▧ 응용논문

Determination of Buffer Stock Levels Using Goal Programming

- 목적 계획법을 이용한 최적 재고 관리 시스템에 관한 연구-

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요지

재고관리 분야는 대부분 생산 제품에 대한 시장변화, 배달지연 등으로 인한 불확실성을 어느 정도 보편화 또는 요인을 제거하여 수리적 모형을 만들어 여러 가지 제약조건들을 변화 있게 가정하여 각 제품별로 최적 재고 관리 시스템을 만들어 왔다. 그러나 이러한 수학적 모형으로부터 나온 해결책을 실제로 적용될 때 자본 능력 한계, 저장 면적 한계 등을 고려하지 않는 것이기 때문에 실현 불가능할 수 있다. 따라서 이러한 문제점을 해결하고자 수요예측, 저장면적의 한계, 자본 한계 등 이러한 제약조건들을 동시에 최대한으로 만족시킬 수 있는 목적계획법(Goal Programming)을 이용하여 각 제품별 안전 재고를 산출하여 실질적으로 적용할 수 있는 재고 시스템을 구성하여, 여러 가지 제약조건 등을 동시에 만족시킬 수 있는 최적 안전재고를 산출하여 최적 재고 관리 시스템을 구축, 현실성을 높이고자 한다.

1. Introduction

Sometimes enterprises are out of stock that makes the complaint of great numbers of businessmen faced with the dilemmas and frustrations of attempting simultaneously to maintain stable operations, provide customers with adequate service, and keep investment in stocks and equipment at reasonable levels. A manager should think of the inventory control problem to escape the shortages or too much inventories. The basic problem of developing inventory policy is to strike a balance between operating savings and the costs and capital requirements associated with larger stocks. In the past, businessmen have been able to achieve a reasonably balanced inventory policy largely through an intuitive understanding of the needs of their business.

However, as the business grows it becomes more complex, and as business executives become more specialized in their jobs or further removed from direct operations, so that achieving an economical balance intuitively is increasingly difficult. That is why more businessmen are finding the concepts and mathematics of the growing body of inventory theory to give direct practical help.

The pressure for capital and the growth of return on investment as a measure of business performance have made business management increasingly conscious of the

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이 연구는 일부 정보통신부의 정보통신 우수시범학교 지원사업에 의하여 수행된 것임

importance of inventories as a cost element. The trend toward heavy fixed investment to reduce direct-labor cost and growing pressure from labor for employment stability have combined to force more careful future planning. Production-planning and logistics problems have received the greatest amount of attention of all classes of business problems in the course of development of industrial operations research since 1950. Universities and other research groups have shown a greatly expended interest, since World WarⅡ, in the development of new quantitative methods for studying industrial problems, and much of this attention has been focused on production-planning problems. These trends have combined to yield a growing body of technique and concept, for studying management's inventory problems and designing efficient systems for resolving them [5].

Inventory and production management problems involve some factors which are relatively exact, such as cost factors; other factors considered are subject to chance errors. The need for consideration of the many uncertain or intangible factors and the necessity for balancing conflicting objective are reasons both for the difficulty which business organizations typically have had in resolving these problems and for the potential value of new methods for attacking them. Businessmen as individuals cannot expect to eliminate entirely the effect of inventory fluctuation when forecasts on which production plans are based are in error. But they can protect their part of the economy from the costs of extreme fluctuations, first, by keeping inventories well under control and, second, by fixing inventory levels and plans based on clear assessment and balancing of risks. Both serve to reduce the likelihood that inventories will absorb excessive assets and to minimize the need for forces inventory liquidation.

Inventories serve important social function [5]. For example, inventories permit the consumer to be the master of an economy made up of highly integrated and often inflexible production units. Inventories give business the flexibility at reasonable cost to meet the design of consumers. The rigidities and lack of consumer freedom have been clear on occasions like war or mobilization build-ups when inventories have been deleted and the economy producer-controlled. Inventories also permit business to help stabilize employment and increase the utilization of skilled employees. Inventories help by absorbing fluctuations in demand-some predictable, like seasonal buying pattern, others unpredictable. The importance of employment stability on community well-being is receiving full attention from political and economic groups. efficient inventory management and production scheduling are an essential part of a business program to achieve goal of company.

2. PROBLEM STATEMENT

Determining safety stock levels is an important part of inventory and production scheduling. The safety (or Buffer) stock is defined as the average level of the net stock just before a replenishment or the amount of inventory kept on hand, on the average, to allow for the uncertainty of demand and uncertainty of delivery schedules over the replenishment lead time [7]. The level of safety stock is controllable in the sense that this investment is directly related to the level of desired customer service(that is, how often customer demand is met from stock).

The theory behind the determination of safety stocks when the company is faced with uncertain demand is fairly straight forward [4]. The determination of buffer stocks are a function of the service, α , set by company policy. Provided the distribution of demand can be identified as a normal distribution, then the actual safety stock level can be established based upon that particular fractile value, K, of the demand distribution which will guarantee that demand can be satisfied α-service level. This safety stock level is the difference between K and the average demand during the lead time for which the order is outstanding. The difference can be expressed in terms of the standard deviation of the demand distribution. Since safety stocks, on the average, remain at a fairly constant level, it is important that the expense associated with maintaining these stocks be controlled. These expenses are reflected in the cost of holdings and maintaining this inventory.

Other dimensions associated with safety stock levels can also be considered. Safety stocks require space for storage and in many cases space is a limited resource. Inventory tax which is reported and paid periodically is based upon inventory levels at a fixed date. The firm cannot only reduce the levels in anticipation of the approaching date by adjusting its order policies; but also by establishing control and reducing the fixed portion of inventory, the safety stock. Another important consideration that must not be overlooked is that safety stocks represents a use of capital and therefore their presence prohibits the use of that capital in other areas of the business operation.

Buffer stock levels may be reduced when actual demand is forecasted and the safety stock decision is based upon the distribution of the forecasting errors which result [5]. This was done by measuring the forecast error after actual sales are recorded and updating the statistics about the distribution of forecasting errors which had been observed from the past use of the error forecasting model. If the forecast error, ε , is defined as the difference between the forecast and the actual sales for the given period, then the buffer stock level for any production be associated with the largest negative error to expected from the use of the forecasting model. If $\bar{\varepsilon}$ and δ_e are respectively, the arithmetic mean and standard deviation of the distribution of forecast errors then the maximum error can be stated as the absolute value of the expression $[\varepsilon - K\delta_e]$. Where K is the appropriate fractile value on the distribution of forecast errors, and is dependent upon the desired service level. If the forecast model yields unbiased forecast, then this safety stock level is simply $K\delta_e$.

After safety stocks are determined independently from this forecasting model for each product group, it can be seen that capital and warehouse space constraints may restrict the actual implementation of the results of the forecasting model. To escape this problem a model is proposed that can be used to evaluate the decisions on safety stock levels simultaneously for all products subject to the restrictions imposed by the constraints upon this decision process. The model presented is an application of the goal programming (GP) algorithm. This GP algorithm, combined with both the results of a forecasting model and a set of constraints imposed by the limited space and capital resources, is used to determine buffer stock levels for a set of product groups.

3. BACKGROUND OF G.P.

Goal programming is a mathematical programming technique to deal with multiple, incommensurate objectives. It operates by minimizing deviational variables. For background on theory and technique, refer to Charnes and Cooper(1955, 1977), Lee(1972), and

Ignizio(1976). There are a number of goal forms possible in goal programming. It may be desired to (1) minimize deviational variables according to some weighting scheme; (2) satisfy goals according to some preemptive priority scheme; or a combination of these form. An alternative form would be to (3) minimize the maximum deviation form a set of goals.

The basic motivation is the fact that the definition of preemptive priority factors implies that higher order goals must be optimized before lower order goals are even considered. In the terminology of multiple objective decision making, our algorithm does provides an efficient solution for a specified ordinal ranking.

The general model for the linear GP problem can be stated as follow [2] :

(1) Min $$z = \sum_{k=1}^{p} p_k \sum_{i=1}^{m} (w_{ik}^- d_i^- + w_{ik}^+ d_i^+)$$

Subject to

(2)
$$\begin{aligned} A_{11}X_1 + A_{12}X_2 + \cdots + A_{1n}X_n + d_1^- - d_1^+ &= b_1 \\ A_{21}X_1 + A_{22}X_2 + \cdots + A_{2n}X_n + d_2^- - d_2^+ &= b_2 \\ A_{m1}X_1 + A_{m2}X_2 + \cdots + A_{mn}X_n + d_n^- - d_m^+ &= b_m \end{aligned}$$

(3)
$$\begin{aligned} b_{11}x_1 + b_{12}x_2 + \cdots + A_{1n}x_n &= c_1 \\ b_{21}x_1 + b_{22}x_2 + \cdots + b_{2n}X_n &= c_2 \\ b_{m1}x_1 + b_{m2}x_2 + \cdots + b_{mn}x_n &= c_n \end{aligned}$$

(4) $x_j \geq 0,\ j = 1,2,\ldots,n;\quad d_i^-, d_i^+ \geq 0,\ i = 1,\ldots,m$

where x_j - the decision variables

b_i - the target for goals I

A_{ij} - the coefficient of x_j in the i_{th} goal constraint

c_i - the resource value for real constraint I

b_{ij} - the coefficient of x_j in the i^{th} real constraint

d_i^- - the underachievement of goal i

d_i^+ - the overachievement of goal i

p_k - the k^{th} preemptive priority factor

$(p_{k-1} >>> p_k$ for *all k*)

w_{ik}^- - the weight assigned to d_i^- at priority p_k

w_{ik}^+ - the weight assigned to d_i^+ at priority p_k

Equation (1) represents the objects function, which minimizes the weighted sum of the deviational variables at each priority. The system of equation (2) represents the goal constraints, relating the decision variables to the targets and the system of equation (3) represents the real constraints, satisfying the equality with the decision variables at first time, and the set of inequalities (4) represents the standard nonnegativity restrictions on all variables.

Initially, all present GP algorithms seek to achieve the goals associated with the highest priority (P_i). Once the maximum level of achievement is obtained, the lower priority goals are considered without destroying the level of achievement for P_i. Hence, while trying to achieve the goals with respect to priority P_1, it is unnecessary to consider the goal constraints of the lower priorities, thus realizing a considerable reduction in problem size. The elimination procedure is based on the fact that in order to maintain the levels of achievement for the higher priority goals, a number of nonbasic variables can be eliminated at each state.

4. APPLICATION OF MODEL

A businessman try to set the buffer stock levels for the demonstration of this application. Consider four product groups for which buffer stock decisions must be determined subject to physical space constraint and a capital constraints. Assume that a suitable forecasting model has been used to predict the demand for these product group has been empirically determined from past use of this forecasting model: that the forecasting model is statistically unbiased and distribution of forecasting errors are approximately normally distributed: that a service level of α.I considered desirable for each of the product group: and the limiting constraints on the determination of the buffer stock levels are a space constraint dictated by warehouse space allocation and a capital constraint related to the amount of capital associate with this inventory.

In Figure 1 the hypothetical data related to this example are presented. Column 1 contains four product group. Column 2 represents the priority of the product groups. Therefore X_1 is the first priority to be satisfied because X_1 group is the most important to this businessman considering lost of good will. Column 3 contains the minimum buffer stock that must be maintained to assure a certain customer service level. Column 4 represents the space requirements(square feet/unit). Column 5 contains the capital requirement ($/unit)

Product Group	Priority	α service level value on error	Space requirement sq. ft./unit	Product value $ per unit
X_1	1	80	1.24	5.56
X_2	3	40	2.45	3.25
X_3	2	50	8.43	4.02
X_4	4	1610	0.25	0.14

Figure 1 -Hypothetical Data

Using these data, we can formulate as follows:

In this formulation X_i is the buffer stock level that can be held for the i^{th} product group.

This level must be determined simultaneously for all product groups subject to the priority of goals and the space and capital requirements. The first two equations are real constraints. the others are goal constraints. The variables which represent deviations from the service level goals for the i^th^ product group are d_i^+ and d_i^-. The variable, d_i^+, is the amount of over achievement of the i^th^ goal, while the variable d_i^- is the amount of under achievement of this goal. In the context of formulation, d_i^- will be greater than zero when the achievement of the i^th^ goal is unattainable. If d_i^+ is greater than zero, then this is an indication that there are enough space and capital resources such that the buffer stock can be increased for i^th^ product group. In this example there are 600 square feet of warehouse space available and $800 limit on the capital investment to be allocated to buffer stock levels across the product groups.

Min Z = $P_1(d_1^+ + d_1^-) + P_2(d_2^+ + d_2^-) + P_3(d_3^+ + d_3^-) + P_4(d_4^+ + d_4^-)$

Subject to:

$$\begin{aligned}
&1.24X_1 + 2.45X_2 + 8.43X_3 + 0.25X_4 \le 600\\
&5.56X_1 + 3.25X_2 + 4.02X_3 + 0.14X_4 \le 800\\
&X_1 + d_1^- - d_1^+ = 80\\
&X_2 + d_2^- - d_2^+ = 40\\
&X_3 + d_3^- - d_3^+ = 50\\
&X_4 + d_4^- - d_4^+ = 1610\\
&X_i,\ d_i^+,\ d_i^- \ge 0 \quad \text{for } i = 1,\ 2,\ 3,\ 4.
\end{aligned}$$

An optimal solution to this problem was identified.

The solution is as follows:

$X_1 = 80$ $d_1^+,\ d_1^- = 0$ $X_2 = 40$ $d_2^+,\ d_2^- = 0$ $X_3 = 0,$ $d_3^- = 50$ $X_4 = 1600$ $d_4^- = 10$

This solution makes a businessman set up the optimal buffer stock level subject to a physical space contraint and a capital constraint.

5. CONCLUSION

The example above was used to demonstrate the application of a goal programming model to the determination of the buffer stock levels for inventory control purposes when a businessman tries to set up the inventory with limited amounts of investment and space. There are advantages to this approach which justify this application. These advantages are the simultaneous analysis, and the consideration of constraints in the dynamic review of assumptions.

The first advantage refers to the simultaneous determination of the buffer stock levels for all products involved. Traditional inventory control models consider inventory strategy decisions independently by product or product group. This application is considered an extension of the inventory models since the decisions are made for all products simultaneously.

Traditional inventory models do not consider outside restrictions placed upon the solutions of the models. The results as indicated by inventory models may not be feasible, particularly when the analysis considers only one product at a time. The proposed model is an extension and an improvement since it explicitly considers the environmental constraints placed upon the safety stock decision, such that a nonfeasible solution cannot result. Considering both the simultaneous and constrained nature of the solution process, this model can be used to identify the conflict between inventory control policies and the resources which are available to enact these policies. The model can also accommodate a priority ranking when original policies cannot be completely satisfied.

As goals, priorities, weights within priorities, or environmental constraints change, the model can be adapted to correct these changes with the flexibility. In addition, the model can be used as part of a sensitivity analysis to analyze the effects that different goals or priorities have upon safety stock levels. Goals may change as either the service level requirements or the results of the forecasting model change. As the accuracy and the precision of the forecasting model change for any product group, these change can be reflected in the buffer stock determination by adjusting the appropriate value in equations. Also, service requirements can be adjusted with slight changing of α to meet the seasonal variations.

The environmental constraints of the model may also change. As storage space or capital available for inventory purposes change, the buffer stock decision can be modified to reflect the present limitations placed upon the safety stock decision by the available resources. Additional environmental constraints can be included in the model to meet special requirements.

The dynamic nature of the model was implied above in the discussion of the flexibility of the model. The dynamics of the model relate to the periodic use of the model to evaluate the safety stock decision as goals, priorities, weights, and constraints change. In effect the model can be used as part of a periodic evaluation of this decision process. In this way decisions that are made at any one time need not be considered static, but can be reviewed and changed to reflect current conditions.

In summary it can be seen that GP model can be applied in the decision process concerning buffer stock decisions. The use of this model is and can be viewed as an extension to the use of the traditional inventory models since simultaneous decisions can be made subject to constraints upon the process. In addition the model is good for a dynamic, flexible review of the decisions that are made. For these reasons, this model has potential for meaningful applications in determination of buffer safety level simultaneously with environmental constraints.