i(f)=\frac{4}{f}F_i^2J_{ai}J_{rt}(f)^2S_v(f)$$

Where F_i is the generalized force and $S_v(f)$ is the Harris spectrum given by:

$$S_v(f)=\frac{4k\eta_1(f)}{(2+\eta_1(f)^2)^{\frac{5}{6}}}\quad\text{where}\quad\eta_1(f)=f(\frac{L_H}{v_{10}})$$

Where L_H is the Harris spectrum reference length and v_{10} is the velocity at the reference height. The term J_{ai} and J_{rt} are the mean wind velocity variation function and the gust effects spatial correlation function, respectively, as recommended DNV Gust Wind Response Analysis.

3.4 Fatigue Damage

From equation (9), the RMS stress for a particular spectrum I, of the fatigue environment can be calculated. where $S_i(f)$ is the spectral density and H(f) is the transfer for the direction being considered.

$$\sigma_{RMS_i}=\sqrt{\int_0^{\infty}H^2(f)\times S_i(f)df}$$

For every RMS stress density there exist an average time, T_z, between zero crossings with a positive slope for a stationary Gaussian process with zero mean. This period called the Zero Crossing Period is given by:

$$T_z=\frac{\sigma_{RMS_i}}{\sqrt{\int_0^{\infty}f^2\times H^2(f)\times S_i(f)df}}$$

For a narrow band process this is the average period or the reciprocal of the average frequency of the process. The expected number of cycles, N, associated with this spectrum during the design life of the structure is:

$$N=\frac{mL}{T_z}$$

Where L is the design life of the structure and m is the fraction of the design life that this spectrum prevails. For a given stress range s, the number of cycles to failure, NF(s), can be found from the S-N curve used. Thus for a stress range between s and s+ds the amount of damage, dD, is:

$$dD=\frac{N}{N_F(s)}p(s)ds \qquad (10)$$

Where p(s) is the probability that the stress range is between s and s+ds.

Standard references show that if a linear system is excited by a Gaussian random process, then the response will also be a Gaussian process, thus in our case, having assumed system linearity and Gaussian excitation, the stress time histories are Gaussian (at least to the order of our approximations). We further assume that each response is narrow banded, that the spectral density of the response is significant only over a narrow range of frequencies. Under these conditions the stress range is a Rayleigh distributed random variable having a probability density function given by:

$$p(s)=\frac{s}{\sigma^2_{RMS}}\exp[-\frac{s^2}{2\times\sigma^2_{RMS}}] \qquad (11)$$

Where s=stress range and σ_{RMS}=RMS value of the stress range, evaluated by eq.(9).

Substituting equation (11) into equation (10), the expected damage from the given spectrum is:

$$D=\frac{N}{\sigma^2_{RMS_i}}\int_0^{\infty}\frac{s}{N_F(s)}\exp\{-\frac{s^2}{2\times\sigma^2_{RMS_i}}\}ds \qquad (12)$$

The total expected damage for all seastates during the life of the structure is the sum of the damages for each individual seastate. The expected fatigue life is equal to the design life divided by the expected damage.