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Passive Vibration Suppression With an Enhanced Shunted Piezoelectric Circuit

강화된 Piezoelectric Shunt Circuit에 의한 수동진동제어 연구

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Key Words : Passive Vibration Suppression(수동진동제어), Piezoelectric Circuit(압전회로), Machanical Passive Viscoelastic Damping(기계적 수동점탄성감쇠)

요 약 : 회로내에 capacitor를 부가 연결시켜 이론과 실험적으로 고찰한 새로운 기법의 연구이다. 종래에 사용되어 온 전자회로는 낮은 주파수의 진동진폭을 억제할 때에 큰 inductance 값을 필요로 하는 결점이 있었다. 이런 문제점을 해결하기 위하여 본 연구에서는 강화된 압전 분권회로에 병렬로 capacitor를 연결하도록 설계하였다. 새로운 기법은 기계적인 analogy 이론에 의해 증명을 하였으며, 알루미늄 보에 대하여 필요한 동조 모드에서 실험적으로 입증하였다. 따라서 이러한 결과들은 electronic passive damping 에 있어서 예전부터 요구되어 온 절반정도의 inductance값만으로도 구조물의 진동응답을 아주 심도 있게 감소시킬 수 있다는 것을 보여주고 있다.

1. Introduction

The passive electronic damping is of interest in areas such as the automotive and aerospace industries since it is not much heavy and temperature independent comparing with mechanical passive viscoelastic damping. Recently, K2 ski designers used a resistive/capacitive(RC) shunt circuit to dissipate the vibration energy absorbed by the piezos and showed a good vibration suppression performance at the first vibration mode. Also many researchers have proved that the use of a tuned electronic damping is a potential way to suppress vibrations.

Forward(1979) carried out a preliminary demonstration of the feasibility of using external electronic circuits to damp mechanical vibrations in optical systems.

Hagood and von Flotow(1991) showed the analytical models of the shunted piezoelectric with the experimental verifications of these models using a resistive(R) and a resonant(LR) electrical shunt to provide damping for beam. Edberg, Bicos and Fechter(1991) showed that a lightweight electronic circuit was used instead of a heavy commercial inductor which was used by Hagood et al. and a possibility to simultaneously dissipate two modes of vibration using a tuned shunt circuit.

Hollkamp(1994) developed a theory to suppress multiple modes using a single piezoelectric material from the experimental model of Edberg et al. However, due to mutual loading effects between multiple shunts, tuning of the inductor for the one specific mode would cause detuning of the other mode in the shunt circuit, this approach was not practical. Wu and Bicos(1996) analyzed a piezoelectric shunt theoretically using a lead zirconate titanate(PZT) element shunted with a parallel resistor and inductor circuit for passive structural damping and vibration control. Their theoretical analysis for parallel shunt circuit might not verify with the experimental results due to ignoring the internal resistance of the entire shunt circuit.

The conventional shunt circuit consists of three electric components, capacitor C(PZT), inductor L and resistor R as shown in Figure 1.

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The two external terminals of the PZT which is regarded as a capacitor, since electrically the PZT behaves similar to a capacitor, are connected with the series inductor and resistor shunt circuit. The piezoelectric ceramic element is used to convert mechanical energy of a vibrating structure to electrical energy by direct piezoelectric effect. This electric energy is dissipated by heating through the shunting resistor efficiently when the electrical resonant frequency matches with mechanical one. At resonance, the reactive components between the inductor $\left(j\omega L = \omega L\angle 90^0\right)$ and capacitor $\left(\frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^0\right)$ cancel each other and the phase between the current and voltage is zero, that means the power factor is one.

2. Theoretical Modeling of Passive Shunted PZT

A passive shunt circuit (Figure 1) has to be tuned to suppress a target mode like a mechanical vibration absorber. Vibration absorber consists of an attached second mass, spring, and damper forming an additional single degree of freedom system.

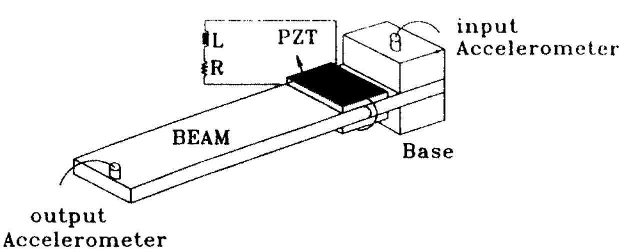

Fig. 1 Experimental setup with shunt circuit

The second spring mass and damper system is then tuned to resonate and hence absorb all the vibrational energy of the system. Like the same principle of a mechanical absorber, the electric resonant frequency should be tuned at the very near or equal to the mechanical resonant one.

Hagood and Von Flotow(1991) give a theoretical model for PZT with shunting inductor and resistor. For this model, the transverse case is assumed, which field is in z or 3 direction and force is in x or 1 direction. With an inductor and a resistor in parallel with the PZT inherent capacitance, the electrical impedance of the shunt and the total electrical impedance are

$$Zsu = Ls + R, \quad Z_{EL} = \frac{Ls + R}{LC_p^T s^2 + RC_p^T s + 1} \tag{1}$$

where the Laplace parameter, L is the shunting inductance, R is the shunting resistance and C_p^T is the inherent capacitance of PZT at constant stress. The electrical impedance can be express as nondimensional form.

$$\bar{Z}_{EL} = \frac{Z_{EL}}{Z^D} \frac{LC_p^T s^2 + RC_p^T s}{LC_p^T s^2 + RC_p^T s + 1} \tag{2}$$

where Z^D is the open circuit electrical impedance, $1/sC_p^T$. The non-dimensional mechanical impedance of the shunted piezoelectric circuit, $\bar{Z}^{ME}$, is done by Hagood and Von Flotow (1991).

$$\bar{Z}^{ME} = Z^{SU} / Z^D = \frac{1 - k_{31}^2}{1 - k_{31}^2 \bar{Z}^{EL}} \tag{3}$$

where k_{31} is the electro-mechanical coupling coefficient of the piezoelectric circuit provided by manufacturer, where the field is applied across the 3 direction and mechanical effects are in the 1 direction. Equation (2) can be substituted into equation (3) and the non-dimensional mechanical impedance of a shunted piezoelectric circuit can be obtained.

$$\bar{Z}^{ME} = 1 - k_{31}^2 \left(\frac{\delta^2}{\gamma^2 + \delta^2 r\gamma + \delta^2} \right) \tag{4}$$

where $\delta = \omega_e / \omega_n^D$, $\gamma = s / \omega_n^E$, $r = RC_p^S \omega_n^E$ and $\omega_e = 1/\sqrt{LC_p^S}$. ω_n^D and ω_n^E the natural frequency of the structure mode of interest with an open a shunted piezoelectric circuit and a shunted piezoelectric circuit which can be obtained from the frequency response function. The capacitance of the PZT should be find out roughly using the following equation: since the capacitance is dependent on frequency.

$$C_p^T = \frac{K_3^T \times \varepsilon_0 \times A_p}{t_p} \tag{5}$$

where C_p^T is the capacitance of the PZT at constant stress, K_3^T is the relative dielectric constant at 1kHz, and the constant ε_0 is 8.85×10^{-12} F/m, A_p is surface area of PZT and t_p is the thickness of the PZT. The product of $K_3^T \varepsilon_0$ is called the permittivity of the dielectric denoted ε. The PZT capacitance at constant strain, C_p^S, is obtained from the $C_p^S = C_p^T(1 - k_{31}^2)$ which is dependent upon the electro-mechanical coupling coefficient, k_{31}.

The other form of mechanical impedance for the open circuit piezoelectric can be written where K^E represents the mechanical stiffness of the shorted piezoelectric.

$$Z^D = \frac{K^E}{s(1 - k_{31}^2)} \tag{6}$$

The generalized electromechanical coupling constant for a piezoelectric bonded to a structure can be obtained from the frequency change of the electric boundary conditions (Hagood and Von Flotow, 1991).

$$K_{31}^2 = \frac{(\omega_n^D)^2 - (\omega_n^E)^2}{(\omega_n^E)^2} \tag{7}$$

The other optimum tuning parameters are calculated from the values sought the above as follows:

$$\delta_{opt} = \sqrt{1 + K_{31}^2} \qquad r_{opt} = \sqrt{2}\frac{K_{31}}{1 + K_{31}^2}$$
$$\omega_e = \frac{1}{\sqrt{LC_p^S}} \qquad R_{opt} = \frac{r_{opt}}{C_p^S \omega_n^E} \tag{8}$$

where δ_{opt} is the r_{opt} optimal tuning ratio, is the electrical damping ratio and ω_e is the electrical resonant frequency. The equation of motion of a single degree of freedom model with shunt circuit can be expressed by

$$x(s) = F(s) / \left(ms^2 + \bar{Z}^{ME} Z^D s + k\right) \tag{9}$$

From the equations (4), (6), (9), the transfer function of the absorber for a undamped structure can be found

$$\frac{x}{x^{ST}} = \frac{\gamma^2 + \delta^2 r\gamma + \delta^2}{(\gamma^2 + 1)(\gamma^2 + \delta^2 r\gamma + \delta^2) + K_{31}^2(\gamma^2 + \delta^2 r\gamma)} \tag{10}$$

One more thing that should be noted is the dissipative power at the resonance is

$$W_d = \frac{v_2^2}{2R} \tag{11}$$

where v_2 is the voltage across the resistor. Also ensure that too much damping destroys the piezoelectric vibration absorber.

3. Concept of ESC (Enhanced Shunt Circuit)

Even though many researchers are struggling to improve the performance of the passive shunt damping, there are still defects to which should be fixed. One of them is that hundreds of Henries are required to suppress relatively low structural frequencies. Though it is possible to generate the inductances on the order of hundreds to thousands of Henries with lightweight electronic inductor circuit, it causes also the large internal resistance which exceeds the optimum resistance value to need suppress the specific vibration amplitude. To solve this problem, a new class of Enhanced Shunt Circuit(ESC) is presented to be an effective means for damping out the vibration of flexible structures.

The proposed ESC combined a conventional shunt circuit with parallel connections of capacitors as shown in Figure 2. Application of KCL (Kirchhoffs current Law) gives

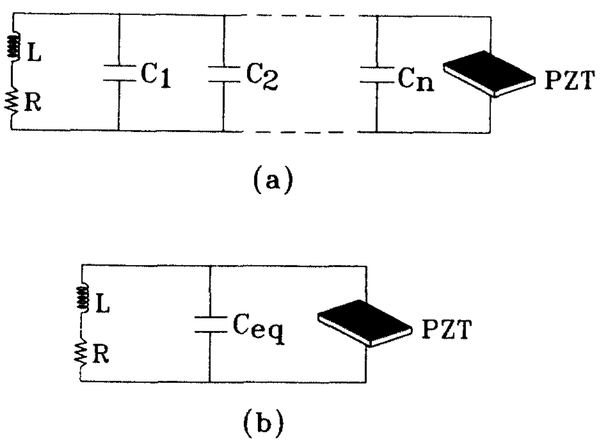

Fig. 2 Parallel connection of (a) N+PZT capacitors (b) Equivalent circuit

$$i = i_{PZT} + i_2 + i_3 + \dots + i_n \tag{12}$$

From the current-voltage relation for PZT

$$i = (C_{PZT} + C_1 + C_2 + \dots C_n)\frac{dv}{dt} = \left(C_{PZT} + \sum_{n=1}^{N} C_n \right)\frac{dt}{dv} \tag{13}$$

and N capacitors, in the circuit of Fig. 2, the current is equal to an equivalent circuit C_{eq}.

$$C_{eq} = (C_{PZT} + C_1 + C_2 + \dots C_n) = \left(C_{PZT} + \sum_{n=1}^{N} C_n \right) \tag{14}$$

Thus the equivalent capacitance of N parallel capacitors is simply the sum of the individual capacitances include the PZT. The voltage would be equal to that which is present across the parallel combination. If two parallel connected capacitors, which is demonstrated by experiment as for example later,

$$C_{eq} = (C_{PZT} + C_1) \tag{15}$$

The physical meaning of a shunt circuit which generates an additional damping as tuning to a target frequency is easily understandable with the consideration of mechanical-electrical analogies. The equation of a mechanical system is

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0 \tag{16}$$

Assuming $x(t)$ is of the form $x(t) = Ae^{st}$ and obtaining the characteristic equation:

$$s^2 + \frac{c}{m}s + \frac{k}{m} = 0 \tag{17}$$

Using simple algebra, the two solutions for s are given by

$$s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} \tag{18}$$

Rewriting the equation (16) using customary notation,

$$\frac{d^2x}{dt^2} + 2\xi\,\omega_n\frac{dx}{dt} + \omega_n{}^2 x = 0 \tag{19}$$

The mechanical natural frequency and damping ratio are defined as follows:

$$\omega_n = \sqrt{\frac{k}{m}}, \quad \xi = \frac{c}{2\sqrt{km}} \tag{20}$$

The electric system equation is

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = 0 \tag{21}$$

The resulting characteristic equation is

$$Ls^2 + Rs + \frac{1}{C} = 0 \tag{22}$$

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \tag{23}$$

and the circuit is underdamped electric resonant frequency is

$$\omega_e = \frac{1}{\sqrt{LC}}, \quad \xi = \frac{R}{2\sqrt{L/C}} \tag{24}$$

Comparing equation (16) and (21), the differential equations for the two systems are of identical form. From equation (24), the inductance L (henry) and capacitance C (farad) are easily adjusted without changing the electric resonant frequency. For example, to decrease the inductance as half as that of required, the capacitance should be increased as double as previous one to be tuned the same electric resonant frequency.

$$\omega_e = \frac{1}{\sqrt{LC}} = \sqrt{\frac{1}{\frac{L}{2} \times 2C}} \tag{25}$$

And the electric damping ratio is changed as follows:

$$\xi_e' = \frac{R}{2\sqrt{\frac{L}{2}/2C}} = \frac{R}{\sqrt{L/C}} \tag{26}$$

Comparing equation (24) and equation (26), the new electric damping ratio is increased as double as previous one.

$$\xi_e' = 2\xi_e \tag{27}$$

A statespace model of the circuit (Figure 3) can supplement the above theory to obtain to same electric resonant with a half inductance and a double conductance.

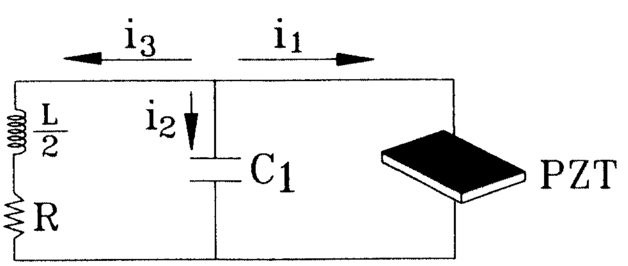

Fig. 3 Shunt circuit with an additional capacitor

Using a simple application of Kirchhoffs voltage and current law:

$$i_1 = -(i_2 + i_3) \tag{28}$$

$$v = \frac{1}{C_{PZT}}\int i_1 dt = \frac{1}{C_1}\int i_2 dt = \frac{L}{2}\frac{di_3}{dt} + Ri_3 \tag{29}$$

$$\int i_2 dt = -\left(\frac{C_{PZT}C_1}{C_{PZT}+C_1}\right)\left(\frac{1}{C_{PZT}}\right)\int i_3\, dt \tag{30}$$

From equations (28)-(30), the current flowing the series LR branch circuit is

$$\frac{L}{2}\frac{di_3}{dt} + Ri_3 + \frac{1}{C_{PZT}+C_1}\int i_3\, dt = 0 \tag{31}$$

Then by defining the state variables,

$$x_1 = \int i_3 dt, \quad x_2 = i_3 \tag{32}$$

the state spae representation of equation (31) is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{L/2(C_{PZT}+C_1)} & -\frac{R}{L/2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{33}$$

Also, equation (16) can be presented in state space form as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{34}$$

If $C_{PZT}=C_1$, equation (33) is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{L/2(2C_{PZT})} & -\frac{R}{L/2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{35}$$

Solving the eigenvalue problem of equation (35), the same electric resonant obtained with a half inductance and a double conductance. This concept will be verified by the experiment as performing with a half of the inductance and the full inductance required to be tune a target mode.

4. Experimental Implementation

The experiments with a shunt circuit are performed for two different inductance values at the first mode frequency and the second mode frequency. For maximum effectiveness, a pair of 6.3cm x 3.8cm PZT tile is bonded to the each side root of the beam which is 30cm long and 3.8cm wide by using Tru-Bond Epoxy Adhesive. The pair of PZT tile are poled through their thickness and elongate lengthwise so that they are operating in transverse mode. The beam is grounded and wired in parallel so as to produce opposite fields in the top and bottom piezoceramics, and thus caused a moment on the beam, which the top PZT contracts as the bottom one expands. Also, if the beam is bent, a voltage generated through the shunt circuit by the direct piezoelectric effect of PZT material.

The internal function generator of the spectrum analyze(DSP Siglab) is used to generate a random base acceleration from 1 Hz to 100 Hz with a spectral resolution 0.065 Hz. This random signal is used to excite a beam which was mounted on an APS shaker(Acoustic Power System Inc.) through amplifying by a power amplifier(APS Model 114). The input signal is measured by an accelerometer (KISTLER 8630A50) which is attached by APS shaker. The tip displacement signal(output signal) of the beam is measured by an accelerometer(PCB 309A) and is fed to the spectrum analyzer to determine its frequency content. The magnitude ratio(dB) and the phase shift(degree) of the system response are automatically displayed on the screen. Thus, the transfer function between the input and output can be obtained.

An active filter (Horowitz and Hill, 1989) is used as synthetic inductor in the shunt circuit. The advantages of this inductor are due to its convenience in using, a light weight and generating various high inductances. However gyrator circuit is not a pure inductor, it creates a resistive component which is not desirable for designing the optimal resistance in the shunt circuit. In figure 4, R_4 is ordinarily a capacitor, with the other impedances being replaced by resistors, creating an inductor $L_{eq} = R^*C$, where $R^* = R_1R_3R_5 / R_2$. As changing the variable resistor R_2, the various inductance can be obtained.

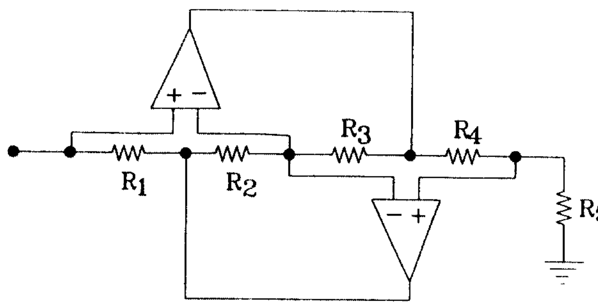

Fig. 4 Circuit diagram of synthetic inductor

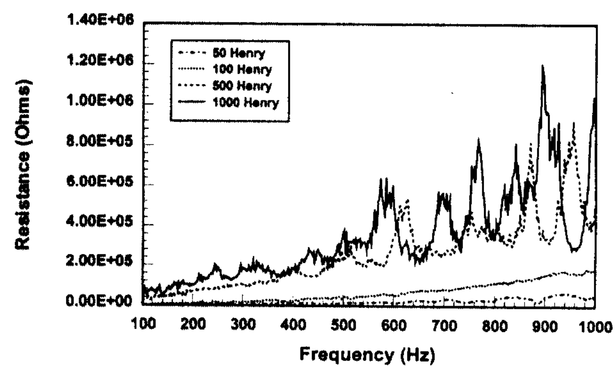

Fig. 5 Internal resistance associated with the synthetic inductor design

The internal resistances of synthetic inductor are shown in Figure 5. This shows that the internal resistances are not only the frequency dependent, but also the inductance dependent. As increasing the amount of inductance and the frequency, the internal resistances are also increased.

5. Experimental Results and Discussions

The experimental results cover two ranges of frequencies. The first range(9-12 Hz) includes the first bending mode and the second(52-66

Hz) covers the second bending resonant frequency. Comparisons are shown between the amplitudes of vibration when the shunt circuit is unactivated(open circuit) and when it is activated using different resistors at a fixed room temperature. In addition, tests on with full and half inductances are also performed (Figure 6-7).

After measuring the frequency response function between output and input voltages, the open and short circuit frequencies measured are 10.625Hz and 10.5Hz for the first mode and 58.625 and 58.247 for the second mode frequency. Using equation (7), the generalized electro-mechanical coupling coefficients calcuated of a PZT pair are 0.15 and 0.11 for the first and second mode frequency respectively.

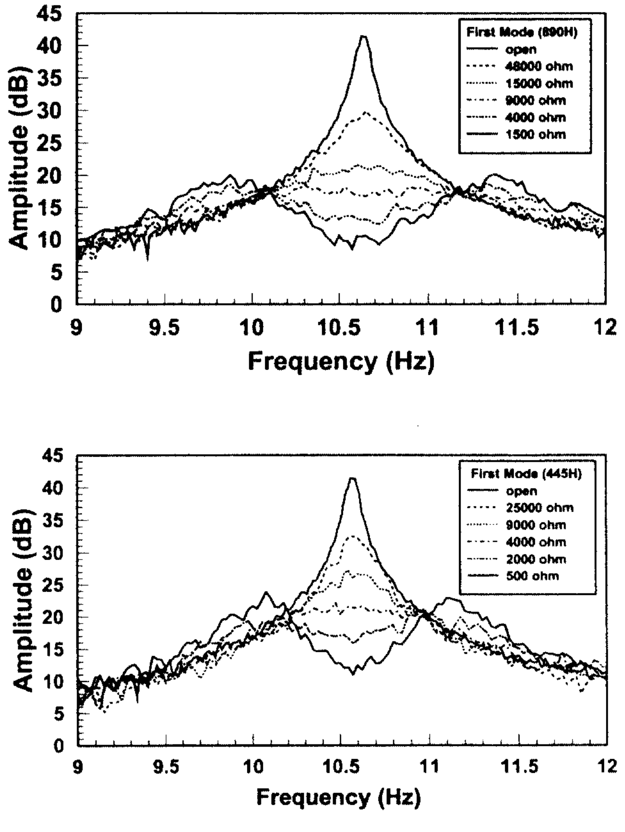

Fig. 6 Frequency response curve with full and half inductance at first mode

The theoretical tuning parameters calculated, inductance and optimum resistance, are compared with the experimental ones. It is difficult to decide these optimal tuning parameters using the conventional shunt circuit theories developed by many researchers since capacitance of PZT and inductance have some internal resistance that is not negligible and the material parameters of capacitors(PZTs) used in the shunt electric circuit have 5-10% manufacture errors and capacitors and inductors are dependent on frequency.

The inductances 819H, 27H and optimal resistances 11760ohm, 1625ohm at the first and the second mode frequency are calculated by the theory. However the actual inductance and resistance values used to suppress the vibration amplitudes are measured 890H, 28H and 10000 ohm, 1000ohm by experiment. It is evident that the tuning parameters can not properly predict by the conventional theory. The experimental parameters are measured by an impedance/gain-phase analyzer(Hewlett-Packard 4194A).

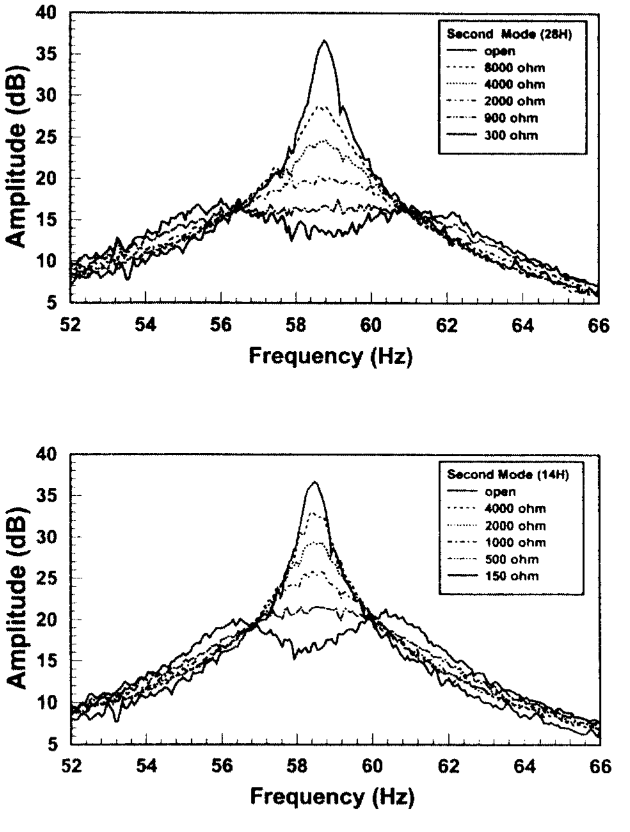

Fig. 7 Frequency response curve with full and half inductance at second mode

The shunt electrical passive absorber is tested experimentally to reduce the vibration amplitudes of the first and the second modes of a cantilever beam with the half and full inductances. It is evident that decreasing resistance results in improving the vibration attenuation characteristics of the first bending mode and the second bending mode. As decreasing below the optimal resistance, the two peaks are rising up, which is a similar phenomenon of mechanical absorber damping. From the Figures 6, the shunted piezoelectric pair is found to produce about 25dB drop from the peak vibration amplitude of the open circuit case at first mode frequency. For the second mode after shunting, a peak amplitude reduction of about 20dB is obtained. The effects of using the shunt circuit with half inductance for the first and the second mode are shown in Figures 6, 7. Only the half resistors are needed to obtain the same vibration amplitude reductions, when full inductances are used at the first and the second mode. This phenomenon is explained by the theoretical analysis in equation (27).

9. Conclusions

A theoretical model of the shunted piezoelectric circuit is reviewed to understand the concept of a mechanical and electrical relationship to suppress the vibration amplitude. A similarity between the transfer functions of a tuned mechanical vibration absorber and a tuned electrical one is found. The difference between two is that the generalized coupling coefficient for the passive electronic damper, K_{31}, plays a same role as the mass ratio, (ma/ms), in the passive mechanical damper.

A technique that was capable of reducing the structure vibration amplitude using the electrical passive absorber with a half or a quarter or even one-tenth inductance is introduced. To prove the effectiveness of this technique, a theory was developed with the consideration of mechanical electrical analogies. Experimentally this technique with a cantilever beam with shunt circuit was tested. The experiments were demonstrated successfully with a full and a half inductance at the first mode and the second mode frequency. The vibration amplitudes of the beam were decreased about 25dB and 20dB at the first mode and the second mode frequency respectively. When the shunt circuit with a half inductance was used, the electric damping was doubled. Hence, the developed technique presents an invaluable tool for suppressing the structure vibration amplitudes that happened in many engineering applications.

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