A New Calculation Method for the Radiation Impedance of Transducer with Regular Square Vibrating Surface

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Abstract

Although the radiation impedance of a transducer with a regular square surface has been studied by many researchers, the formulas are still very complicated, which results in long computation time and low accuracy. In this paper, we propose a new algorithm for the calculation of acoustic radiation impedance in which the regular square vibrating surface of a transducer is divided into small elements and duplicate calculations are eliminated in the process of calculating mutual effects of the elements. Using this algorithm, shorter computation time and higher accuracy of results can be obtained. As a demonstration, the self and the mutual radiation impedance of transducers with a regular square surface are calculated and the accuracy of the results is evaluated.

I. Introduction

The radiation impedance of an ultrasonic transducer is an important factor for determining its acoustic radiating characteristics[1-3]. In general, ultrasonic transducers are arrayed in a sonar and the radiation beam pattern of the sonar has been influenced by the self and the mutual radiation impedance[4]. Although the radiation impedance has been studied by R. Pritchard[5], G. Chertock[6], C. Wallace[7], etc. , the mathematical presentation is still so complex that the analytical solution for a general shape of transducer is hardly obtained. The self radiation impedance of a transducer with regular square vibrating surface, commonly used in the underwater sonar system, has been calculated and approximate solutions were suggested by G. Swenson[8], P. Tournois[9], W. Thompson[10], S. Burnett[11], D. Stepanishen [12], and H. Levine[13], etc. However, because those calculation methods include a lot of approximation and numerical integration, they require enormous calculation or their accuracy is limited.

In this paper, we propose a new algorithm for the calculation of acoustic radiation impedance of a transducer with a regular square vibrating surface that is divided into small elements and the duplicate calculations are eliminated in the process of computing the mutual effects of elements. Using the algorithm proposed, quadruple loops due to quadruple integration are simplified to single or double loops, and shorter computation time and higher accuracy can be obtained.

II. Theory

2-1. Self radiation impedance

In order to calculate the radiation impedance of a regular square vibrating surface, we divide the surface into $n \times n$ vibrating elements and consider mutual influence among each elements as shown in Figure 1. The transfer function $f(r)$ which means the influence between e_{pq} and e_{lm} in the vibrating surface is represented as follows[14] :

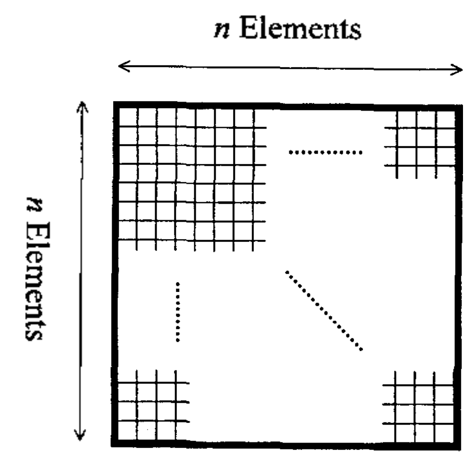

Figure 1. Vibrating surface divided into n×n elements.

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$$f(r)=\frac{e^{-jkr}}{r}. \tag{1}$$

Quadruple calculation loops are necessary to calculate with regard to indices p, q, l, and m, indicating the locations of elements. Large n results in good accuracy but enormous calculating process. Since Eq. (1) is a function only of distance r between two elements, under assumption of uniform vibrating amplitude, there are a lot of cases that the distance between elements is same. For rectangular vibrating surfaces consisting of 10×10 elements, the number of cases of identical distances between elements is obtained in Table 1, where the numbers in parentheses mean the relative position between the elements. For example, (4,3) represents relative distance of $(4^2+3^2)^{1/2}=5$, and the number of same distance cases is 336 as in the table. When the relative coordinate of elements is (x, y) for the vibrating surface divided into $n\times n$ elements, the equation for obtaining such numbers can be deduced explicitly from the terms in numerical series for three coordinate conditions as follows:

$$n_{id}=\begin{cases}\{(2x)^2 \mid x=1\sim n-1\} & for \quad x=y,\\ \{4nx \mid x=1\sim n-1\} & for \quad x=1 or y=1,\\ \{8xy \mid y=2\sim n-1, x=1\sim y-1\} & for \quad x\neq y.\end{cases} \tag{2}$$

Therefore the total amount of calculation is reduced by multiplication of n_{id} and the transfer function $f(r)$. Using Eq. (1) and (2), the self-radiation impedance for each coordinate condition are obtained as follows:

(1) for $x=y$,

$$s_1=\sum_{x=1}^{n-1}(2x)^2\frac{\exp\left\{-jk\sqrt{2\left(\frac{n-x}{n}\right)^2}\right\}}{\sqrt{2\left(\frac{n-x}{n}\right)^2}}, \tag{3}$$

(2) for $x=1$ or $y=1$,

$$s_2=\sum_{x=1}^{n-1}4xn\frac{\exp\left\{-jk\left(\frac{n-x}{n}\right)\right\}}{\left(\frac{n-x}{n}\right)}, \tag{4}$$

(3) for $x\neq y$,

$$s_3=\sum_{y=2}^{n-1}\sum_{x=1}^{y-1}8xy\frac{\exp\left\{-jk\sqrt{\left(\frac{n-x}{n}\right)^2+\left(\frac{n-y}{n}\right)^2}\right\}}{\sqrt{\left(\frac{n-x}{n}\right)^2+\left(\frac{n-y}{n}\right)^2}}, \tag{5}$$

where k is a wave number.

Thus, the total self-radiation impedance Z_s of a rectangular vibrator divided into $n\times n$ elements can be obtained as follows :

$$\frac{Z_s}{\rho c}=\frac{jk}{2\pi}\left(\frac{a}{n}\right)^4(s_1+s_2+s_3), \tag{6}$$

where a is the length of a side of a vibrating surface.

Table 1. Number of cases of identical distance for 10?10 elements.

(9,9) 4	(9,8) 16	(9,7) 24	(9,6) 32	(9,5) 40	(9,4) 48	(9,3) 56	(9,2) 64	(9,1) 72	(9,0) 40
(8,8) 16	(8,7) 48	(8,6) 64	(8,5) 80	(8,4) 96	(8,3) 112	(8,2) 128	(8,1) 144	(8,0) 80	
(7,7) 36	(7,6) 96	(7,5) 120	(7,4) 144	(7,3) 168	(7,2) 192	(7,1) 216	(7,0) 120		
(6,6) 64	(6,5) 160	(6,4) 192	(6,3) 224	(6,2) 256	(6,1) 288	(6,0) 160			
(5,5) 100	(5,4) 240	(5,3) 280	(5,2) 320	(5,1) 360	(5,0) 200				
(4,4) 144	(4,3) 336	(4,2) 384	(4,1) 432	(4,0) 240					
(3,3) 196	(3,2) 448	(3,1) 504	(3,0) 280						
(2,2) 256	(2,1) 576	(2,0) 320							
(1,1) 324	(1,0) 360								

2-2. Mutual radiation impedance

To calculate mutual radiation impedance, we consider two neighbor transducers with regular square vibrating surfaces. The influence between an element in a transducer and one in the other transducer shown in Figure 3 is calculated. Two vibrating surfaces, each surface being divided into $n\times n$ elements as shown in Figure 1, are separated by d, and the effect between element e_{pq} and element e_{lm} can be represented by Eq. (1), i.e. in the same

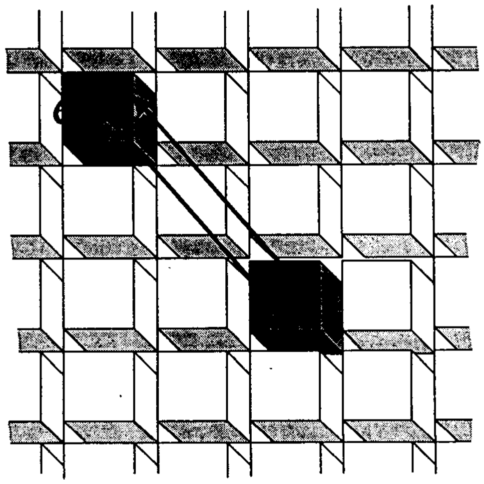


Figure 2. Mutual effects between elements.

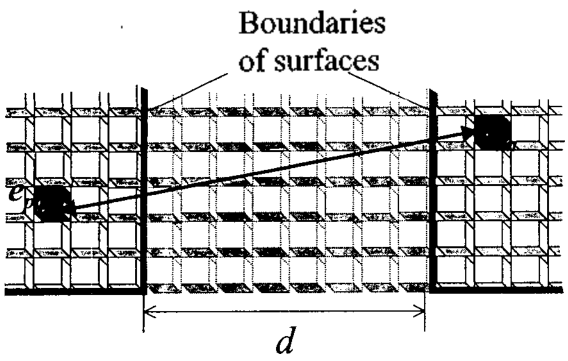


Figure 3. Mutual effects between elements in neighboring surfaces.

manner as the self-radiation impedance. For simplification of the required calculation, if the distance d is multiple times the length of an element, i.e., where m is an integer, the mutual radiation impedance for each coordinate condition can be obtained as follows:

(1) for $x=y$,

$$m_1 = \sum_{x=1+d}^{n-1} \frac{4(n-x)(x-d)\exp\left(-jk\frac{x\sqrt{2}}{n}\right)}{\frac{x\sqrt{2}}{n}}, \qquad (7)$$

(2) for $x=1$ or $y=1$,

$$m_2 = \sum_{x=1+d}^{2n-1+d} \frac{\left(2n^2 - 2n|x-n-d|\right)\exp\left(-jk\frac{x}{n}\right)}{\frac{x}{n}}, \qquad (8)$$

(3) for $x \neq y$ and $x, y<n$ and $x \leq d$ (or $y \leq d$),

$$m_3 = \sum_{x=1}^{d}\sum_{y=d+1}^{n-1} \frac{\{36-4(x-1)\}(y-d)\exp\left(-jk\frac{\sqrt{x^2+y^2}}{n}\right)}{\frac{\sqrt{x^2+y^2}}{n}}, \qquad (9)$$

(4) for $x \neq y$,

i) $x, y<n$ and $x>d$ (or $y>d$),

$$m_4 = \sum_{x=1+d}^{n-2}\sum_{y=x+1}^{n-1} \frac{\left[4(n-d)(x-d)+8\left\{\frac{n-d}{2}-(x-d)\right\}(y-d)\right]}{\frac{\sqrt{x^2+y^2}}{n}} \exp\left(-jk\frac{\sqrt{x^2+y^2}}{n}\right), \qquad (10)$$

ii) otherwise

ii-1) if $d \leq n$

$$m_5 = \sum_{x=1}^{n-1}\sum_{y=n}^{2n+d-1} \frac{4(n-x)(n-|n+d-y|)\exp\left(-jk\frac{\sqrt{x^2+y^2}}{n}\right)}{\frac{\sqrt{x^2+y^2}}{n}}, \qquad (11\text{-}1)$$

ii-2) if $d>n$

$$m_5 = \sum_{x=1}^{n-1}\sum_{y=d}^{2n+d-1} \frac{4(n-x)(n-|n+d-y|)\exp\left(-jk\frac{\sqrt{x^2+y^2}}{n}\right)}{\frac{\sqrt{x^2+y^2}}{n}}, \qquad (11\text{-}2)$$

where (x, y) represents the relative coordinates between the two elements.

Consequently, the total mutual radiation impedance Z_m of two square vibrating surfaces can be represented as follows:

$$\frac{Z_m}{\rho c} = \frac{1}{2}\frac{jk}{2\pi}\left(\frac{a}{n}\right)^4 (m_1+m_2+m_3+m_4+m_5). \qquad (12)$$

III. Result

The self-radiation impedance and the mutual radiation impedance for regular square vibrating surfaces with a side length a are calculated to evaluate the efficiency of the method suggested in this paper.

3-1. Self radiation impedance of a regular square vibrating surface

Figure 4 shows the results of self radiation impedance for various of n. In the figure, solid lines of radiation

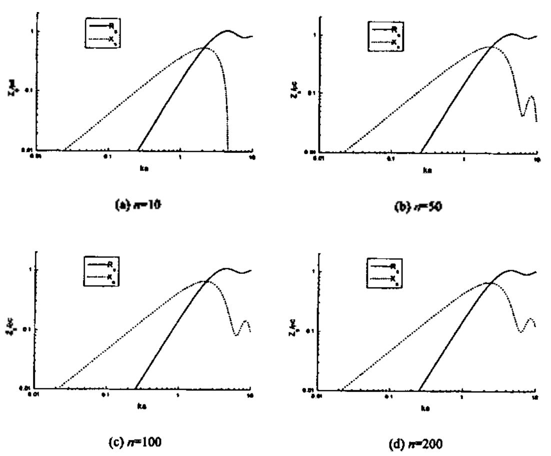


Figure 4. Self radiation impedance as function of element number.

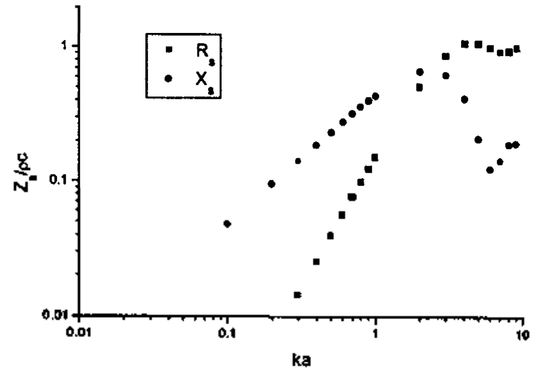


Figure 5. Result of self radiation impedance calculated by D. Burnett[11].

resistance R_s remains almost the same regardless of n, but dotted lines of radiation reactance X_s vary with n. The accuracy of the results can be estimated by comparison with the result by D.S.Burnett[11] as shown in Figure 5. For 10×10 vibrating elements, the results of the radiation resistance R_s in Figure 4(a) are similar to those in Figure 5 calculated by the integration formula, whereas the results of the radiation reactance X_s for the two methods mentioned above show large differences as ka increases.

Using the proposed method, it takes only a few seconds for a PC(IBM Compatible Pentium) to carry out the calculation and obtain the results in Figure 5, while it takes a few hours with the conventional integration

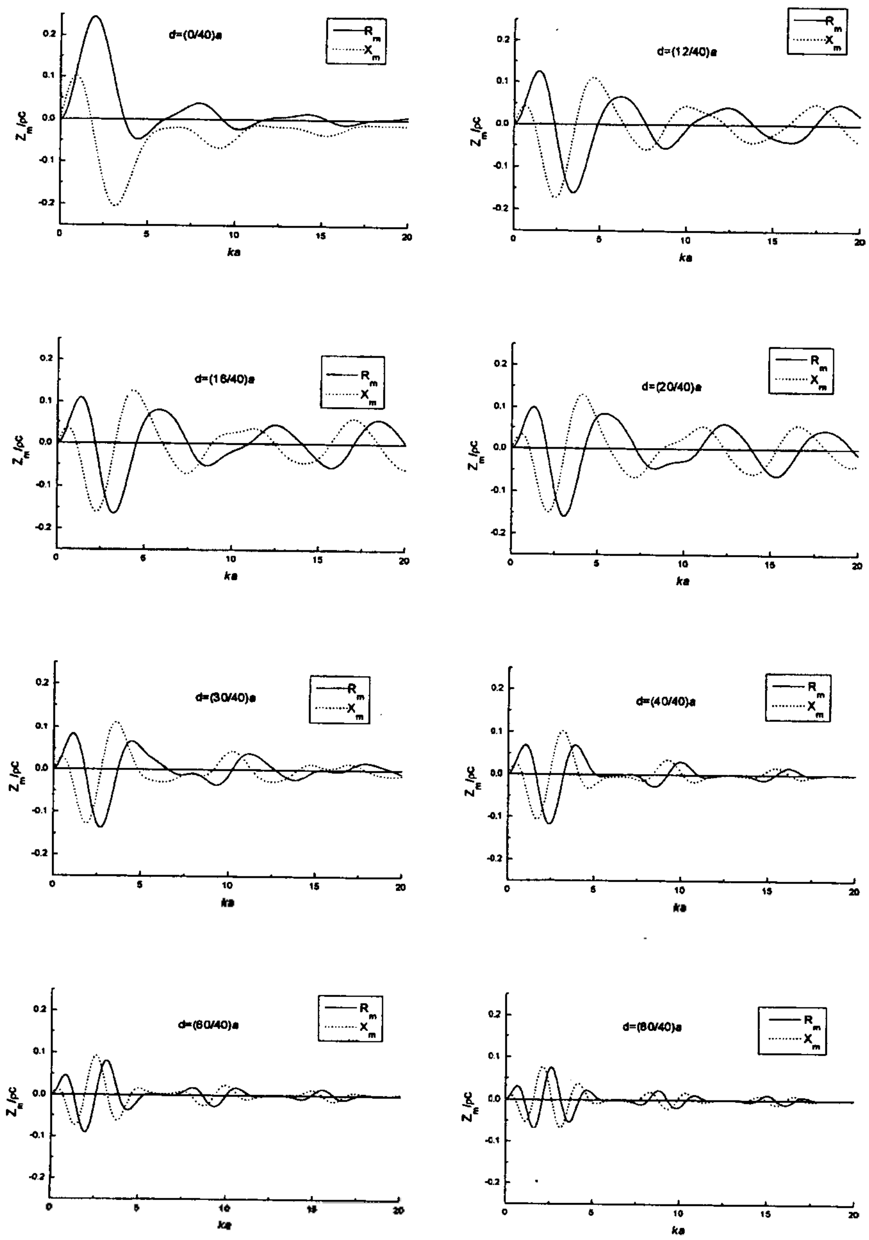


Figure 6. Calculated results of mutual radiation impedance of regular square surface.

method. Figure 4(b) represents the results for 50×50 vibrating elements. The figure shows that the results of the radiation reactance X. have been improved though they still show large errors when *ka* is larger than 4. In this case, the calculation takes 20 to 30 seconds to carry out the calculation. From Figure 4(c), the results for n=100 are in good agreement with Figure 5, but there exists 17% error of radiation reactance as ka increases over 4. It takes a few minutes to calculate for n=100. To estimate the different accuracies with different n, the results with the number of elements 200×200 were given in Figure 4(d). The results in Figure 4(d) coincide with those of Figure 5 down to 5 places of decimals, and it takes 20 to 30 minutes to obtain the results. Comparing Figure 4 with Figure 5, it is noted that the accuracy improves as n increases, but even for small n, the accuracy is quite high when *ka* is small. Thus the optimal number of elements can be determined.

3-2 The mutual radiation impedance

Figure 6 represents the mutual radiation impedance of two vibrators with regular square vibrating surfaces with n=40. The two vibrators are separated by multiples of a side length of an element. This figure shows that the mutual radiation impedance decreases as the distance between the two vibrators increases. Especially, within the distance range from 0 (the two vibrators are put together) to a (the two vibrators are separated by the side length of a surface), both the radiation resistance(the solid line) and the radiation reactance(the dot line) change dramatically. The variation of radiation impedance versus *ka* for various values of d is shown in Figure 7 Figures 6 and 7 show that the mutual radiation impedance has significant effect within the distance range from 0 to a even for large ka values. It should be noticed that acoustic field is affected by the mutual radiation impedance depending on the distance between vibrating surfaces when we array vibrating surfaces in underwater sonar system.

To confirm the effectiveness of the algorithm proposed for mutual radiation impedance, we compare the results of Figure 6 with those calculated by integral method shown in Figure 8. They are same to the sixth decimal places. However, the method proposed requires only a few seconds while the one suggested by J. Lee[15] takes a few hours. Therefore, it could be said that the former is more efficient than the latter. In addition, it is expected that highly accurate calculation can be done by the former method in relatively short time by increasing number of element.

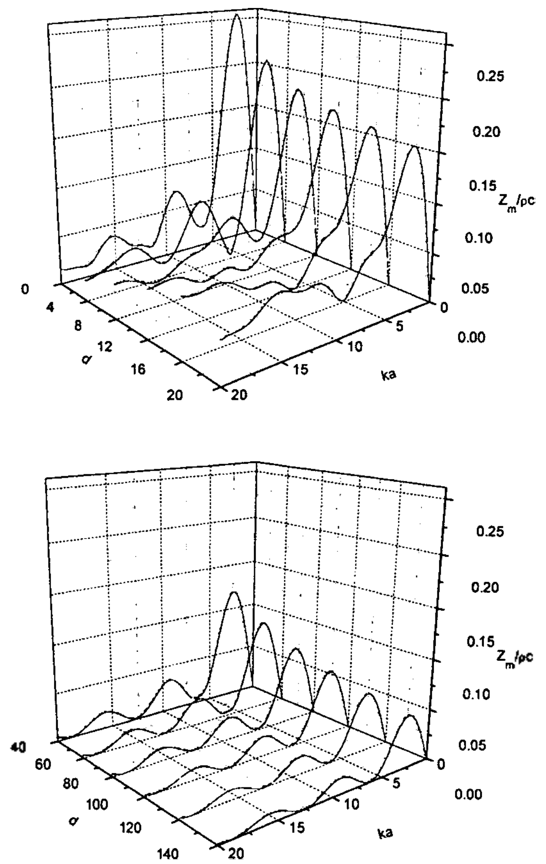
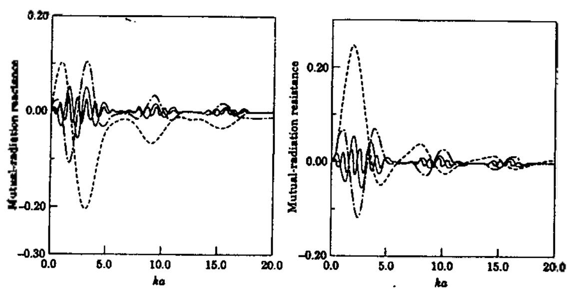


Figure 7. Mutual radiation impedance as function of distance d.



(a) (b)

Figure 8. Results of mutual radiation impedance calculated by J. Lee[15].

(a) mutual radiation resistance, (b) mutual radiation reactance.

d:– –,(0/40);– · –, (40/40)a;——, (80/40)a;——, (160/40)a

Ⅳ. Conclusion

We proposed a new calculation method to analyze the radiation impedance of a transducer with a regular square surface used in sonar. In the method, the vibrating surface is divided into small elements and the mutual influence between the elements is calculated. The method is efficient for estimating the self and the mutual radiation impedance. The proposed algorithm can avoid the repeated computation by recognizing the number of duplicate calculations in the computation of mutual influence between each element.

Evaluation of the effectiveness by comparing the proposed method with the conventional numerical integration method shows that shorter computation time and higher accuracy are expected by the proposed method.

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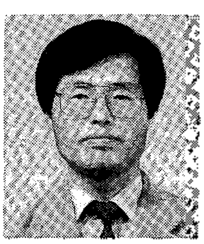

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