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◉ 論　文

A Method for Vibration and Sensitivity Analysis of Structure Systems with Non-linear Characteristics

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비선형 특성을 가진 구조시스템의 진동과 감도해석 방법

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Key Words : Dynamics of Structural System(동적구조시스템), Non-Linear Vibration (비선형진동), Substructure Synthesis Method(부분구조합성법), Perturbation Method(섭동법), Sensitivity Analysis(민감도해석)

초 록

본 논문에서는 대형구조물의 해석에 있어서 부분구조합성법과 섭동법을 이용하여 복잡한 비선형 시스템의 해석방법을 제안하였다. 해석방법은 전체시스템을 먼저 몇 개의 분계로 분할한다. 각 분계의 운동방정식에 비선형항이 존재하여도 전체시스템의 지배적 진동모드는 선형모드라는 가정하에 이 시스템의 각 분계를 모드좌표로 변환한다. 이때, 비선형항은 근사적으로 변환한다. 그리고 섭동법을 이용하여 각 분계의 모드좌표방정식은 섭동좌수별로 정식화되어 순차적으로 구해진다. 비선형의 감도는 비선형계수로 정의되고, 그에 상응하는 강성에 의해 구해진다. 제안된 해석방법으로 비선형 회전체, 비선형 베어링-페데스탈로 구성된 대형시계구조물의 진동을 해석하였다. 해석방법의 유효성을 평가하기 위해 응답의 정도와 계산소요시간을 유한요소법의 결과와 비교·분석하였다.

1. Introduction

Dynamic behavior of a structural system has been investigated extensively by linear analysis procedures. Most of the components that comprise a structural dynamic system can be accurately modeled as linear. However, the strongly non-linear components can make the system possess characteristics that are substantially different from those of a linear system. For nonlinear system, exact solutions are generally not possible and approximate solutions can only be obtained numerically. In structural

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dynamics, numerical integration of equations of motion is used frequently in order to find the steady state response of structure. This can be extremely time consuming, especially for large order system. Therefore, in many branches of structural dynamics, approximate analytical techniques are used as an alternative to numerical integration procedures for the steady state response analysis of nonlinear MDOF structures. A number of analytical methods for nonlinear system have been investigated. These include the averaging method[1], Ritz-Galerkin Method[2]. Harris and Crede[3] used perturbation method as an analytical method for nonlinear systems. In recent years, reduction of computation time with keeping accuracy for linear and nonlinear dynamic problem has become the focus of intense research efforts. The dynamic analysis of multiple structural systems can require the solution of large order sets of linearized differential equations of motion. These linear equations are frequently diagonal zed by means of an eigenvector transformation, which greatly simplifies the analysis. The sub-structure synthesis method developed herein allows for significant reduction in the size of the overall system problem which retaining the essential dynamic characteristic. The authors[4,5] proposed a new method to analyze the dynamic problem of MDOF systems with nonlinearity by using substructure synthesis method (SSM). Applying the SSM technique reduced the overall size of the problem for the nonlinear structure, and obtained the approximate solutions of nonlinear rotor system with perturbation method. However, in rotating machinery, ball bearing clearances, squeeze film damper, journal bearings, seals, frictional forces, etc. contribute to the nonlinearity. When such system is subjected to periodic external excitation, they respond with a variety of complex dynamic behaviors in certain parameter ranges. Therefore, this paper developed more expanded analytical technique of the nonlinear structure vibration including nonlinearity on assembling region by applying the SSM and perturbation theory using the modes of the vibration, which are obtained by linear part of equation of motion. This method is applied to rotor system in order to demonstrate the performance of the method in respect to accuracy and computation time. And the results are investigated for the effect of bearing and rotor nonlinearity to the overall system by observing the sensitivity of each component.

2. System Modeling

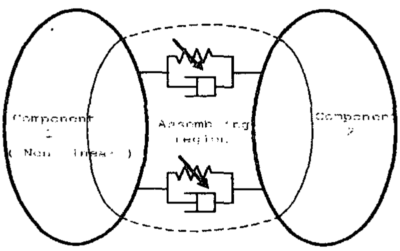

Fig. 1 Substructuring of a nonlinear system

A nonlinear structural system, which consists of several components, is considered. For the analysis, it is divided into three kinds of component according to each characteristic, for instances, linear components, nonlinear components and assembling components as shown in Fig.1. The disassembling can give two benefits; 1) the system size to be handled at one is reduced, that gives great advantages in the modeling and computation, 2) the separation of linear and nonlinear subsystem, which results in economical computation.

The vibration modes of substructures are obtained by FEM. Thus the original system is described in the modal space. The nonlinear components are formulated in modal space approximately by using constant mode shapes not changed by nonlinearity. Then the modal equations of nonlinear components are expanded by perturbation method with small parameters for nonlinearity. Now, each component should be assembled mathematically, this is made through constraint equations.

2.1 Substructuring a Rotor System

For convenience in developing this methodology, a rotor system, which has a shaft, a pedestal and bearings, is presented. It is disassembled into three kinds of components; nonlinear component (shaft), assembling components (bearing), linear component (pedestal). The bearing is nonlinear components. Here nonlinear component is assigned as component 1 and linear component is assigned as component 2.

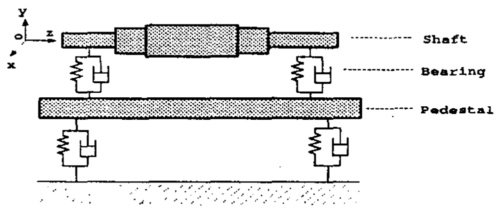

Fig. 2 Rotor-bearing-pedestal system

The o-xyz coordinate systems of rotor are shown in Fing.2. X-axis is horizontal of shaft, y-axis is vertically upwards. Eccentricities of the rotor, the acceleration of gravity, any non-uniformity in the cross-section along its length are ignored.

2.2 Component 1 – Shaft with nonlinearity

The shaft is analyzed by using finite element method. The nonlinear restoring force of shaft is described by a nonlinear relationship between stress and strain as follows;

$$\sigma = E(\widetilde{\varepsilon} + \gamma \cdot \widetilde{\varepsilon}^3) \qquad (1)$$

where $\sigma, \widetilde{\varepsilon}$ and γ denote the stress, strain and the material coefficient, respectively. To deter-mine the strain energy accompanying the deformation of elastic rotor due to bending, let us consider the strain-stress relationship of Eq.(1) disregarding axial deformation. The nonlinear stiffness term for the rotor element is determined approximately as same way as the linear stiffness matrix by considering the minimum strain energy. Here, internal force term $\{F^{in}_1\}$ which is the interaction force between components, is considered because the components 1 can be synthesized through this internal force with the other component. Adding the external force and considering the boundary conditions, the equation of motion for the rotor as component 1 can be written as follows;

$$[M_1]\{X_1\}+[C_1]\{X_1\}+[K_1]\{X_1\} + \varepsilon[K_N]\{X^3_1\}=\{F_1\}+\{F^{in}_1\} \qquad (2)$$

where $[M_1]$, $[K_1]$ and $[C_1]$ are mass matrix, stiffness matrix and proportional damping matrix. $[K_N]$ and $\{F_1\}$ are a nonlinear stiffness term and exciting force vector, respectively. ε is small parameter which is defined in terms of γ. The displacement vector $\{X_1\}=\{x_i, y_i, \theta_{xi}, \theta_{yi}\}^T$, $(i=1,2,\cdots m)$ is consist of the displacements and rotations for the x-direction and y-direction for the i-th nodal point.

m is the number of node. In order to obtain modal coordinate system, the linear homogeneous equation of Eq.(2) is analyzed where the governing vibration mode of the system is considered to be linear in spite of nonlinearity. And the damping term is neglected in this step for simplicity. By using the modal matrix $\{\Phi_1\}$, the displacement $\{X_1\}$ can be transformed into the modal coordinates $\{\xi_1\}$ as follows:

$$\{X_1\} \equiv [\Phi_1]\{\xi_1\} \qquad (3)$$

Substituting Eq.(3) into Eq.(2) and pre-multiplying by $[\Phi_1]^T$, Eq.(2) can rewritten by the modal equations with respect to the linear and nonlinear term with weak external force

$$\{\ddot{\xi}_1\}+[{}^{\backslash}\omega^2{}_{\backslash}]\{\xi_1\}+\varepsilon[\Phi_1]^T[K_N]\{X^3_1\} = \varepsilon\{f_{\xi 1}\}+\varepsilon\{f_{h1}\} \qquad (4)$$

where $\varepsilon\{f_{\xi 1}\}=[\Phi_1]^T\{F_1\}$ and $\varepsilon\{f_{h1}\}$ is internal force term in modal coordinates. Usually $[\Phi_1]^T[K_N]\{X^3_1\}$ is not diagonal term. However, the 1st mode is governing the vibration of the system comparing with other higher modes when the system is excited around the 1st natural frequency. In this study, dynamic behavior of around the primary resonance is mainly investigated considering the nonlinear characteristic and dynamic behaviors caused by the mode coupling are not treated such as the multi-mode resonance. Thus, the nonlinear term can be approximated as diagonal matrix form disregarding the non-diagonal term. Therefore, every component of Eq.(4) become diagonal matrix. Then, the damping term is regarded as proportional weak damping $\varepsilon 2\zeta_{1i}\omega_{1i}$ in modal coordinates to apply the perturbation method. i th element of Eq.(4) can be obtained as follows

$$\ddot{\xi}_{1i} + \varepsilon 2\zeta_{1i}\omega_{1i}\dot{\xi}_{1i} + \omega_{1i}^2\xi_{1i} + \varepsilon\,\omega_{n1i}^2\xi_{1i}^3 = \varepsilon f_{\xi 1i} + \varepsilon f_{h1} \qquad (i = 1,2,\cdots n) \tag{5}$$

where ω_{n1i}^2 is nonlinear stiffness term in modal coordinates.

Here, the perturbation method is introduced to solve the nonlinear equation. In Eq.(5), the small variant $\varepsilon\,\omega_{n1i}^2$ can be regarded as the perturbation parameter term, because the variant $\varepsilon\,\omega_{n1i}^2$ is relatively smaller than ω_{1i}^2. The modal responses ξ_{1i} can be expanded in term of a series of the perturbation parameter ε expressed as follows:

$$\xi_{1i} = \xi_{1i}^{(0)} + \varepsilon^2\xi_{1i}^{(2)} + \cdots \tag{6}$$

Substituting Eq.(6) into Eq.(5), perturbed equations are obtained by rearranging it in terms of ε^n. Because the perturbation parameter ε is chosen arbitrarily, the coefficients terms of each power of ε must be zero. Neglecting the terms involving ε^2 , ε^3 and ε^4 , we obtain the following equations.

$$\ddot{\xi}_{1i}^{(0)} + \omega_{1i}^2\xi_{1i}^{(0)} = f_{h1}^{(0)}$$

$$\ddot{\xi}_{1i}^{(1)} + \omega_{1i}^2\xi_{1i}^{(1)} = f_{\xi_{1i}} + f_{p1}(\xi_{1i}^{(0)}, \dot{\xi}_{1i}^{(0)}) + f_{h1}^{(1)} \tag{7}$$

The term f_{p1} is the nonlinear term, $f_{h1}^{(0)}, f_{h1}^{(1)}$ are perturbed internal force terms.

2.3 Component 2 – pedestal

The pedestal, which is the linear system, is also analyzed by using finite element method. The internal force term $\{F_2^{in}\}$ is considered because the component 2 can be synthesized through this internal force with the other component. The equation of motion can be written as

$$[M_2]\{\ddot{X}_2\} + [C_2]\{\dot{X}_2\} + [K_2]\{X_2\} = \{F_2\} + \{F_2^{in}\} \tag{8}$$

where $[M_2]$, $[K_2]$ and $[C_2]$ are the mass, stiffness matrix and proportional damping matrix, respectively. After the eigenvalue analysis, the Eq.(8) is changed into modal coordinate as same way as the component 1 under the assumption of proportional weak damping $\varepsilon 2\zeta_{2i}\omega_{2i}$ and weak external force $\varepsilon f_{\xi 2i}$ as follow,

$$\ddot{\xi}_{2i} + \varepsilon 2\zeta_{2i}\omega_{2i}\dot{\xi}_{2i} + \omega_{2i}^2\xi_{2i} = \varepsilon f_{\xi 2i} + \varepsilon f_{h2} \tag{9}$$

where εf_{h2} is internal force term in modal coordinates. The modal responses ξ_{2i} can be expanded by the perturbation parameter ε as same as component 1.

2.4 Assembling region – Bearing with nonlinearity

In rotor dynamics the nonlinear component, such as fluid-film bearings, ball bearing clearances, squeeze-film dampers, dead band supports, rubs, etc. can make the overall system posses characteristics that are substantially different from those of a linear system, such as self-excited vibrations and jump discontinuities. When vibrations of large amplitude occur, bearings cannot be modeled by linear spring and damping. More accurate approach is needed to compute the nonlinear bearing forces for multi-degree-of freedom system. Referring to Fig. 2, the displacement of the rotor due to the bearing displacements can be expressed as X_h .

$$[K_h]\{X_h\} + \delta\{N_h\} = \{F_h\} \tag{10}$$

where $[K_h]$ is the bearing stiffness matrix and $\{F_h\}$ is bearing force. $\delta\{N_h\}$ is a displacement dependent nonlinear term of bearing where δ is defined in terms of nonlinear characteristic. The nonlinear bearings are modeled as ball bearings in this case, such that they have a cubic nonlinear term, where the force and displacement expressions are given in matrix form

$$\begin{bmatrix} k_{h11} & k_{h12} \\ k_{h21} & k_{h22} \end{bmatrix}\begin{Bmatrix} x_{h1} \\ x_{h2} \end{Bmatrix} + \varepsilon\begin{bmatrix} k_{B11} & k_{B12} \\ k_{B21} & k_{B22} \end{bmatrix}\begin{Bmatrix} x_{h1}^3 \\ x_{h2}^3 \end{Bmatrix} = \begin{Bmatrix} f_{h1} \\ f_{h2} \end{Bmatrix} \quad (11)$$

where f_{h1}, f_{h2} are internal forces. k_{hjk}, k_{Bjk} and x_{hj} (j,k=1,2) are bearing coefficients and physical coordinates, respectively, Generally, there is a damping term in the bearing, but it is ignored in this study. In order to solve the overall system equation, the small nonlinear parameter is regarded as the perturbation parameter. δ is a small nonlinear parameter of bearing. Accordingly, it can be expressed as perturbation parameter ε in accordance with the component 1.

The equation of assembling region can be expressed as the linear combination of the component eigenvectors. Nonlinear Eq.(11) can be rewritten to the linearized equations by using perturbation method as follows;

$$\{x_{hj}\} = \{x_{hj}^{(0)}\} + \varepsilon\{x_{hj}^{(1)}\} \cong [\phi_{hj}]\{\{\xi_j^{(0)}\} + \varepsilon\{\xi_j^{(1)}\}\} \quad (12)$$

where $[\phi_{hj}]$, (j=1,2) are eigenvectors of assembling region. Internal force of components 1, 2 is formulated as follows;

$$\{f_{h1}\} = k_{h12}\{x_{h2}^{(0)}\} + k_{h11}\{x_{h1}^{(0)}\} + \varepsilon\{k_{B11}\{x_{h1}^{(1)}\} + k_{B12}\{x_{h2}^{(1)}\} + k_{h11}\{x_{h1}^{(0)3}\} + k_{h12}\{x_{h2}^{(0)3}\}\}, \quad (13)$$

$$\{f_{h1}\} = -\{f_{h2}\}$$

The internal force $\{f_{h2}\}$ of component 2 is expressed with the internal force $\{f_{h1}\}$ of component 1, same value with opposite direction by the relation of symmetric bearing as assembling region in SSM. By substituting the internal forces, which are eliminated by assembling into each component equations and arranging with ε, the perturbed each component equation and assembling region equations can be obtained.

$$\{f_{hj}\} = \{f_{hj}^{(0)}\} + \varepsilon\cdot\{f_{hj}^{(1)}\} \ (j = 1,2) \quad (14)$$

By substituting Eq.(14) into each component as internal force $(f_{hj}^{(0)}, f_{hj}^{(1)}, (j = 1,2))$, the variables of each component ξ_{1i}, ξ_{2i} are written to perturbation form. We obtain the sets of perturbed equations for component 1 and component 2. Therefore, the equation of assembling region is expressed with the 0th order and 1st order of perturbation equations. The characteristic of this analysis is to perturb the each nonlinear component in matrix form, performing the mode synthesis at bearing part with component 1 and component 2, which is expressed in linear equation.

3. Synthesis of Non-Linear System

This section is concerned with reducing the number of degrees of freedom for each component by modal substitution. All the components are then assembled together and the complete structure analyzed as shown in Fig.1. The equations of motion of the typical linearized nonlinear rotor system in the generalized modal coordinates are obtained by truncating the vibration modes of the shaft and pedestal.

$$\{\ddot{\xi}\} + [K]\{\xi\} = \{F(\xi^{(0)}, \xi^{(1)}, t)\} \quad (15)$$

where $[K]$ is stiffness matrix of the overall system which is composed of each component and assembling region

$$[K] = \begin{bmatrix} \omega_{1i}^2 & & & & & & & 0 \\ & \omega_{1i}^2 & & & & & & \\ & & k & 0 & -k & 0 & & \\ & & 0 & k & 0 & -k & & \\ & & -k & 0 & k & 0 & & \\ & & 0 & -k & 0 & k & & \\ & & & & & & \omega_{2i}^2 & \\ 0 & & & & & & & \omega_{2i}^2 \end{bmatrix}$$

$$\{\xi\}=\{\xi_1^{(0)},\ \xi_1^{(1)},\ x_{h1}^{(0)},\ x_{h1}^{(1)},\ x_{h2}^{(0)},\ x_{h2}^{(1)},\ \xi_2^{(0)},\ \xi_2^{(1)}\}^T,$$

$$\{F(\xi,t)\}=\begin{Bmatrix} f_{\xi_1},\ f_{p_1},\ -f_{h1}^{(0)},\ -f_{h1}^{(1)}, \\ f_{h2}^{(0)},\ f_{h2}^{(1)},\ f_{\xi 2},\ f_{p2} \end{Bmatrix}^T \quad (16)$$

In order to reduce the number of degrees of freedom, transformation matrix, which is composed with substructure's eigenvectors, is applied.

$$\begin{Bmatrix} \xi_1^{(0)} \\ \xi_1^{(1)} \\ x_{h1}^{(0)} \\ x_{h1}^{(1)} \\ x_{h2}^{(0)} \\ x_{h2}^{(1)} \\ \xi_2^{(0)} \\ \xi_2^{(1)} \end{Bmatrix} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ \phi_{h1} & 0 & 0 & 0 \\ 0 & \phi_{h1} & 0 & 0 \\ 0 & 0 & \phi_{h2} & 0 \\ 0 & 0 & 0 & \phi_{h2} \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{Bmatrix} \xi_1^{(0)} \\ \xi_1^{(1)} \\ \xi_2^{(0)} \\ \xi_2^{(1)} \end{Bmatrix} \equiv [P]\{\eta\} \quad (17)$$

$[\phi_{h1}]$, $[\phi_{h2}]$ are eigenvector matrix of assembling region, which is derived from the each component eigenvectors corresponding to bearing. By substituting Eq.(17) into Eq.(15) and pre-multiplying with the above transformation matrix $[P]^T$ yields

$$\{\ddot{\eta}\}+[K_r]\{\eta\}=\{f_r(\xi^{(0)},\xi^{(1)},t)\} \quad (18)$$

where $\{f_\eta(\xi^{(0)},\xi^{(1)},t)\}$ is the displacement dependent term and $[K_r]$ is reduced stiffness matrix.

3.1 Periodic Forced Vibration Response

The number of equation depends on the number of adopted modes in Eq.(15). After obtaining the reduced order equation, that equation can be arranged in terms of perturbation order. Accordingly, the perturbation 0^{th} order equation and the perturbation 1^{st} order equation obtained, which are to be solved sequentially;

$$\begin{Bmatrix} \ddot{\xi}_{1i}^{(l)} \\ \ddot{\xi}_{2i}^{(l)} \end{Bmatrix} + \begin{bmatrix} \omega_{1i}^2 + a_1 & a_2 \\ a_3 & \omega_{2i}^2 + a_4 \end{bmatrix} \begin{Bmatrix} \xi_{1i}^{(l)} \\ \xi_{2i}^{(l)} \end{Bmatrix} = \begin{Bmatrix} f_{\eta 1}^{(l)} \\ f_{\eta 2}^{(l)} \end{Bmatrix} \quad (19)$$

$$a_1=\phi_{h1}^T k_{h11}\phi_{h1},\quad a_2=\phi_{h1}^T k_{h12}\phi_{h2},$$
$$a_3=\phi_{h2}^T k_{h21}\phi_{h1},\quad a_4=\phi_{h2}^T k_{h22}\phi_{h2}$$

where the $f_{\eta 1}^{(l)}$, $f_{\eta 2}^{(l)}$ $(l=0,1)$ are displacement dependent external force term which are remained after eliminating the internal force. The exciting force by an unbalance of rotor is assumed as $mr\Omega^2\cos(\Omega t)$, where Ω is rotating speed(rad/s)
and mr is unbalance. In this study, we adopted the formulation that nonlinear restoring force is transformed into modal coordinate assuming that the exciting frequency Ω is around the 1^{st} natural frequency of the system. Because of it can be easily observed the nonlinear vibration characteristics around the critical speed. It is convenient to change the time scale for the simplicity of response analysis of periodic oscillation.

To this end, $\Omega t=\tau+\phi$, $\frac{d}{dt}=\Omega\frac{d}{d\tau}$, where τ is the new time variable and ϕ is a phase angle which is considered in perturbation 1^{st} order response. Perturbation 0^{th} order respónse can be obtained as follows,

$$\xi_i^{(0)}=A_i^{(0)}\cos\Omega t \quad (20)$$

where $A_i^{(0)}$ is modal amplitude vector. By introducing the perturbation 0^{th} order solution into the perturbation 1^{st} order equation, we obtain the perturbation 1^{st} order equation. Considering the periodicity condition, the solution of perturbation 1^{st} order equation become

$$\xi_i^{(1)}=A_{1i}^{(1)}\cos\Omega t+A_{2i}^{(1)}\cos 3\Omega t \quad (21)$$

where $A_{1i}^{(1)}$, $A_{1i}^{(1)}$ are modal amplitude vector. The same procedure can be used to derive the higher-order approximations, although this is seldom necessary. Introducing Eq.(20) and Eq(21) into the modal coordinates, and changing into physical coordinates, responses of the overall system can be obtained.

3.2 Frequency Domain Response

Here an analytical frequency domain technique based on the perturbation theory in the SSM is presented in the context of obtaining the response of rotor system with nonlinearity. The equation of motion of a nonlinear rotor system in the modal coordinates can be expressed as same as time domain analysis in the form of perturbation 1st order equation. The relation between the frequency of the nonlinear oscillations and the exciting frequency around the 1st natural frequency (ω_{n1}) of the overall system is defined as follows:

$$\Omega^2 = \omega_{n1}^2 + \varepsilon\omega_0 \tag{22}$$

By regarding the first approximation to be Eq.(20) and its substitution into the perturbation 1st order equation results in a polynomial expression of trigonometry

$$\Omega^2\ddot{\eta}_i^{(1)} + \omega_{ii}^2\eta_i^{(1)} = f_{i1}\dot{\xi}_i^{(0)} + f_{i2}\xi_i^{(0)3} + f_{i3}\cos(\tau+\phi) \tag{23}$$

where $f_{i1}^{(1)}, f_{i2}^{(1)}, f_{i3}^{(1)}$ are determined by solving the perturbation 0th order equation. From the Eq.(23), we obtain the term to prevent the secular term by equating the coefficient of $\cos\tau, \sin\tau$ to zero. By equating the coefficients of $\cos\tau$ and $\sin\tau$ and squaring these results, the relationship between the frequency, amplitude and force terms are obtained by using the trigonometric relations.

3.3 Sensitivity

It is estimated that the nonlinear dynamic behavior of a subsystem influences the neighboring subsystem. To observe an effect of nonlinearity to the other subsystem, we introduce a sensitivity of nonlinear subsystem. It is very important to observe the effect of nonlinearity in the dynamic design of a structure with nonlinearity. By observing the sensitivity, effective range of nonlinear analysis can be estimated. After obtaining the responses of the perturbation 0th -order equation and perturbation 1st-order equation of overall system, those are changed into physical coordinates. The response of subsystem 1 is described as follows:

$$\begin{aligned}\{X_1\} &= [\Phi_1]\{\xi_1\}\\ &= X_{1i}^{(1)}\cos\Omega t + \varepsilon_{1i}^{(1)}\cos\Omega t + \varepsilon X_{2i}^{(1)}\cos 3\Omega t\end{aligned} \tag{24}$$

where $X_{1i}^{(0)}$, $X_{1i}^{(1)}$, $X_{2i}^{(1)}$ are amplitude. The sensitivity is defined with perturbation parameter and perturbation 1st order amplitude. The sensitivity of the subsystem 1 can be expressed as follows:

$$S_1 = \frac{\Delta\{X_1^{(1)}\}}{\Delta\varepsilon} \tag{25}$$

4. Numerical examples

The rotor system whose properties are tabulated in Table 1 is considered in this analysis with cubic stiffness type nonlinearity in shaft and bearing. The material coefficient γ in Eq.(1) and bearing coefficient δ in Eq.(10) are 0.1. The perturbation parameter ε is set as follows; ε=0.09

Table 1 Properties of the rotor system

Length of rotor	L(mn)	800
Pedestal length	L(mn)	800
Diameter of rotor	D_R(mn)	16
Diameter of pedestal	D_P(mn)	50
Young's modulus of rotor	E(N/m^2)	2.1×10^{11}
Density of shaft casing	ρ (kg/m^3)	7.81×10^3
Bearing coeff.	k_b(N/m)	1.0×10^6
Nonlinear bearing coeff.	k_B(N/m)	1.0×10^6

The rotor and pedestal are to be analyzed as uniform beams modeled by twenty finite elements, respectively. The material density and the modules of elasticity of rotor and pedestal are same value. The damping ratio of the rotor and pedestal in each normal mode is given by ζ = 0.01. The pedestal is constrained to foundation. The damping of the bearing and constraint is ignored.

Table 2 Natural frequency system (unit: Hz)

Mode No.	FEM (DOF=168)	SSM Adopted mode number (Shaft/Casing element)		
		10/10	20/20	40/40
1	73.29	73.10	73.22	73.26
2	164.26	162.52	164.17	164.54
3	310.64	312.13	310.53	310.81
4	488.22	487.27	488.34	488.25
5	668.84	668.54	668.38	668.78

Table 2 shows the natural frequency of rotor system in Hz by FEM and SSM where the adopted component modes are changed. The results obtained by considering 20 modes of each component in SSM are in very good agreement with those obtained by FEM.

Therefore, 20 modes of each component are considered for the further response analysis. In this analysis model, the response is considered at the representative nodal point (x-direction) of rotor system such as at the middle of shaft.

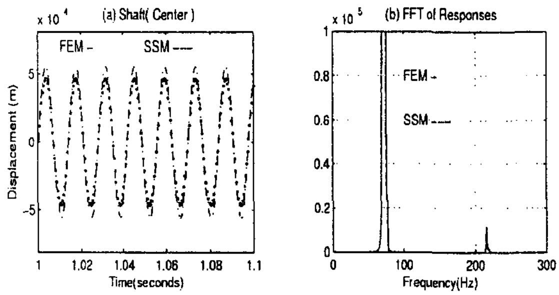

Fig. 3 Response by FEM and SSM

Figure 3 compares time domain responses of SSM with FEM when the center of rotor is excited at 451 rad/s by unbalance of rotor (unbalance = 88.3 g/mm) where the first natural frequency of the system is 460.52 rad/s. It can be observed at the selected point that by using only 20 modes relatively accurate responses of the rotor system can be simulated comparing with response of FEM and its FFT, as shown in Fig. 3 (a),(b). The difference of the responses is regarded as the influence of discarding the higher modes of the system.

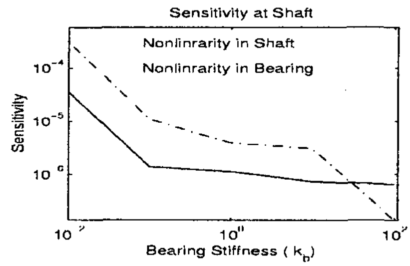

Fig. 4 Non-linearity Sensitivity at shaft

Figure 4 shows sensitivity at the rotor where the bearing nonlinearity and shaft nonlinearity are adopted to the overall system with changing the nonlinear parameter from 0.01 to 0.2.

The calculated sensitivities are presented in Fig. 4 as a function of the normalized support stiffness ($k^* = k/k_b$), where the factor k_b is the stiffness of the bearing. The two sensitivities vary with bearing stiffness. Investigation of the sensitivity reveals that the bearing nonlinearity is mainly more sensitive than shaft one. It is regarded that the nonlinearity in the bearing had a more significant effect on the rotor's response as compared to the nonlinearity in the shaft system. But bearing sensitivity is more insensitive than shaft one around ($k^* = 10^2$). It is regarded that support stiffness is too hard to vibrate ($k = 10^8 N/m$).

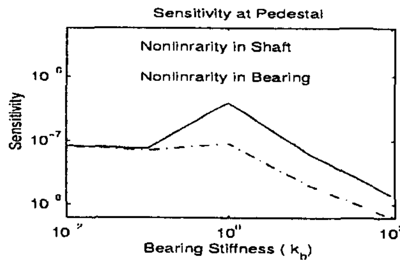

Fig. 5 Non-linearity Sensitivity at pedestal

Figure 5 shows sensitivity at the pedestal where the bearing nonlinearity and shaft nonlinearity are adopted to the overall system with the same conditions of shaft response. Investigation of the sensitivities, as shown in Fig.5, reveals that the sensitivity of shaft is bigger than bearing one. It is regarded that the nonlinearity in the shaft had a more significant effect on the pedestal's response while vibrating in same mode with near 1st natural frequency.

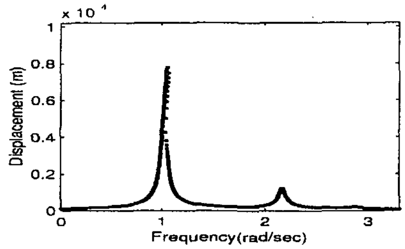

Fig. 6 Nonlinear frequency response at near the middle of shaft

Figure 6 shows the nonlinear frequency response at near the middle of shaft as a function of the frequency in non-dimensional form, which is equal to the rotating speed divided by the 1st critical speed. It can be observed that the resonance peak of each selected point is not straight, but slopes to the right for hardening nonlinear system, as shown in Fig.6.

Table (3) shows the computing time with changing the adopted modes number by *SparkStation 10, SunMicrosystem Co.* in time domain analysis based on one frequency by FEM and SSM. It can be observed that drastic reduction in computational time can be obtained when adopting only lower modes what is a critical factor in the analysis of structural dynamics.

Table 3 Computing time comparison

Condition No.	Adopted mode Number	Time(sec)
1	10	28
2	20	33
3	40	51
FEM	168(DOF)	760

5. Conclusions

In this paper, the dynamic analysis method of a nonlinear rotor, nonlinear bearing and linear pedestal system were theoretically formulated by using the perturbation method and the SSM as a case of large mechanical nonlinear structure. The formulation is concerned with reducing the number of degrees of freedom for each component by modal substitution. All the components are then assembled together and the complete structure analyzed in time and frequency domain near the 1st natural frequency. It was shown that nonlinear responses could be efficiently calculated according to the selected number of vibration mode for economical calculation. Sensitivity is defined by changing the perturbation parameter to investigate the influence of component vibration to the overall system. It is observed that the nonlinearity in the bearing had a more significant effect on the rotor's response as compared to the nonlinearity in the shaft system. And at the pedestal the nonlinearity of shaft is more influential than the bearing one.

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