한국에너지공학회지(1999), 제8권 제2호
Energy Engg. J(1999), Vol. 8, No. 2, pp. 293~297

The Curvature and Shear Effects on the Eddy Viscosity

Hyo Jae Lim
Department of Mechanical Engineering, Hoseo University

Abstract — Direct comparisons are made between curvature-corrected eddy viscosity models and the present experimental data. The results show that the curvature effects can be quantified through a curvature parameter R_f or S_c and a non-equilibrium value of p/ε. The data reveal a significant dependence of the eddy viscosity on the curvature and strain history for a fluid in a stabilizing curvature field, S_c>1.0. Especially, experimental result shows that the eddy viscosity coefficient ratio at S_c=3 changes from 10 to -10 although shear rate preserved constant. It is therefore suggested that proper curvature modifications, particularly the strain history effect, must be introduced into current eddy viscosity models for their application to turbulent flows subjected to curvature straining field for a non-negligible period of time.

1. Introduction

Turbulent flows having streamline curvature is of importance in many power plant and energy system applications like cooling and heating coils of heat exchanger, bends in air or fuel supply piping system, flow over compressor and turbine blades. The complete understanding about the role of streamline curvature particularly in heat exchangers is necessary because the efficiency depends on largely the separation and reattachment zone where heat transfer decreases seriously.

The effect of streamline curvature on turbulence has attracted continuing interests of turbulence researchers since the advent of early mixing length model by Prandtl in 1930. Experimentally it has been widely known that the streamline curvature affects strongly the length and velocity scales of turbulence [Ramaprian and Shivaprasad[1]], and consequently, the eddy viscosity changes appreciably depending on the curvature: When the streamline curvature is such that the tangential momentum decreases away from the center of curvature, the curvature has a destabilizing effect on the fluid motion and thus the transport of momentum is enhanced (or the eddy viscosity is increased). However, when the tangential momentum decreases toward the center of curvature, it has a stabilizing effect on the fluid motion and the eddy viscosity is decreased.

In a turbulent flow field with curved streamlines, a number of theories [So[2]; Leschziner and Rodi[3]; Pourahmadi and Humphrey[4]; Cheng and Farokhi[5]] reveal that the eddy viscosity depends on the ratio of production to dissipation (p/ε), and a certain type of curvature parameters. However, a recent experiment [Holloway and Tavoularis[6]] shows clearly that the history of curvature strain to which the fluid particles are exposed also plays a significant role in determining the eddy viscosity. The present experimental study is carried out to further explore the curvature strain history effect by analyzing the present data as well as those obtained by Holloway and Tavoularis[6].

2. Curvature Parameters for the Eddy Viscosity

CFD (Computational Fluid Dynamics) method has been used in the analysis of flow field encountered in many engineering design. Because most of flows are turbulent, the turbulence effect would be considered necessarily in calculation. Generally, a model has good accuracy and high convergence is the standard k-ε model. In this model, the Reynolds shear stress expressed by $-\overline{uv}=\nu_t\ (\partial U/\partial y)$, where $\nu_t=C_\mu\ (k^2/\varepsilon)$. Here constant C_μ is eddy viscosity coefficient and has value of 0.09 in plane flows. However, many previous research works show that C_μ must be treated as a variable to get a better result in the flow having streamline curvature.

A theoretical curvature modification of the eddy viscosity $\nu_t=-\overline{uv}/(\partial U_s/\partial r-U_s/R_c)$ can be derived from the algebraic stress model [Gibson[7]] that consists of a set of algebraic equations which relate the Reynolds stresses algebraically with mean velocity gradients and other turbulent quantities. The curvature-dependent eddy viscosity model obtained in this way differs in the governing parameters that depend on the simplifying assumptions in the derivation procedure. Introduction of local equilibrium assumption of turbulent kinetic energy expressed by $p=\varepsilon$ leads $S_c=(U_s/R_c)/(\partial U_s/\partial r-U_s/R_c)$ [So[2]], or $\Phi=(k/\varepsilon)^2(\partial U_s/\partial r+U_s/R_c)U_s/R_c$ [Leschziner and Rodi[3]], where R_c is the local radius of streamline curvature, U_s is the local velocity magnitude taking the same sign as R_c and r is in the direction parallel to R_c (where $R_c>0$, r points radially outward). In contrast, retaining the non-equilibrium value of $p/\varepsilon\neq 1$ in the algebraic relation between the production tensor P_{ij} and the Reynolds stresses $\overline{u_iu_j}$, Cheng and Farokhi[5] derived a model for C_μ in terms of p/ε and the flux Richardson number R_f as

$$C_\mu=\frac{2}{3}\Phi\left[1-R_f-\Phi\frac{p}{\varepsilon}\frac{R_f^2+4R_f+1}{1-R_f}\right] \quad (1)$$

where $\Phi=0.24/(0.5+p/\varepsilon)$ and $R_f=2(U_s/R_c)/(\partial U_s/\partial r+U_s/R_c)$. Taking account of the effects of streamline curvature and the wall-damping simultaneously, Pourahmadi and Humphrey[4] obtained C_μ in terms of a curvature parameter $(U_s/R_c)/(\partial U_s/\partial r)$, a time scale ratio $(k/\varepsilon)/(\partial U_s/\partial r)$, wall function f, the ratio p/ε and three other ratios of velocity gradients.

An experiment of Holloway and Tavoularis[6] about the effect of interaction between streamline curvature and shear on uniformly sheared turbulent flows in a curved duct revealed that the evolutions of turbulence quantities such as the integral length and time scales, the turbulent kinetic energy and the Reynolds shear stress can be described by two non-dimensional parameters; one is a curvature parameter $S=(U_s/R_c)/(\partial U_s/\partial r)$ and the other is the non-dimensional development time $\tau=(s/U_s)|\partial U_s/\partial r|$ where s is the distance along the streamline.

3. Experimental Apparatus and Procedure

Figure 1 shows a schematic layout of the present experimental apparatus. An adjustable shear generator was equipped at the exit of the contraction to provide a uniform shear flow at any desired shear rate. The basic design concept is similar to that of Chung and Kyung[8]. The streamwise turbulent fluctuations at x/D=8 were nearly homogeneous in the transverse direction with about 3~5% of the centerline mean velocity. Standard hot-wire technique was used with TSI-IFA100 constant temperature anemometer, TSI-1246T1.5 cross-wire probe for u and v components. The sensing elements of the hot wires were made of tungsten wire of diameter 5 μm and their length 1.2 mm. The wire overheat ratio was adjusted to 1.7 and the frequency response was set at 40 kHz. The analog bridge voltage was amplified with gain 2, low-pass filtered with the cut-off frequency of 5 kHz and subsequently digitized by a 12-bit analog to digital converter (TSI-IFA200). The sampling rate was 10 kHz and the digitized data were stored in a computer. In the curved duct flow, the probe alignment with the stream direction is one of the difficult problems. In the present study, the maximum error of yaw angle was estimated to be about 1°. The relative uncertainties of the present data were estimated by the standard method [Yavuzkurt[9]; Moffat[10]]. The results are ±1.5%, ±5%, ±12% and ±12% for measurements of U, $\overline{u^2}$, $\overline{v^2}$ and $\overline{uv}$, respectively.

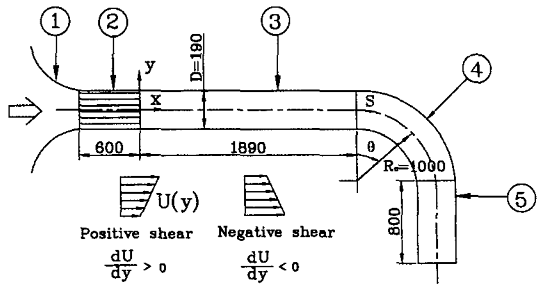

Fig. 1. Schematic diagram of wind tunnel. (1) contraction nozzle, (2) shear generator, (3) developing straight duct, (4) curved duct, (5) redeveloping straight duct. Duct height=600 mm. All dimensions are in mm.

Five sets of initial shear conditions at the entrance to the curved duct were selected for the present experiment. They are: dU/dy=28.5, 17.8, 0.0, -15.3 and -25.3 sec^{-1}. Mean velocity, turbulent kinetic energy and Reynolds stresses were measured in the whole flow field. The dissipation rate ε was estimated by calculating the Taylor microscales which can be obtained from velocity fluctuation data. More detail and useful data may be available in Lim *et al.*[11].

4. Analysis of the Eddy Viscosity Data

Figure 2 displays the measured values of $C_\mu/C_{\mu o}$ plotted against the flux Richardson number R_f. Here C_μ is defined by the relation $-\overline{uv} \equiv C_\mu (k^2/\varepsilon)(\partial U_s/\partial r - U_s/R_c)$ and $C_{\mu o}=0.09$ for plane flows (or $R_c=\infty$). The solid and dotted lines represent the model predictions by Eq. (1) for $p/\varepsilon=0.5$ and 1.5, respectively. For $R_f<0$, the model equation predicts qualitatively correct dependency of C_μ on R_f. Note that most data have the ratio p/ε in a range $0.2<p/\varepsilon<2.0$. Closer examination reveals that the role of p/ε implied by the model is also in agreement with the experimental observation (see Fig. 4). But in the range of $R_f>1.0$, the model dictates decreasing values of C_μ, eventually becoming negative with increasing R_f whereas most of the data beyond $R_f=1$ are positive. For R_f close to but smaller than unity, $C_\mu/C_{\mu o}$ varies very widely from -10 to 15 which can not be explained by the model (1). When $R_f=1$, the mean velocity field is that of the solid body rotation, and the mean shear rate $(\partial U_s/\partial r - U_s/R_c)$ is zero. Consequently, there is no production of turbulent kinetic energy, and only significant mechanism of turbulence is the spatial transport of turbulent kinetic energy in the plane of streamline curvature at a rate

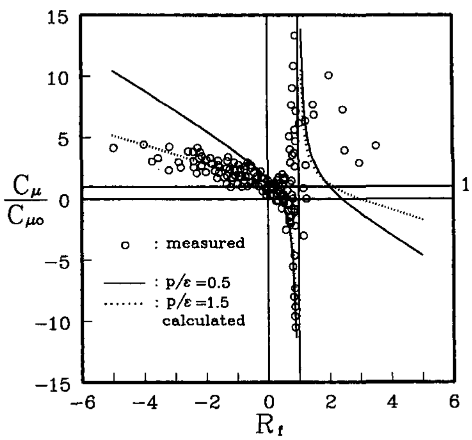

Fig. 2. Comparison between predicted and measured eddy viscosity coefficient.

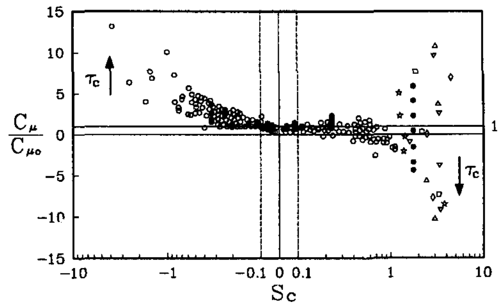

Fig. 3. Variation of eddy viscosity coefficient with curvature parameter S_c. open symbols: present data, closed symbols: data of Holloway and Tavoularis[6].

$2\overline{uv}U_s/R_c$ per unit fluid mass and time. Such a flow situation may be termed as 'overwhelmingly curvature dominated flow'.

In order to examine the scattering of data in the range $0<R_f<1$, all data points in Fig. 2 are re-plotted in Fig. 3 as a function of the curvature parameter S_c proposed by So[2]. Note that since $S_c=0.5R_f/(1-R_f)$, all data in the range $0<R_f<1$ appear in the half plane $S_c>0$. The data from Holloway and Tavoularis[6] are also represented in Fig. 3 by closed symbols. For convenience, S_c in the range $-0.1<S_c<0.1$ is represented by a linear scale. For $S_c<0$, there is a clear tendency of increasing C_μ which means the augmentation of momentum transfer with increasingly negative S_c. At $S_c=0$, C_μ takes the plane flow value of 0.09. For positive S_c up to about unity, C_μ decreases with increasing S_c. This implies that the momentum transfer is suppressed by the stabilizing positive curvature. For larger positive value of S_c, the data seem to scatter very widely. But close examination of the scattered data reveals a significant behavior of C_μ. For the region of $S_c>1$, the variation of C_μ of the fluid particles following the same streamline, i.e. shear rate keeps up constant, is depicted by the same symbols. The closed symbols forming each vertical column are those obtained along the wind tunnel centerline under each respective shear condition in the experiment of Holloway and Tavoularis[6]. During the initial stage of the curvature strain, the Reynolds shear stress $-\overline{uv}$ has a positive value. By the definition of C_μ, a positive S_c (i. e., positive strain, $\partial U_s/\partial r - U_s/R_c>0$) produces a positive C_μ. Having been exposed to the curvature strain field for a sufficiently long time, the experimental

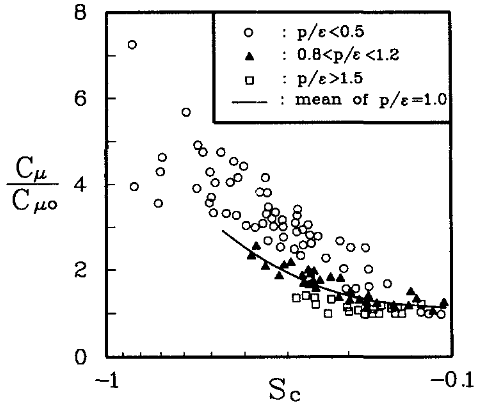

Fig. 4. The dependency of measured eddy viscosity coefficient on p/ε.

data show that the Reynolds shear stress $-\overline{uv}$ becomes negative with the downstream distance, and, consequently, positive S_c results in a negative C_μ. Therefore, C_μ crosses the line C_μ=0 to become negative with the increasing non-dimensional development time τ_c defined by $\tau_c \equiv (s/U_s) \mid \partial U_s/\partial r - U_s/R_c \mid$. From the above observation, it is certain that the curvature and strain history affect the eddy viscosity very significantly for S_c>1. Even for the case S_c<-0.1, such history (or memory) effect is still noticeable.

Finally, the effect of p/ε on C_μ in the range S_c<-0.1 can be inferred from Fig. 4 where all measured data are grouped into three ranges; namely p/ε<0.5, 0.8<p/ε<1.2 and p/ε>1.5. The solid line represents the mean of the data with p/ε of near unity. Evidently, smaller p/ε produces larger C_μ and the curvature dependency of C_μ is stronger for p/ε<1 than p/ε>1.

5. Conclusions

The curvature effect on the eddy viscosity ν_t has been experimentally investigated. The present data and those of Holloway and Tavoularis[6] revealed that such effect can be quantified through a curvature parameter R_f or S_c and a non-equilibrium value of p/ε. When the streamline curvature persists for a sufficiently long period, the eddy viscosity is significantly affected by the curvature strain history τ_c. Experimental result shows that the eddy viscosity coefficient ratio at S_c=3 changes from 10 to -10 although shear rate preserved constant. It is therefore suggested that proper curvature modifications, particularly the strain history effect, must be introduced into current eddy viscosity models for their application to turbulent flows subjected to curvature straining field for a non-negligible period of time.

Nomenclature

C_μ	: eddy viscosity coefficient in curved flow
$C_{\mu o}$	: eddy viscosity coefficient in plane flow
p	: turbulent energy production rate (m^2/s^3)
R_c	: radius of curvature (m)
R_f	: flux Richardson number
r	: coordinate of radial direction (m)
S_c	: curvature parameter
s	: coordinate of streamwise direction (m)
U_s	: streamwise mean velocity (m/s)
P_{ij}	: production tensor
$\overline{u_i u_j}$	: Reynolds normal and shear stresses (m^2/s^2)
ε	: turbulent energy dissipation rate (m^2/s^3)
ν_t	: eddy viscosity (m^2/s)
τ, τ_c	: non-dimensional development time

Acknowledgments

This work was supported by Korea Research Foundation, contract no. 1998-003-E00023

References

1. Ramaprian, B.R. and Shivaprasad, B.G.: "The structure of turbulent boundary layers along mildly curved surfaces", J. Fluid Mechanics, **85**, 273-303 (1978).
2. So, R.M.C.: "A turbulence velocity scale for curved shear flows", J. Fluid Mechanics, **70**, 37-57 (1975).
3. Leschziner, M.A. and Rodi, W.: "Calculation of annular and twin parallel jets using various discretization schemes and turbulence model variations", ASME J. Fluids Eng., **103**, 352-360 (1981).
4. Pourahmadi, F. and Humphrey, J.A.C.: "Prediction of curved channel flow with an extended k-ε model of turbulence", AIAA, **21**, 1365-1373 (1983).
5. Cheng, G.C. and Farokhi, S.: "On turbulent flows dominated by curvature effects", ASME J. Fluids Eng., **114**, 52-57 (1992).
6. Holloway, A.G.L. and Tavoularis, S.: "The effects of curvature on sheared turbulence", J. Fluid Mechan-

ics, **237**, 569-603 (1992).
7. Gibson, M.M.: "An algebraic stress and heat-flux model for turbulent shear flow with streamline curvature", Int. J. Heat Mass Transfer, **21**, 1609-1617 (1978).
8. Chung, M.K. and Kyong, N.H.: "Measurement of turbulent dispersion behind a fine cylindrical heat source in a weakly sheared flow", J. Fluid Mechanics, **205**, 171-193 (1989).
9. Yavuzkurt, S.: "A guide to uncertainty analysis of hot-wire data", ASME J. Fluids Eng., **106**, 181-186 (1984).
10. Moffat, R.J.: "Describing the uncertainties in experimental results", Experimental Thermal and Fluid Science, **1**, 3-17 (1988).
11. Lim, H.J., Chung, M.K. and Sung, H.J.: "The evolution of initially uniform shear flow through a nearly two-dimensional 90° curved duct", Experiments in Fluid, **19**, 173-187 (1995).