# 선로저항을 반영하는 에너지함수 유도를 위한 등가시스템 기법의 개발

論 文 48A - 10 - 1

## The Development of Equivalent System Technique for Deriving an Energy Function Reflecting Transfer Conductances

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Abstract - This paper shows that a well-defined energy function can be developed to reflect the transfer conductances for multi-machine power systems under an assumption that all transmission lines have uniform R/X ratios. The energy function is derived by introducing a pure reactive equivalent system for the given system. In this study, a static energy function reflecting transfer conductances is also derived as well as the transient energy function. The proposed static energy function is applied to voltage stability analysis and tested for various sample systems. The test results show that the accuracy of voltage stability analysis can be considerably improved by reflecting transfer conductances into the energy function.

Key Words: Equivalent system technique, Energy function, Transfer conductances, Voltage stability

#### 1. Introduction

The direct energy method has been developed for transient stability analysis of power systems to avoids the conventional time simulation which requires time-consuming numerical integration. Although considerable progresses have been recently reported, it still involves formidable problems to be solved for field applications such as reflecting the effects of transfer conductances and the control effects of exciters and speed governors[2-11]. Recently it has been attempted to apply direct energy method to voltage analysis[12-13], which is another encouraging event in the research area of the direct energy method. In the application to voltage stability analysis, it has become more important to reflect the effects of transfer conductances since the voltage instability is rather a local problem sensitively influenced by the transmission resistances. Actually, local transmission lines of low voltage level have relatively high R/X ratios. In this respect, this study aims to develop a new energy function to reflect the effects of transmission conductances for multi-machine systems, which will be applied to voltage

stability analysis.

Many authors have been contributed development of an energy function to reflect the effects of transfer conductances[4,5,8,10,11]. However, most of them could succeed in dealing with two-generator systems only. Gudaru[5] and Uemura[3] challenged the task with multi-machine systems. They made some progress with partial success but failed to provide a well-defined energy function for general use. Later, Chiang[6] discussed the non-existence of general energy function for any lossy system with more than 3 machines. However, Moon et al[14] proved that a well-defined energy function can be developed to reflect transfer conductances for multi-machine systems under the assumption of uniform R/X ratios for all transmission lines by introducing several theorems regarding the energy integral, in which complicate mathematics is included.

This study is the extension of the previous research[14], and provides a simpler approach to derive an energy function reflecting transfer conductances for multi-machine systems by introducing an equivalent pure-reactance system for the lossy system with uniform R/X ratios. The equivalent system can be easily obtained, from which a well-defined transient energy function can be directly set up. In this study, a static energy function reflecting transfer conductances is also derived as well as the transient energy function. The proposed static energy function is applied to voltage stability analysis and tested for various sample systems. The test results show that

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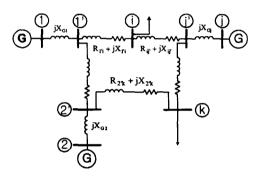
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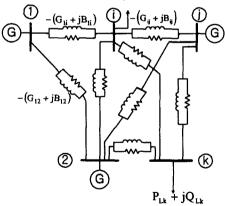
the accuracy of voltage stability analysis can be considerably improved by reflecting transfer conductances into the energy function.

## II. Derivation of An Energy Function for Multi -Machine Power Systems with Uniform R/X Ratios

In order to derive a well-defined energy function for the multimachine system, we have adopted a classical model for the generators and an assumption that the system has uniform R/X ratios after removing all the generator terminal nodes. Fig.1 (a) shows the original system. Fig.1 (b) shows the reduced system, where it is noted that a generator can be denoted as an ideal generator with zero internal impedances. We will consider the reduced system Fig.1 (b) rather than the original Fig1. (a) to derive an energy function.



(a) Original System



(b) Reduced System

Fig. 1 System representation

In Fig 1.(b), all generators can be considered as ideal generators with zero-internal impedances. Actually, we will assume that all transmission lines in Fig 1.(b) have uniform R/X ratios. That is

$$\frac{\mathbf{r}_{ij}}{\mathbf{x}_{ij}} = \mathbf{K} \tag{1}$$

where  $r_{ij}$ ,  $x_{ij}$  being resistance and reactance of a line

connecting bus i and j

The system admittance matrix  $Y_{bus}$  can be expressed as follows:

$$\mathbf{Y}_{\mathbf{bus}} = [\mathbf{G}_{ij} + \mathbf{j}\mathbf{B}_{ij}] \tag{2}$$

Then, We can easily find the following relationships between line impedances and elements of  $Y_{\text{bus}}$ .

$$\mathbf{g}_{ij} + j\mathbf{b}_{ij} = \frac{1}{\mathbf{r}_{ij} + j\mathbf{x}_{ij}} = \frac{\mathbf{r}_{ij} - j\mathbf{x}_{ij}}{\mathbf{r}_{ij}^2 + j\mathbf{x}_{ij}^2}$$
(3.a)

$$G_{ij} = -g_{ij}$$
,  $B_{ij} = -b_{ij}$ ,  $i \neq j$  (3.b)

$$G_{ii} = -\sum G_{ij}$$
,  $B_{ii} = -\sum B_{ij}$  (3.c)

From the above relationships, the elements of  $Y_{bus}$  also keep the following uniform ratios:

$$\frac{G_{ij}}{B_{ii}} = -\frac{r_{ij}}{x_{ii}} = -K \tag{4.a}$$

$$\frac{G_{ij}}{B_{ij}} = \frac{-\sum_{i \neq i}^{N} G_{ij}}{-\sum_{i \neq i}^{N} B_{ij}} = -K$$
 (4.b)

The swing equations for the system in Fig 1.(b) can be given by

$$(P_{mi} - D_i \omega_i - M_i \dot{\omega}_i) - P_{Li}^{sp} = \sum_{j=1}^{N} V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij})$$

$$(5.a)$$

$$Q_{Gi}^{sp} - Q_{Li}^{sp} = \sum_{i=1}^{N} V_i V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij})$$
 (5.b)

With the use of (4.a) and (4.b), the above equations can be rewritten as

$$(P_{mi} - D_i \omega_i - M_i \dot{\omega}_i) - P_{Li}^{sp}$$

$$= \sum_{j=1}^{N} V_i V_j (-KB_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \qquad (6.a)$$

$$Q_{Gi}^{sp} - Q_{Li}^{sp} = \sum_{i=1}^{N} V_i V_j (-KB_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij})$$
 (6.b)

By manipulating the above equations, we can derive the following equations, which are just the same form of the swing equation as the pure reactance system yields.

● (6.a) - K\*(6.b) yields:

$$P_{mi}' - D_{i}\omega_{i} - M_{i}\dot{\omega}_{i} = \sum_{\substack{j=1\\j\neq i}}^{N} V_{i}V_{j}((1+K^{2})B_{ij}\sin\theta_{ij})$$
 (7)

where 
$$P_{mi} = P_{mi} - KQ_{Gi}$$
 (8.a)

$$P_{Li} = P_{Li}^{sp} - KQ_{Li}^{sp}$$
 (i = 1,2,,N) (8.b)

$$Q_{Gi} - Q_{Li} = -(1 + K^2)B_{ii}V_i^2 - \sum_{\substack{j=1\\j \neq i}}^{N} V_iV_j(1 + K^2)B_{ij}\cos\theta_{ij}(9)$$

where 
$$Q_{Gi} = Q_{Gi} + K(P_{mi} - D_i \omega_i - M_i \omega_i)$$
 (10.a)

$$Q_{Li} = Q_{Li}^{sp} + KP_{Li}^{sp}$$
 (i = 1,2,,N) (10.b)

By using the above result, we can establish an equivalent pure-reactive network for the system in Fig.1 (b) with uniform R/X ratios. Fig.2 shows the equivalent network.

Now, one can easily check that the swing equation for the above equivalent system are just the same as (7) and (9), which reconfirms the equivalence of the two systems.

For convenience, say that generator buses are numbered first up to m and load buses next. Since the equivalent system is a pure reactance network, the energy function can directly be set up as follows:

$$\begin{split} E &= \frac{1}{2} \sum_{i=1}^{m} M_{i} \omega_{i}^{2} \\ &- \frac{1}{2} \sum_{i=1}^{N} \left[ (1+K^{2}) B_{ii} V_{i}^{2} + \sum_{\substack{j=1 \\ j \neq i}}^{N} (1+K^{2}) B_{ij} V_{i} V_{j} \cos \theta_{ij} \right]_{\{V_{0}, \theta_{0}\}}^{\{V, \theta\}} \\ &- \sum_{j=1}^{m} c \int_{\theta_{0}}^{\theta_{i}} P_{mi}^{'} d\theta_{i} + \sum_{j=1}^{N} c \int_{\theta_{0}}^{\theta_{i}} P_{Li}^{'} d\theta_{i} \\ &- \sum_{i=1}^{N} c \int_{V_{0}}^{V_{i}} \underbrace{Q_{Gi}^{'} - Q_{Li}^{'}}_{V_{i}} dV_{i} \end{split} \tag{11}$$

In the energy function above,  $V_i$  (i=1,2,...,m) denotes the generator internal voltage which is kept constant. Consequently, the last term vanishes for generator buses.

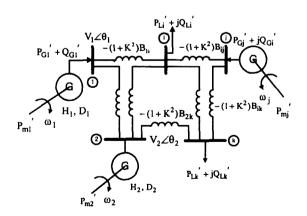


Fig. 2 Equivalent pure reactance network

That is

$$\sum_{i=1}^{N} c \int_{V_{o}}^{V_{i}} \frac{Q'_{Gi} - Q'_{Li}}{V_{i}} dV_{i} = -\sum_{i=m+1}^{N} c \int_{V_{o}}^{V_{i}} \frac{Q'_{Li}}{V_{i}} dV_{i}$$
 (12)

By substituting (8.a), (8.b), (10.a), (10.b) and (12) into (11), we can obtain the following energy function:

$$\begin{split} E &= \frac{1}{2} \sum_{i=1}^{m} M_{i} \omega_{i}^{2} \\ &- \frac{1}{2} \sum_{i=1}^{N} \left[ (1 + K^{2}) B_{ii} V_{i}^{2} + \sum_{\substack{i=1 \ j \neq i}}^{N} (1 + K^{2}) B_{ij} V_{i} V_{j} \cos \theta_{ij} \right]_{\{V_{0}, \theta_{0}\}}^{\{V_{0}, \theta_{0}\}} \\ &- \sum_{i=1}^{m} {}_{c} \int_{\theta_{0}}^{\theta_{i}} (P_{mi} - KQ_{Gi}) d\theta_{i} + \sum_{i=1}^{N} {}_{c} \int_{\theta_{0}}^{\theta_{i}} (P_{Li}^{sp} - KQ_{Li}^{sp}) d\theta_{i} \\ &+ \sum_{i=1}^{N} {}_{c} \int_{V_{0}}^{V_{i}} \frac{Q_{Li}^{sp} + KP_{Li}^{sp}}{V_{i}} dV_{i} \end{split}$$
(13)

Since it is derived for the pure-reactance system in Fig.2, the above energy function should satisfy the semi-negativeness of its time derivative. However, we can directly check the semi-negativeness of the the energy function (13) by using the partial derivative chain rule.

$$\frac{d\mathbf{E}}{dt} = \sum_{i=1}^{N} \left( \frac{\partial \mathbf{E}}{\partial \omega_{i}} \frac{d\omega_{i}}{dt} + \frac{\partial \mathbf{E}}{\partial V_{i}} \frac{dV_{i}}{dt} + \frac{\partial \mathbf{E}}{\partial \theta_{i}} \frac{d\theta_{i}}{dt} \right)$$

$$= -\sum_{i=1}^{m} \mathbf{D}_{i} \omega_{i}^{2} \leq 0 \tag{14}$$

The proof is given in Appendix. Here, it is noted that the proof in the appendix shows that the original swing epuations (5.a) and (5.b) govern the system so that the energy function in (13) exactly satisfy the semi-negativeness in (14). This means that the the function given by (13) is suitable for the energy function for the original system described by (5.a) and (5.b), although it is an energy function derived for the equivalent system in Fig.2, which verifies the validity of the equivalent system.

#### III. Application to Voltage Stability Analysis

Voltage stability analysis concerns only the static stability due to parameter changes, which makes the proposed energy function well applied. For voltage stability analysis, a new static energy function is derived to reflect the transfer conductances, of which the effects are not negligible in the local voltage stability problems.

In order to set up a static energy function, we first remove the dynamic part from the energy function (13) by letting

$$\omega_{i} = 0 \tag{15.a}$$

$$P_{mi} = P_{Gi}^{sp} \tag{15.b}$$

This is the first study to show the applicability of the proposed energy function, and thus we adopt the assumption that all loads be constant power loads so as to make the problem simplest. That is

$$P_{Li} = P_{Li}^{sp} \tag{16.a}$$

$$Q_{Ti} = Q_{Ti}^{sp} \tag{16.b}$$

Since  $Q_{Gi}$  can be determined by (6), the energy function (13) can be simplified as

$$\begin{split} \mathbf{E} = & -\frac{1}{2} \sum_{i=1}^{N} \left[ (1 + \mathbf{K}^2) \mathbf{B}_{ii} \mathbf{V}_i^2 + \sum_{\substack{j=1 \ j \neq i}}^{N} (1 + \mathbf{K}^2) \mathbf{B}_{ij} \mathbf{V}_i \mathbf{V}_j \cos \theta_{ij} \right]_{\{\mathbf{V}_0, \, \boldsymbol{\theta}_0\}}^{[\mathbf{V}, \, \boldsymbol{\theta}]} \\ & - \sum_{i=1}^{m} \mathbf{P}_{Gi}^{\mathsf{sp}} (\theta_i - \theta_{i0}) + \sum_{i=1}^{m} {}_{c} \int_{\theta_0}^{\theta_i} \mathbf{K} \mathbf{Q}_{Gi} (\mathbf{V}, \, \boldsymbol{\theta}) \mathrm{d} \theta_i \\ & + \sum_{i=1}^{N} (\mathbf{P}_{Li}^{\mathsf{sp}} - \mathbf{K} \mathbf{Q}_{Li}^{\mathsf{sp}}) (\theta_i - \theta_{i0}) \\ & + \sum_{i=1}^{N} (\mathbf{Q}_{Li}^{\mathsf{sp}} + \mathbf{K} \mathbf{P}_{Li}^{\mathsf{sp}}) \log(\mathbf{V}_i / \mathbf{V}_{i0}) \end{split} \tag{17}$$

Here, a new energy function is proposed for the multi-machine system with uniform R/X ratios for all transmission lines. However, the evaluation of the integral term in (17) has some problem for general applications since  $Q_{Gi}$  is a function of state variables ( $\theta$ , V). The integration of the integral term should require some approximation to avoid the problem of path-dependency. One of the approximation methods is to take the integral along a straight line which connects the starting point and the end point of the integral]. The approximation procedures can be found in Reference[14]

#### Check of the Validity of the Static Energy Function

In order to show the validity of static energy function (17), it must be proven that its local extremum or saddle points satisfy the load flow equations. With the careful treatment of the double summation, the partial derivatives of the energy function (17) can be easily calculated as follows:

$$\begin{split} &\frac{\partial \mathbf{E}}{\partial \theta_{i}} \\ &= \sum_{j \neq i}^{N} \left[ (1 + \mathbf{K}^{2}) \mathbf{B}_{ij} \mathbf{V}_{i} \mathbf{V}_{j} \sin \theta_{ij} \right] - \left( \mathbf{P}_{Gi}^{sp} - \mathbf{K} \mathbf{Q}_{Gi} \right) + \left( \mathbf{P}_{Li}^{sp} - \mathbf{K} \mathbf{Q}_{Li}^{sp} \right) \\ &= 0 \quad (i, j = 1, 2, 3, \dots, N \text{ and } i \neq \text{slack}) \quad (18.a) \\ &\frac{\partial \mathbf{E}}{\partial \mathbf{V}_{i}} \end{split}$$

$$= -(1 + K^{2})B_{ii}V_{i} + \sum_{j \neq i}^{N} (1 + K^{2})B_{ij}V_{j}\cos\theta_{ij} + \frac{Q_{Li}^{sp} + KP_{Li}^{sp}}{V_{i}} = 0$$
(i = m+1, m+2, ..., N and j = 1, 2, ..., N) (18.b)

Here, one can easily check that [(18.a) + K\*(6)] and [(18.6) - K\*(18.a)] yield the following load flow equations respectively.

$$G_{ii}V_{i}^{2} + \sum_{j\neq i}^{N} (G_{ij}V_{i}V_{j}\cos\theta_{ij} + B_{ij}V_{i}V_{j}\sin\theta_{ij}) - P_{Gi}^{sp} + P_{Li}^{sp} = 0$$
(i, j = 1, 2, 3, ..., N and i \(\pm \) slack) (19.a)

$$-B_{ii}V_{i}^{2} + \sum_{j\neq i}^{N} V_{i}V_{j}(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) + Q_{Li}^{sp} = 0$$

$$(i = m+1, m+2, \dots, N \text{ and } j = 1, 2, \dots, N)$$
 (19.b)

Therefore, the proposed energy function can be well applied to the direct energy method for voltage stability analysis which concerns only the static stability due to parametric changes.

In this study, the proposed static energy function (17) is examined to determine the maximum loadability and to provide an appropriate voltage stability index. It is well-known that the energy function provides the most reliable voltage stability index for the system with pure reactive transmission lines. However, its application has been limited since the conventional energy function can not reflect the effects of transfer conductances, which may crucially affect the voltage stability for the heavily loaded local sub-systems. The proposed energy function alleviates these disadvantages of the energy function direct method by taking into consideration partially the effects of transfer conductances.

In the practical point of view, the proposed energy function can be well applied to local voltage stability analysis with the assumption of uniform R/X ratios. Numerical results in the next section show that the maximum loadability can be corrected to a considerable extent by reflecting the transfer conductances with uniform R/X ratio.

#### IV. Numerical Result

The proposed static energy function has been applied to voltage stability analysis and tested for the 6-bus, 9-bus sample systems and New England 39-bus system with the use of various R/X ratios.

The purpose of this study is to show that the proposed energy function improves the reliability of the proximity index by reflecting the transfer conductances when the energy margin is adopted as the proximity index to indicate the proximity to the collapse point. The energy margin is defined to be

$$E_{\text{margin}} = E(\mathbf{x}_{\text{sen}}) - E(\mathbf{x}_{\text{nen}})$$
 (20)

where x<sub>sep</sub>: Stable Equilibrium Point

xuep: Unstable Equilibrium Point

The load level index  $\lambda$  is defined to indicate the ratio of the present load to the base-case load, so that the present loads and present generations can be denoted by

$$P_{Li} = \lambda P_{Li0}$$

$$Q_{Li} = \lambda Q_{Li0} \qquad (i = 1, 2, \dots, N)$$

$$P_{Gi} = \lambda P_{Gi0}$$
(21)

Voltage stability analysis has been performed to determine the proximity to the collapse point by using the time-consuming CPflow (Continuation Power flow) technique. With the continuous increase of the load level  $\lambda$ , the CP flow algorithm searches a stable solution and unstable solutions. With the use of the CP flow results, we examine the nose curves at weak points and the energy margin by the proposed energy function. With the use of R/X ratios, the effects of transfer conductances are examined with the discussion of the application of the energy margin to practical system operation.

#### El-Abiad 4-Gen 6-Bus System [7]

First, the P-V nose curves are examined for load buses in case of K=0, i.e. pure resistive case. The results are shown in Fig. 3, from which we can find the maximum load level and the weakest bus.

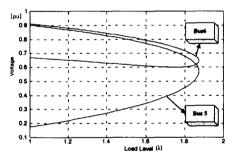


Fig. 3 P-V nose curves at load buses with K=0.0

Since Bus 5 is the week bus, we intensively investigate the P-V nose curves at Bus 5 with various R/X ratios(K=0.0, 0.05, 0.15, 0.3) as given in Fig. 4.

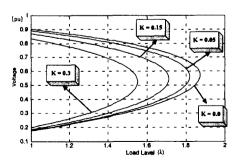


Fig. 4 P-V nose curves at Bus 5

With the increase of load level, the energy margin is plotted in Fig. 5, where the R/X ratio is also changed from 0.0 to 0.3.

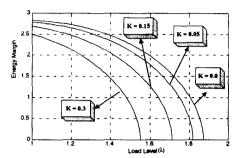


Fig. 5 Energy margin curves

#### New England 10-Gen 39-Bus System

By using similar procedures as given above, the following results are obtained for the New England 39-Bus system. With the comparison of Fig. 6 to Fig. 9, one can easily find that the proposed energy function can well reflect the effects of line resistances to correct the load margin to a considerable extent.

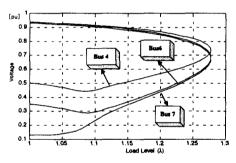


Fig. 6 P-V nose curves at load buses with K=0.0

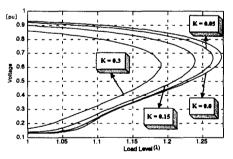


Fig. 7 P-V nose curves at Bus 7

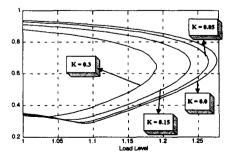


Fig. 8 P-V nose curves at Bus 6

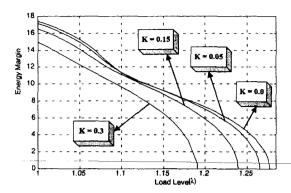


Fig. 9 Energy margin curves

In the above results, it should be noted that the P-V nose curves have strange shapes at low load level with relatively small  $\lambda$ . This is because there exist multi unstable solutions for the highly loaded system and each of them may provide a different P-V curve for the range of  $\lambda < 1.09$ . However, the lower parts of P-V curves for the range of relatively small  $\lambda$  have little meaning to determine the proximity to the collapse point. Fig. 9 also shows unusual trend in the energy margin for the range of  $\lambda < 1.09$ , which can be interpreted in the same way.

Comparisons of the P-V Nose Curve and Energy Margin Methods

- Both of the P-V nose curve and the energy margin have irregular behaviors for the lightly loaded range(See Fig.4, Fig.6, Fig.7, Fig. 8 and Fig.9). Consequently, both curves are available for the heavily loaded range to estimate the proximity to the collapse point.
- Fig. 5 and Fig. 9 show that the proposed energy function well reflects the effects of transfer conductances. Examining the curves of energy margin, one can find that the conventional energy function approach, which cannot reflect the transfer conductances, may result in considerable errors in the calculation of voltage stability margin.
- The energy margin is conventionally considered to be a most reliable index but has limited applications due to its crucial drawback of reflecting the effects of transfer conductances. However, the proposed energy function can solve the problem not completely but to the applicable level in the practical point of view.
- Energy margin provides a unified voltage stability index in contrast to the P-V curves (Each P-V curve at each bus may provide a different index).
- The curve of energy margin behaves very nice well fitted to a quadratic curve in the heavily loaded range. Therefore, it can be used to estimate the maximum loadability by using the approximated relation as follows:

$$\lambda_{\text{max}} - \lambda = \frac{1}{2} \alpha E_{\text{margin}}^2$$
 (22)

where is the quadratic constant

With the above observations, the authors would like to strongly recommend to use the energy margin as the index of the voltage stability margin.

#### VI. Conclusions

This paper has shown that a well-defined energy function can developed to reflect the transfer be conductances for multi-machine power systems under an assumption that all transmission lines have uniform R/X ratios. The energy function is derived by introducing a pure reactive equivalent system for the given system. In this study, a static energy function reflecting transfer conductances is also derived as well as the transient energy function. The proposed static energy function is applied to voltage stability analysis and tested for various sample systems. The test results show that the accuracy of voltage stability analysis can be considerably improved by reflecting transfer conductances into the energy function.

The method of energy margin provides a unified voltage stability index while the method of P-V curve always involves the problem of selecting the weakest bus. In this respect, the energy margin method is strongly recommendable as the voltage stability index.

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#### **Appendix**

## Semi-negativeness of Time-Derivative of Energy Function

The semi-negativeness of the time derivative of the proposed energy function will be shown by using the chain rule for the partial derivatives. For the proposed energy function (13), the partial derivatives in (14) can be evaluated as follows:

$$\frac{-\partial E}{\partial \theta_{i}} = \sum_{i=1}^{N} \{V_{i}V_{j}(1+K^{2})B_{ij}\sin\theta_{ij}\} - (P_{mi} - KQ_{Gi}^{sp})$$

$$\begin{split} &+\left(P_{Li}^{\text{sp}}-KQ_{Gi}^{\text{sp}}\right)=-D_{i}\omega_{i}-M_{i}\dot{\omega}_{i}\\ &\qquad \qquad (i=1,\ 2,\ \cdots,\ m) \end{split} \tag{A.1}\\ &\frac{\partial E}{\partial\theta_{i}}=\sum_{i=1}^{N}\{V_{i}V_{i}(1+K^{2})B_{ij}\sin\theta_{ij}\}-\left(P_{mi}-KQ_{Gi}^{\text{sp}}\right)\\ &+\left(P_{Li}^{\text{sp}}-KQ_{Gi}^{\text{sp}}\right)=0\ (i=m+1,\ m+2,\ \cdots,\ N) \tag{A.2}\\ &\frac{\partial E}{\partial V_{i}}=-\sum_{i=1}^{N}\{V_{i}(1+K^{2})B_{ij}\cos\theta_{ij}\}-\frac{(KP_{Gi}^{\text{sp}}+Q_{Gi}^{\text{sp}})}{V_{i}}\\ &+\frac{(KP_{Li}^{\text{sp}}+Q_{Li}^{\text{sp}})}{V_{i}}=0\ (i=m+1,\ m+2,\ \cdots,\ N) \tag{A.3} \end{split}$$

$$\frac{dV_i}{dt} = 0 \qquad (i = 1, 2, \dots, m) \qquad (A.4)$$

$$\frac{\partial \mathbf{E}}{\partial \omega_i} = \mathbf{M}_i \omega_i \qquad (i = 1, 2, \dots, m) \tag{A.5}$$

Here, it is noted that the last step in (A.1) is obtained by using the swing equation (9), and that the conditions of the real/reactive power balances at each bus make (A.2) and (A.3) zero. For the generator buses, can be replaced by since is the phase angle of the internal induced voltage for generator i, and Eq.(A.4) comes from the assumption that the generator internal voltage be constant. By substituting the (A.1) (A.5) into (14), one can easily show the following semi-negativeness of the time-derivative.

$$\frac{d\mathbf{E}}{dt} = \sum_{i=1}^{N} \left( \frac{\partial \mathbf{E}}{\partial \omega_{i}} \frac{d\omega_{i}}{dt} + \frac{\partial \mathbf{E}}{\partial \mathbf{V}_{i}} \frac{d\mathbf{V}_{i}}{dt} + \frac{\partial \mathbf{E}}{\partial \theta_{i}} \frac{d\theta_{i}}{dt} \right)$$

$$= -\sum_{i=1}^{m} \mathbf{D}_{i} \omega_{i}^{2} \le 0(14)$$

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