Journal of the Korean Institute of Industrial Engineers
Vol. 25, No. 3, pp. 336-341, September 1999.

Determination of Optimum Process Mean and Screening Limits for Production Processes with Multi-Decision Alternatives

[1]Sung-Hoon Hong · [2]Hyuck-Moo Kwon · [3]Sang-Boo Kim · [4]Min-Koo Lee
[1]Department of Industrial Engineering, Chonbuk National University/
[2]Department of Industrial Engineering, Pukyong National University/
[3]Department of Industrial Engineering, Changwon National University/
[4]Department of Business Administration, Seowon National University

다결정 대안을 갖는 생산공정에서 최적공정평균 및 스크리닝 한계선의 결정

홍성훈[1] · 권혁무[2] · 김상부[3] · 이민구[4]

The problem of jointly determining the optimum process mean and screening limits for each market is considered in situations where there are several markets with different price/cost structures. The quality characteristic is assumed to be a normal distribution with unknown mean and known variance. A quadratic loss function is utilized for developing the economic model. Methods of finding the optimum process mean and screening limits are presented and a numerical example is given.

1. Introduction

Consider the problem of selecting the optimum mean value for a continuous production process. All items are inspected to determine whether its quality characteristic meets a predetermined lower specification limit. Conforming items are sold at a regular price, whereas all others are reprocessed or sold at a discounted price. Typical quality characteristics under consideration are weights, volume, number and concentration. Items produced by a production process may deviate from the process mean because of variations in materials, labor and operation conditions. The process mean may be adjusted to a higher value in order to reduce the proportion of the nonconforming items. Using a higher process mean, however, may result in a higher production cost. Consequently, the decision of selecting a process mean should be based on the tradeoff among production cost, payoff of conforming items, and the costs incurred due to nonconforming items.

This problem has been studied by several researchers. Bettes(1962) treated the problem of simultaneously selecting an optimum process mean and controllable upper limit where the rejected items are reprocessed at a fixed cost. Hunter and Kartha(1977) solved the problem of selecting the optimum process mean when the nonconforming items are sold at a reduced price. Bisgaard *et al.*(1984) extended Hunter and Kartha's model to a situation where the nonconforming items are sold at a price proportional to the amount of ingredient used. Golhar(1987) considered the problem of selecting the optimum process mean in a canning process. Carlsson(1989), Boucher and Jafari(1991), and Al-Sultan(1994) discussed situations in which the items are subjected to lot-by-lot acceptance sampling rather than complete inspections. Arcelus and Rahim(1994) developed a model for jointly selecting optimum target means for both a variables and an attributes quality characteristics, and Chen and Chung(1996) considered an economic model for determining the most profitable target value and the optimum measuring precision level for a production process.

In all the previous work, the inspected items are classified into two quality grades only; conforming items are accepted and nonconforming items are rejected. But, it is common practice to grade the outgoing items on the basis of the quality and then sell them in different markets. This practice has been used for chemical materials and primary materials such as lumber, wheat, cotton, and butter(England and Leenders, 1975). Different grades of a product may be sold at different selling prices under different names or marketed in different chain stores or areas under the same brand name (Tang, 1990). Several authors, Tang(1990), Bai and Hong(1992), Kim *et al.*(1994), and Lee and Jang(1997) considered economic inspection procedures with multi-decision alternatives. In this paper, an economic model is developed for jointly determining the optimum process mean and the screening limits for each market in situations where there are several markets with different price/cost structures. Hong *et al.* (1999) proposed a similar model. They considered a situation where the inspection is performed on a surrogate variable and a constant loss function is used.

The loss caused by imperfect quality may include loss of goodwill, warranty, replacement cost, and handling cost. Classical concept in the field of optimum target value determination assumes that this loss is a constant when an item does not conform to product specifications and is zero otherwise. However, Taguchi(1984) argued that this cost concept was incorrect. Instead, he suggested that a quadratic function of the deviation from the product target value could better measure the true loss. In this paper, a quadratic loss function is utilized for developing the expected profit function model. All items are inspected prior to shipment and the decision for disposing the product is made depending on their quality and price/cost structures.

2. Model

Let Y be a performance variable representing the quality characteristic of interest and τ denote the target value of Y. We assume that Y is a 'larger is better' variable and normally distributed with unknown mean value μ_y and known variance σ^2_y. Suppose that a product can be sold to several different markets. When products are sold to market i, the selling price is A_i, and the item with $y<\tau$ causes a loss of $C_i(y,\tau)=a_i(\tau-y)^2$ which is a quadratic function. a_i is a positive constant, and y is the observed value of Y. This function was strongly advocated by Taguchi(1984) and has received widespread attention. If $y\geq\tau$, $C_i(y,\tau)=0$. Now consider the case where $A_i>A_j$ and, $C_i(y,\tau)\leq C_j(y,\tau)$, or $A_i=A_j$ and $C_i(y,\tau)<C_j(y,\tau)$. It is easy to verify that market j is dominated by market i and thus a product should be sold to market i rather than market j. Therefore only the markets which are not dominated need to be considered. Assume that there are m markets which are not dominated. It is not profitable to ship the low quality products to an ordinary market because of the penalty cost $C_i(y,\tau)$. Therefore, market m is considered as an alternative with one of following modes; sell the products at a discounted price, scrap the products, etc. Without loss of generality, it is assumed that $A_i>A_j$ and $C_i(y,\tau)>C_j(y,\tau)$ for all $i<j$. The condition $C_i(y,\tau)>C_j(y,\tau)$ is equivalent to the condition $a_i>a_j$.

Since market i requires higher quality products than market j for $i<j$, an appropriate inspection procedure is as follows.

i) Take measurement y for each incoming item.
ii) Let δ_i, $i=1, 2,..., m$, be real numbers such that $\delta_1\geq\delta_2\geq...\geq\delta_m=-\infty$ and $\delta_0=\infty$.
If $\delta_i\leq\delta_{i-1}$, $i=1, 2,..., m$ ship the item to market i. Note that if $\delta_i=\delta_{i-1}$, the item will not be shipped to market i.

The item is shipped to market i whenever $\delta_i\leq\delta_{i-1}$, $i=1, 2,..., m$. Therefore, the expected revenue for market i is

$$A_i\int_{\delta_i}^{\delta_{i-1}} g(y)\,dy\ , \qquad (1)$$

where $g(y)$ is the probability density function of Y which is a normal density function with mean μ_y and variance σ^2_y. The production cost per item is c_0+c_1y where c_0 and c_1 are positive constants(Bisgaard *et al.*, 1984). The expected production cost per item thus becomes $\int_{-\infty}^{\infty}(c_0+c_1y)g(y)dy=c_0+c_1\mu_y$. The expected penalty cost for market i caused by imperfect quality is

$$\int_{\delta_i}^{\delta_{i-1}} C_i(y,\tau)\,g(y)\,dy. \qquad (2)$$

Therefore, the expected profit per item is given by

$$EP=-s_y-c_0-c_1\mu_y+\sum_{i=1}^{m}\int_{\delta_i}^{\delta_{i-1}}k_i(y)g(y)dy, \quad (3)$$

where s_y is the inspection cost of Y and $k_i(y)=A_i-C_i(y,\tau)$.

The optimum values of $(\mu_y, \delta_1, \delta_2, \ldots, \delta_m)$ can be obtained by maximizing equation (3). We first determine the optimum screening limits $\delta^*_i = \delta^*_i(\mu_y)$, $i=1, 2, \ldots, m$, for given μ_y and then determine μ^*_y maximizing the expected profit. An upper bound of the fourth term in equation (3) for given μ_y is $\int_{-\infty}^{\infty}\{\max_i k_i(y)\}g(y)dy$. This value is attained by shipping the item to market i whenever $y \in I_i$, $i=1, 2, \ldots, m$, where $I_i = [\delta^*_i, \delta^*_{i-1}]$, $\delta^*_0 = \infty$, $\delta^*_m = -\infty$, and δ^*_i, $i=1, 2, \ldots, m-1$, are the smallest real numbers satisfying the inequalities $k_i(y) \geq k_j(y)$ for all $j > i$, simultaneously. If I_i is empty, we let $\delta^*_i = \delta^*_{i-1}$. For a detailed derivation of δ^*_i, see Hong (1996).

Since $k_i(y)$ is the function of the parameters (A_i, a_i, τ), it is clear that the optimum values of $(\delta^*_1, \delta^*_2, \ldots, \delta^*_m)$ also depend on the same parameters and do not depend on the value of μ_y. Inserting the optimum values of $(\delta^*_1, \delta^*_2, \ldots, \delta^*_m)$ into equation (3), we obtain

$$EP = -s_y - c_0 - c_1\mu_y + \int_{-\infty}^{\infty}\{\max_i k_i(y)\}g(y)dy. \quad (4)$$

Setting the partial derivative of equation (4) with respect to μ_y to zero, the following equation is obtained

$$\int_{-\infty}^{\infty}\{\max_i k_i(y)\}(y-\mu_y)g(y)dy - c_1\sigma_y^2 = 0 \quad (5)$$

It is difficult to find closed form expressions for the solution of equation (5). Numerical studies over a wide range of parameter values of $(\tau, A_i, a_i, \sigma_y, c_1)$ indicate that equation (5) has unique solution and it represents a maximum point. A search algorithm such as Muller's method can be used for finding the value of μ^*_y.

3. A Numerical Example

Consider a packing plant of some chemical industry. The plant consists of two processes; a filling process and an inspection process. Each chemical product processed by the filling machine is moved to the loading and dispatching phases on a conveyor belt. Inspection is performed on the weight Y of the chemical product. From theoretical considerations and past experience, it is known that the weight of the chemical product follows a normal density function with $\sigma_y^2 = (1.25kg)^2$. The target value τ of the weight is $40kg$ which is marked on each product.

The chemical product can be sold to foreign, domestic, or discount markets. The low quality product is scraped because of the penalty cost. The selling price in the foreign market is higher than that of the domestic market. The cost caused by imperfect quality in the foreign market is also higher than that of the domestic market, because of differences in the costs of identifying and handling a nonconforming item, labor cost, transportation cost, etc. We will consider the foreign market as market 1, the domestic market as market 2, the discount as market 3, and the scrap as market 4. The selling prices and the estimated penalty cost coefficients in dollars are as follows:

	Foreign market	Domestic market	Discount	Scrap
Price	40	39	24	0
Penalty cost coefficient	10.5	6.5	0.75	0

The production cost in dollars is $6.0+0.6y$ which is proportional to the quantity y, and the inspection cost is $s_y = \$4.0$.

Using these values, we obtain $(\mu^*_y, \delta^*_1, \delta^*_2, \delta^*_3) = (41.74, 39.50, 38.38, 34.34)$. Hence the chemical products are sold to the foreign market if $y \geq 39.50$, to the domestic market if $38.38 \leq y < 39.50$, or to the discount market if $34.34 \leq y < 38.38$. If $y < 34.34$, they are scraped. In this case the expected profit per item is \$4.633. <Figure 1> graphically shows the functions of $k_i(y)$ and $\{\max k_i(y)\}$ for $i=1, 2, 3, 4$.

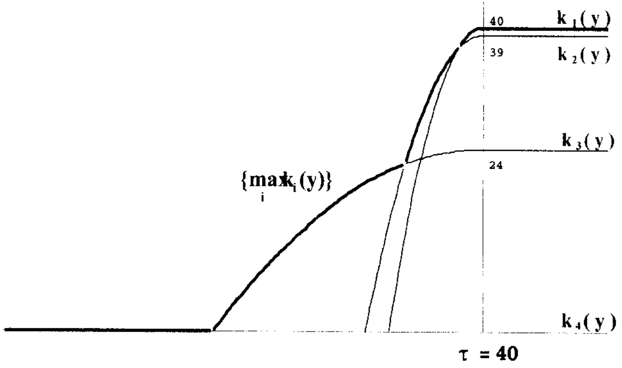

Figure 1. Graphs of $k_i(y)$ and $\max k_i(y)$.

The optimum values of μ_y^* and their expected profits are given in <Figures 2> and <Figures 3> for selected values of σ_y for 0.50 (0.25) 2.50. They show that μ_y^* can be set significantly lower as σ_y decreases. These results agree with our intuition that μ_y^* can be set τ if σ_y is zero. <Figure 3> also shows that the expected profit increases as σ_y decreases. <Table 1> gives the results of varying c_l from 0.4 to 0.8. Both μ_y^* and the expected profit tend to increase as c_l decreases.

While we can obtain accurate values of the selling price for each market, it is sometimes difficult to obtain accurate penalty cost information. To study the sensitivity of this model to the changes of cost parameters, the percentage decreases (PD) are given in <Table 2> for selected combinations of a_1, a_2, and a_3

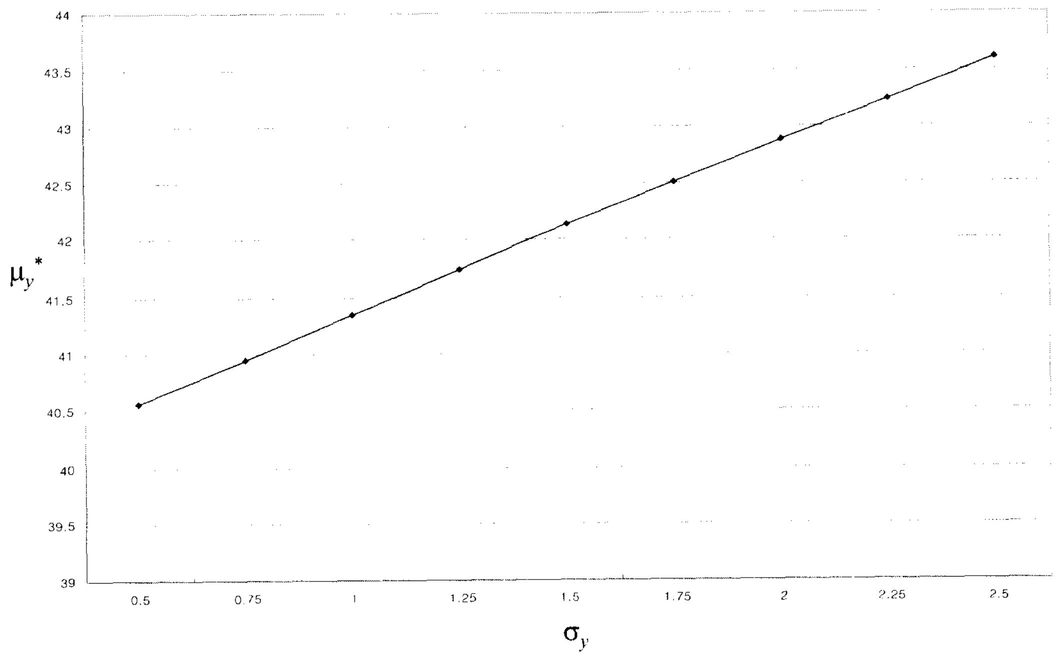

Figure 2. Optimum Values of μ_y^* as a Function of σ_y.

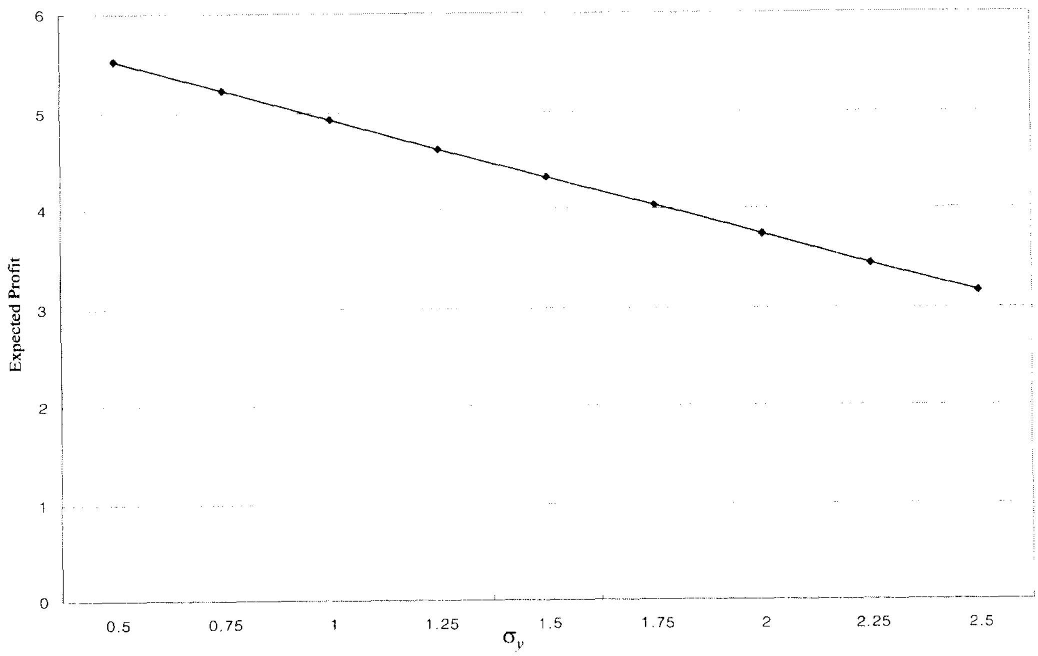

Figure 3. Expected Profit as a Function of σ_y.

Table 1. Effects of c_1

No	c_1	μ_y^*	δ_1^*	δ_2^*	δ_3^*	Expected Profit
1	0.4	41.99	39.50	38.38	34.34	13.005
2	0.5	41.86	39.50	38.38	34.34	8.813
3	0.6	41.74	39.50	38.38	34.34	4.633
4	0.7	41.65	39.50	38.38	34.34	0.464
5	0.8	41.56	39.50	38.38	34.34	-3.696

Table 2. Optimum solutions and percentage decreases obtained by using improper cost parameters

No	a_1	a_2	a_3	μ_y^*	δ_1^*	δ_2^*	δ_3^*	PD (%)
1	8.4	5.2	0.6	41.65	39.441	38.194	33.68	0.161
2	8.4	5.85	0.675	41.69	39.374	38.297	34.04	0.097
3	8.4	6.5	0.75	41.73	39.275	38.385	34.34	0.140
4	9.45	7.15	0.825	41.77	39.341	38.460	34.61	0.075
5	9.45	7.8	0.9	41.80	39.222	38.526	34.84	0.248
6	9.45	5.2	0.675	41.66	39.515	38.179	34.04	0.140
7	10.5	5.85	0.75	41.71	39.536	38.285	34.34	0.032
8	10.5	6.5	0.825	41.75	39.500	38.374	34.61	0.001
9	10.5	7.15	0.9	41.78	39.454	38.451	34.84	0.026
10	11.55	7.8	0.6	41.80	39.484	38.557	33.68	0.088
11	11.55	5.2	0.75	41.66	39.603	38.164	34.34	0.187
12	11.55	5.85	0.825	41.71	39.581	38.272	34.61	0.052
13	12.6	6.5	0.9	41.74	39.595	38.363	34.84	0.026
14	12.6	7.15	0.6	41.78	39.572	38.487	33.68	0.046
15	12.6	7.8	0.675	41.81	39.544	38.550	34.04	0.101

with remaining parameters fixed. PD is expressed as

$$PD = \frac{EP^* - EP'}{EP^*} \times 100(\%), \qquad (6)$$

where EP^* and EP' are the expected profit obtained by using the actual cost parameters and the expected profit obtained by using the improper cost parameters, respectively. The values of a_i, $i=1, 2, 3$, are within $\pm 10\%$ or $\pm 20\%$ to represent overestimated or underestimated values of the actual value. For example, the 5^{th} cost parameters in < Table 2> are $a_1=9.45$, $a_2=7.8$, and $a_3=0.9$ which are obtained by 10% underestimation of a_1, 20% overestimation of a_2, and 20% overestimation of a_3. In this case, PD is 0.140 %. <Table 2> shows that the model is very robust to the changes of cost parameters.

5. Conclusions

We have considered the problem of jointly determining the optimum process mean and screening limits for each market in situations where there are several markets with different price/cost structures. A quadratic loss function is utilized for developing the economic model for determining the optimum process mean and screening limits. There is no closed form expression for the optimum process mean. Hence, a numerical search such as Muller's method is used. Extensive sensitivity analyses show that the optimum process mean and screening limits are very insensitive to the changes of cost parameters. Numerical results also show that the optimum process mean tends to increase and the expected profit tends to decrease as the process variation σ_y increases. Numerical studies are performed by using FORTRAN and IMSL (International Mathematical and Statistical Libraries) subroutines on a 586PC. In most cases the results can be obtained within a few minutes. A possible area of further investigation would be the extension of the model to the cases where the variance σ_y^2 is unknown.

References

Al-Sultan, K. S. (1994), An algorithm for the determination of the optimal target values for two machines in series with quality sampling plans, *International Journal of Production Research,* 12, 37-45.

Arcelus, F. J. and Rahim, M. A. (1994), Simultaneous economic selection of a variables and an attribute target mean, *Journal of Quality Technology,* 26, 125-133.

Bai, D. S. and Hong, S. H. (1992), Economic screening procedures using a correlated variable with multi-decision alternatives, *Naval Research Logistics,* 39, 471-485.

Bettes, D. C. (1962), Finding an optimum target value in relation to a fixed lower limit and an arbitrary upper limit, *Applied Statistics,* 11, 202-210.

Bisgaard, S., Hunter, W. G. and Pallesen, L. (1984), Economic selection of quality of manufactured product, *Technometrics,* 26, 9-18.

Boucher, T. O. and Jafari, M. A. (1991), The optimum target value for single filling operations with quality sampling plans, *Journal of Quality Technology,* 23, 44-47.

Carlsson, O. (1989), Economic selection of a process level under acceptance sampling by variables, *Engineering Costs and Production Economics,* 16, 69-78.

Chen, S. L. and Chung, K. J. (1996), Selection of the optimal precision level and target value for a production process: The lower-specification-limit case, *IIE Transactions,* 28, 979-985.

England, W. B. and Leenders, M. R. (1975), *Purchasing and Materials Management*, 5th Edition, Richard D. Irwin, Homewood, IL.

Golhar, D. Y. (1987), Determination of the best mean contents for a canning problem, *Journal of Quality Technology,* 19, 82-84.

Hong, S. H. (1996), Economic complete inspection plans for grading product quality, *Journal of the Korean Institute of Industrial Engineers,* 22, 473-483.

Hong, S. H., Elsayed, E. A. and Lee, M. K. (1999) Optimum mean value and screening limits for production processes with multi-class Screening, *International Journal of Production Research,* 37, 155-163.

Hunter, W. G. and Kartha, C. P. (1977), Determining the most profitable target value for a production process, *Journal of Quality Technology,* 9, 176-181.

Kim, C. T., Tang, K. and Peters, M. (1994), Design of a two-stage procedure for three-class screening, *European Journal of Operational Research,* 79, 431-442.

Lee, M. K. and Jang, J. S. (1997), The optimum target values for a production process with three-class screening, *International Journal of Production Economics,* 49, 91-99.

Taguchi, G. (1984), *Quality Evaluation for Quality Assurance,* Romulus, MI: American Supplier Institute.

Tang, K. (1990), Design of product grading procedures, *Decision Sciences.* 21, 434-445.