⊠연구논문

Thurstonian Modeling for Triangular Method toward Analysis of Rating Data

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Abstract

Products are often evaluated on rating scales to measure and quantify their attributes of interest. In case that one wishes to compare multiple rating datasets simultaneously, there must be a standardized scale with which one can discriminate relative differences among corresponding scale means. In this regard, the concept of Thurstonian modeling applied to various discrimination tests including the triangular method has been recently being reconsidered. In this paper we extend previous researches on the triangular method and evaluate the effect of unequal variances and correlated variables upon the probability of correct response using Monte-Carlo simulation. We observed that the probability of correct response depends on dimensionality, variances, and correlation structure of stimulus sets. But it does not depend on the relative orientation in a multidimensional space.

1. Introduction

Products and concepts are often evaluated on rating scales to quantify degree of liking, level of purchase interest, intensity of an attribute, or degree of difference. Rating scales are constructed in many different ways, but usually involve a number of options labeled with numbers, words or symbols.

One problem with rating data is that in case of using, in particular, different rating scales to evaluate several products of one kind, we cannot compare these data directly unless a standardized scale is provided. This kind of problem is prevalent, for instance, in the fields of food sciences where sensory evaluation methods are frequently required. In this regard, recently, the idea of Thurstonian modeling applied to various discrimination methods including the triangular method, the duo-trio method, *etc.*, has been being reconsidered.

Thurstone(1927) formulated a model through a subjective scale on which sensations can be ordered for stimuli of varying and unknown physical amounts. An individual is assumed to receive a sensation in response to a stimulus. The amount of sensation varies for stimuli of the same strength.

The basic idea behind Thurstonian modeling is that each time a product is evaluated, it will vary in its intensity. This can be a result of physiological effects like sensory adaptation, or it can even be due to lack of homogeneity in the samples of the products themselves. For this reason, it is better to think of rating values in terms of continuous distributions rather than discrete points.

In other words, sometimes the intensity of a stimulus will be stronger, sometimes it will be weaker. There will be, however, an average intensity which will occur most commonly. Such variation in intensity can be represented by a continuous distribution. And the normal distribution is the most common choice.

Various discrimination methods such as duo-trio and triangular methods are widely used in the area of sensory research and the main purpose of using these discrimination methods is to determine the distance between mean stimuli selected from normal distributions. See, *e.g.*, O'Mahony(1992, 1995) and David and Trivedi (1962).

In the triangular method, the subject is instructed to select out of three stimuli (two drawn from one stimulus set and one from another stimulus set) the one which is perceptually different from the other two. In the duo-trio method, one of the three stimuli is a designated standard and the subjects's task is to identify which of the other two stimuli is perceptually most similar to the standard.

In this paper, we mainly focus to the triangular method and evaluate its performance under various conditions, which is an extension of Ennis and Mullen (1985, 1986).

2. Triangular method

The normal Thurstonian model for the triangular method was first given by Frijters(1979) in which the sensory values were assumed to be drawn from normal density of equal variances. In general, we assume that there are two sets of stimuli S_x and S_y. We sample from both stimulus sets and at least two stimuli are drawn from at least one of the stimulus sets.

The stimuli S_{x_i} and S_{y_j} give rise to corresponding sensory values of magnitudes $\boldsymbol{x}_i$ and $\boldsymbol{y}_j$, where $\boldsymbol{x}_i' = (x_{i1}, \cdots, x_{in})$, and $\boldsymbol{y}_j' = (y_{j1}, \cdots, y_{jn})$ and n is the number of dimensions. The sensory values are mutually independently distributed with $\boldsymbol{x}_i$ having density function $f(\boldsymbol{x})$ and $\boldsymbol{y}_j$ having density function $f(\boldsymbol{y})$. The L_p-distance between $\boldsymbol{x}_i$ and $\boldsymbol{y}_j$ is defined by

$$\delta = \left[\sum_{k=1}^{n} |x_{ik} - y_{jk}|^p \right]^{1/p}.$$

Assume that the probability densities $f(\boldsymbol{x})$ and $f(\boldsymbol{y})$ are multivariate normal with means $\boldsymbol{\mu}_x$ and $\boldsymbol{\mu}_y$. In the multivariate Euclidean model with $p=2$ for the triangular method, a correct response will be obtained if

$$\text{(i)} \quad \sum_{k=1}^{n}(x_{1k} - x_{2k})^2 < \sum_{k=1}^{n}(x_{1k} - y_k)^2 \text{ and } \sum_{k=1}^{n}(x_{1k} - x_{2k})^2 < \sum_{k=1}^{n}(x_{2k} - y_k)^2$$

for triangles composed of S_{x_1}, S_{x_2}, and S_y; or if

$$\text{(ii)} \quad \sum_{k=1}^{n}(y_{1k} - y_{2k})^2 < \sum_{k=1}^{n}(y_{1k} - x_k)^2 \text{ and } \sum_{k=1}^{n}(y_{1k} - y_{2k})^2 < \sum_{k=1}^{n}(y_{2k} - x_k)^2$$

for triangles composed of S_{y_1}, S_{y_2}, and S_x.

The distance between $\boldsymbol{\mu}_x$ and $\boldsymbol{\mu}_y$ will be denoted as d', which is sometimes called a level of discriminal distance or a sensory difference. We note that mathematical formulation for the correct response was given by Ennis and Mullen (1986).

The extent to which a sensory difference exists, namely, $d' > 0$, has been quantified in various ways. One obvious way is to use the probability of correct response (P_c) for a particular method. Ennis and Mullen(1986) conducted a Monte-Carlo simulation study to evaluate the effect of d' on P_c for different numbers of independent sensory variables (upto 10 dimensions). In addition, they evaluated the effect of correlation structure and unequal variances (upto 2 dimensions) on P_c.

From the result, we see that when the number of sensory dimensions is greater than 1, the P_c for the triangular method is not monotonically related to d' but depends in a particular way on dimensionality, correlation structure and the relative orientation of the sensory values in a multidimensional space. But the effect of unequal variances on the correlated sensory values is still unknown.

3. Unequal variances and correlated variables

In order to evaluate the performance of the triangular method by P_c under various correlation structures with unequal variances, we performed an extensive Monte-Carlo simulation by generating 100,000 sets of triangles from multivariate normal distributions for each case. We used a random number generator builtin SAS 6.12 package.

3.1 Unequal variances

<Table 1> shows the effect of unequal variances of the stimulus sets on P_c, when d'=0. P_c values are almost the same when the variance-ratio is close to 1. In this case it is evident that dimensionality has a minimal effect. Theoretically, when the triangular method is performed between identical stimulus sets, P_c is 1/3 regardless of dimensionality.

As one might expect, P_c increases as the variance-ratio increases. Under a fixed variance-ratio, increasing dimensionality results in an increase in P_c. This effect becomes more evident as the variance-ratio increases. This result shows that for the triangular method the multidimensionality itself is an important determinant of discrimination between stimulus sets.

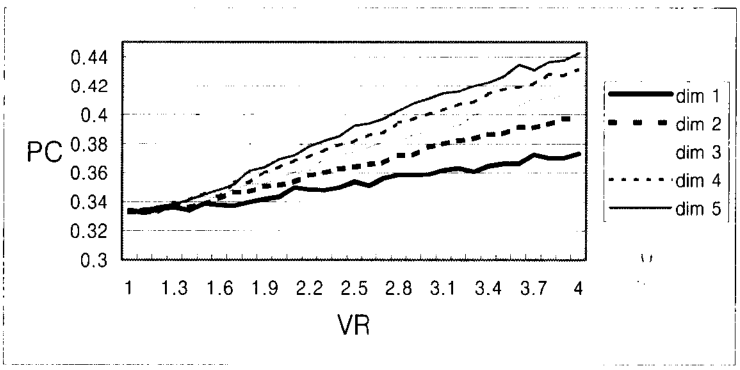
In <Figure 1>, all five lines gradually increase as the variance-ratio increases but have different slopes depending on their dimensions. The slopes become steeper as dimension increases. This means that the higher dimension we have the more influenced is P_c by the variance-ratio.

3.2 Unequal variances under correlated variables

When the variables are independent, the probability of a correct response will depend on the distance between the means of the stimulus sets and the number of variables involved in the distance estimate. It will not depend on the relative orientation of the stimulus sets in the multidimensional space.

< Table 1 > Estimated probability of a correct response, P_c, as a function of variance-ratio when $d'=0$

variance ratio	number of dimensions				
	1	2	3	4	5
1.0	0.33403	0.33308	0.33378	0.33393	0.33357
1.5	0.33881	0.33830	0.34346	0.34562	0.34484
2.0	0.34329	0.35162	0.35621	0.36396	0.36944
2.5	0.35391	0.36408	0.37437	0.38197	0.39255
3.0	0.35878	0.37733	0.38788	0.40044	0.41132
3.5	0.36619	0.38677	0.40462	0.41744	0.42694
4.0	0.37343	0.39751	0.41333	0.43175	0.44267



< Figure 1 > Plot of P_c values given in Table 1

But when the sensory values are correlated, P_c also depends on the degree of correlation and on the relative orientation of the difference between sets. In order to illustrate this effect, consider Figures 2-3 in which stimulus sets differ by 3 standard units (d'=3) in two dimensions and for which correlation coefficient (ρ) is 0.4 and 0.8, respectively. Each of them represents a weak correlation and a strong correlation, respectively.

In the Figures 2-3, we observe that periodicity of the P_c curves are very similar to sine curves with period of 180°. Moreover the patterns of the four P_c curves are almost the same. And, all P_c curves keep regular intervals among them regardless of relative orientation. This means that variance-ratio behaves independently of the relative orientation.

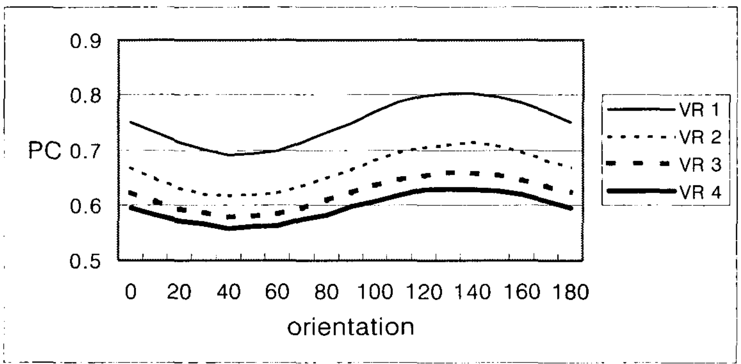
P_c curves attain the minimum at between 40° and 50° and the maximum at between 130° and 140°. Therefore we chose 45° and 135° as the standards of discrimination.

On each P_c curve with the same variance-ratio, the maximum value of P_c for $\rho = 0.8$ is larger than that for $\rho = 0.4$. In contrast, the minimum value of P_c for $\rho = 0.8$ is smaller than that for $\rho = 0.4$. This result shows that the effect of orientation increases as ρ increases. We confirmed this pattern by another simulation and the results are plotted in Figures 4-5.

As expected, when the orientation is 45°, the minimum value of P_c curve decreases as ρ increases and when orientation is 135°, the maximum value of P_c curve increases as ρ increases. In this case of $d' = 3$, the boundary of two sets becomes ambiguous as the variance-ratio increases. Hence the P_c decreases as the variance-ratio increases.

< Table 2 > P_c for different relative orientations of stimulus coordinates in a 2-dimensional space when correlation coefficient (ρ) is 0.4 and discriminal distance (d') between stimuls sets=3.

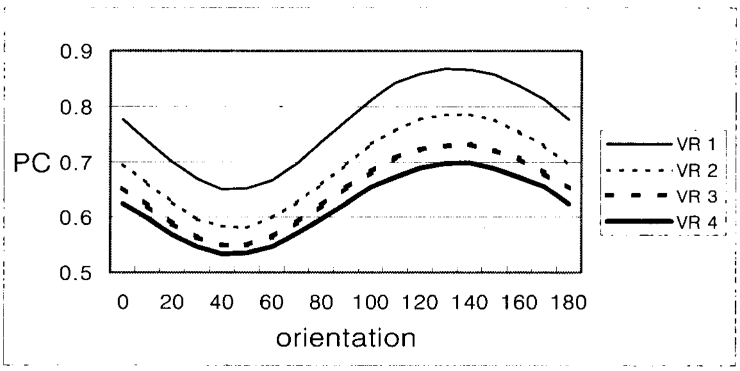
orientation (unit: degree)	variance ratio (VR)			
	1	2	3	4
0	0.75018	0.66774	0.62298	0.59481
30	0.70110	0.61981	0.58657	0.56611
60	0.69869	0.62210	0.58455	0.56276
90	0.74877	0.66469	0.62299	0.59770
120	0.79841	0.70471	0.65311	0.62799
150	0.79656	0.70848	0.65557	0.62586



< Figure 2 > Plot of P_c values given in Table 2

< Table 3 > P_c for variance-ratios and relative orientations of stimulus coordinates in a 2-dimensional space when ρ=0.8 and d'=3.0.

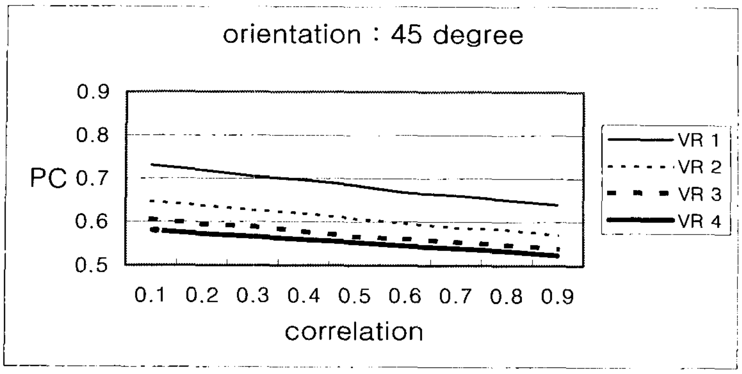
orientation (unit: degree)	variance ratio (VR)			
	1	2	3	4
0	0.77660	0.69556	0.65402	0.62391
30	0.67093	0.59522	0.56274	0.54639
60	0.66825	0.59931	0.56399	0.54613
90	0.77470	0.69567	0.65495	0.62485
120	0.85835	0.77717	0.72317	0.68892
150	0.85689	0.77538	0.72109	0.68827



< Figure 3 > Plot of P_c values given in Table 3

< Table 4 > P_c for variance-ratios and correlations between variables when relative orientations of stimulus coordinates in a 2-dimensional space is 45° and d'=3.0.

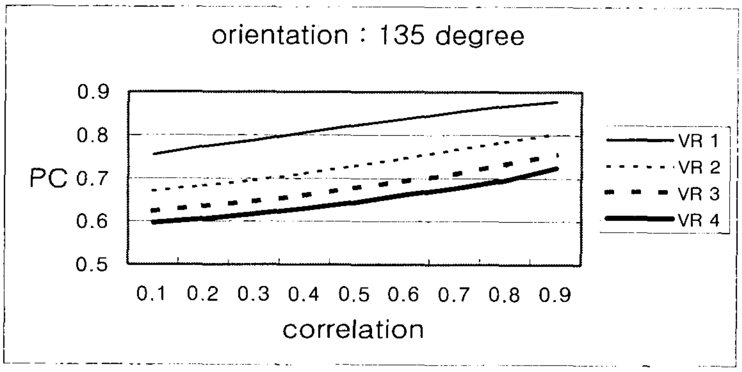
correlation	variance ratio (VR)			
	1	2	3	4
0.1	0.73011	0.64619	0.60584	0.57976
0.2	0.71778	0.63660	0.59473	0.57167
0.3	0.70379	0.62598	0.58979	0.56553
0.4	0.69589	0.61971	0.57795	0.55851
0.5	0.68414	0.60723	0.56660	0.55331
0.6	0.66796	0.59653	0.56148	0.54458
0.7	0.66049	0.58633	0.55445	0.53951
0.8	0.65007	0.58247	0.54723	0.53316
0.9	0.63951	0.56964	0.53938	0.52343



< Figure 4 > Plot of P_c values given in Table 4

< Table 5 > P_c for variance-ratios and correlations between variables when relative orientations of stimulus coordinates in a 2-dimensional space is 135° and $d'=3.0$.

correlation	variance ratio (VR)			
	1	2	3	4
0.1	0.75532	0.67035	0.62286	0.59666
0.2	0.77365	0.68246	0.63515	0.60479
0.3	0.78812	0.69461	0.64634	0.61737
0.4	0.80504	0.71132	0.66036	0.62969
0.5	0.82209	0.72937	0.67765	0.64408
0.6	0.83751	0.74718	0.69403	0.66144
0.7	0.85396	0.76696	0.71040	0.67722
0.8	0.86764	0.78567	0.73357	0.69646
0.9	0.87797	0.80254	0.75579	0.72367



< Figure 5 > Plot of P_c values given in Table 5

4. Concluding Remarks

Extending the previous work by Ennis and Mullen(1986), we evaluated the P_c of the triangular method under various conditions. Even when d' is zero, the possibility of choosing the odd stimulus correctly increases as the variance-ratio increases. The effect of the variance-ratio increases as dimensionality goes higher. However, when d' is not zero, namely, when the stimulus sets are not identical, the boundary between sets becomes ambiguous as the variance-ratio increases so that one cannot discriminate one set from the other easily. Hence increasing variance-ratio results in an decrease of P_c.

When variables are correlated within stimulus sets P_c depends on the degree of correlation and on the relative orientation between stimulus sets. We also observed the periodicity according to orientations. Further, it turned out that the variance-ratio does not have any interaction with orientation and correlation. We confirmed it by the regular interval between curves or lines given in Figures 2-5.

In summary, we observed the dependence of P_c on dimensionality, the variances of stimulus sets, and the relative orientation of the stimulus sets.

Finally we note that this type of study can be applied similarly to other discrimination methods to assess characteristics of P_c.

Acknowledgements

One of the authors, Yu, wishes to acknowledge the financial support from *Excellent Research Scholarship* awarded by the Graduate School, Ewha Womans University, in the academic year of 1997.

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