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Social Construction of Mathematics Understanding among Student Peers in Small Group Settings

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The purpose of this review of literature is to investigate what kinds of research have been done on social construction of mathematics understanding among elementary students in small groups. Only empirical studies were reviewed, and then grouping was done in terms of the purpose of the study. This grouping identified three categories:

1) Social and mathematical norms in mathematics classroom,
2) Teaching productive communication behaviors for active learning in small group, and
3) Participation roles and communication behaviors in different group structure.

To enhance social construction of mathematics understanding in small group settings two suggestions are made: the importance of the selection of collaborative tasks or problems and teachers' beliefs about mathematics and the teaching an learning of mathematics.

0. INTRODUCTION

The mathematics classroom envisioned by the reform movement is one in which the teacher's role is to facilitate students' working on worthwhile mathematical tasks that engage them in mathematical problem solving, reasoning, and communication (Wood & Cobb, 1992). Accomplishing the reform goals for learning and teaching in mathematics requires that both teachers and students learn new modes of interaction and develop new knowledge and skills. Moreover, this revised view of teaching and learning is more interwoven with features of the social context (e. g., classroom interactions, the social organization of the school) (Brown, Stein & Forman 1996).

More recently, perspectives that take into account the role of social and cultural factors in the processes of teaching and learning have begun to appear within mathematics education research (Bishop 1988; Cobb & Bauersfeld 1995; Nunes, Schliemann & Carraher 1993; Saxe 1991a, 1991b). One of the perspectives that has been gaining attention within the mathematics education research community is sociocultural theory which emphasizes that the construction of knowledge occurs in a social, institutional, and cultural context (Forman, Minick & Stone 1993; Jonathan 1994; Saxe, Gearhart, Note & Paduano 1993).

In order to accomplish this reform of the teaching and learning of mathematics, this review of literature is to investigate what kinds of research have been done on social construction of mathematics understanding among elementary students in small groups. Based on the theoretical framework such as constructivism, Vygotsky's sociocultural theory of cognitive development, only empirical studies were selected and reviewed. Empirical studies meeting the purpose of the study were critically reviewed. Each study was reviewed, and then grouping was done in terms of the purpose of the study. This grouping identified three categories:

1) Social and mathematical norms in mathematics classroom,
2) Teaching productive communication behaviors for active learning in small group, and
3) Participation roles and communication behaviors in different group structure.

1. Social and Mathematical Norms in Mathematics Classroom

In order for students to construct meaningful understanding, what roles should teachers do in the mathematics classroom? According to four studies in this section establishing social and mathematics norms is important for social construction of mathematics understanding (Lampert 1990; Lo, Wheatley & Smith 1994; Yackel, Cobb & Wood 1991; Yackel & Cobb 1996).

In an one-year study with third graders of understanding the role of mathematics class discussion in the construction of mathematics understanding, Lo et al. (1994) concluded that a student's participation in class discussion is influenced by his mathematics knowledge, beliefs, social competence, and how other students see him as a participant, both socially and mathematically.

Similarly, Yackel, Cobb and Wood (1991) suggested the importance of social norms in mathematics classroom. Considering small-group interactions as a source of learning opportunities in second-grade mathematics, Yackel et al. (1991) focused on the construction of classroom norms for cooperation and on the learning opportunities that arise in the course of interactions when children work in small groups to complete mathematical activities. According to their first-year study in a three-year project, learning opportunities arose after social norms were established.

Another similar study was also conducted by Yackel and Cobb (1996). There should be distinct differences between general classroom social norms and mathematics classroom norms. Yackel and Cobb (1996) focused on the mathematical aspects of students' activity in order to establish classroom environments that facilitate students' mathematical conceptual development. Through a process of developing sociomathematical norms, what counts as acceptable explanation and justification in mathe-matics classroom was interactively established.

Taking a similar approach (i. e., emphasis on the importance of social norms in mathematics classroom) on one hand, but a very different perspective (i. e., from the history and philosophy of mathematics), Lampert (1990) as a fifth-grade teacher and researcher examined whether it is possible to make knowing mathematics in the classroom more like knowing mathematics in the discipline. Lampert pointed out that the students would not learn a different way of thinking about what it means to know mathematics simply by being told what to do. She noted that changing students' ideas about what it means to know and do mathematics was in part a matter of creating a social situation that worked according to rules different from those that ordinarily pertain in classrooms, and in part respectfully challenging their assumptions about what knowing mathematics entails.

The commonality of four studies is that students' learning opportunities arise once social norms were established and that classroom discussion or discourse either in small group or in whole class as an instructional strategy contributes students' construction of mathematics understanding. In addition, a classroom teacher plays an essential role in establishing social norms and encouraging students' participation in classroom discussion or discourse.

From the four studies a little different teacher's role could be identified: that is, the teacher in Lampert's study was a representation of what is means to know mathematics. Lampert (1990) pointed out that the teacher should show them what it means to know mathematics and should do mathematics with students in the classroom. But in Lo et al. (1994), Yackel et al. (1991), and Yackel and Cobb (1996) studies the teacher is to facilitate and encourage students' participation in class discussion through negotiating and establishing social or sociomathematical norms.

One possible hypothesis from the four studies is that we could enhance students' construction of mathematics understanding through classroom discussion or discourse if we could identify productive social or sociomathematical norms.

2. Teaching Productive Communication Behaviors for Active Learning in Small Group

Unlike the studies on constructing social or mathematics norms in classroom, the studies that will be discussed here have focused on identifying and teaching productive communication behaviors in small groups (Cooper 1980; King 1990; King 1994; Nattiv 1994; Webb, Troper & Fall 1995). These studies seem to assume that, since just grouping students does not promote their learning through peer interaction, students need to learn certain communication skills to enhance their interaction that leads to active and constructive learning in small groups.

Two studies conducted by King (1990; 1994) approached the same purpose — to enhance peer interaction for knowledge construction in classroom, but took a quite different perspective from social norm studies. What King hypothesized was that students' need to learn productive communicative skills (e. g., the skills of questioning and explaining). This idea clearly opposes the studies for establishing social and mathematical norms because King taught directly the communicative skills without considering a variety of classroom cultures. The subjects were divided into two groups (i. e., a reciprocal peer-questioning group and a discussion group) and the reciprocal peer-questioning group only received training for peer questioning. The findings were obvious: that is, guided reciprocal peer-questioning group outperformed, gave and received significantly more explanations in response to their questions, gave and received fewer low-level elaboration responses, and asked significantly more critical thinking questions and fewer recall questions.

Like King's study, Webb, Troper and Fall (1995) taught seventh-grade general mathematics classes basic communication skills to help them work effectively in groups. The students received training to develop their ability to help each other while working on problems in small groups such as asking clear and precise questions, being persistent in asking for help, giving explanations instead of the answer, and giving feedback to other students about their work. Results showed that continuing to work on problems after receiving help was important for achievement. The more engaged a student was in generating the solution to a problem, the more likely the student was to obtain a high score on the posttest.

Similarly, Nattiv (1994) tried to confirm the relationship between helping behaviors and math achievement and generalize it to elementary-aged students such as third, fourth, and fifth graders. Students were assigned to groups of five to six students so that each team included students of both genders and the three ability levels (i. e., high-, medium-, and low-achieving students). All students received instruction on helping behaviors for three weeks. Results revealed that helping behaviors were related to achievement. In particular, giving (and receiving) explanations or other meaningful help were related positively to achievement. The researcher suggested that it would be beneficial to teach students in cooperative learning groups how to help one another, how to ask for, and receive help.

According to the four studies, it could be said that responsiveness to the partner or caring a partner (e. g., helping behaviors, questioning and explaining) and a shared goal on a task are important communication behaviors in a collaborative task in small group settings. One apparent implication drawn from these five quantitative studies is that identifying productive communication skills for social construction of knowledge would be a valuable attempt to help students actively engage in classroom discussion and discourse. As discussed earlier from the studies on social norms, teaching specific communication skills should be taught once appropriate social norms are established. Without considering classroom norms that are slightly different from each classroom because of different teachers' different view of teaching and learning mathematics, transfer of such communication skills would not be possible as King (1994) showed that there was no significant effect of questioning and explaining strategy on retention test.

3. Participation Roles and Communication Behaviors in Different Group Structure

The major question for this paper is focused on facilitating social construction of mathematics understanding. Possible answers to this question, which were suggested in the previous two sections were either establishing social and mathematical norms in mathematics classroom or explicitly teaching how to question and explain or how to help each other. However, the studies in the previous two sections did not pay much attention to group structure. In other words, the question still remains: "Is grouping with similar ability students better to promote social interaction rather than with different ability students?" Five studies (Fuchs, Fuchs, Hamlett & Karns 1998; Gauvain & Rogoff 1989; Jones & Carter 1994; Mulryan 1992; Tudge 1992) shed light on the differences of how students communicate and how they participate in small-group interaction, when they were paired or grouped with similar ability peers or more knowledgeable peers.

With nine science classes and three science teachers from the fifth grade, Jone and Carter (1994) examined verbal and nonverbal characteristics of low- and high-achieving dyads and how interactions and behaviors of high-low dyads differed from high-high and low-low dyads. According to the results, when high- and low-achieving students worked together, high-achievers drew the low-achievers' attention to the learning task. High achievers sometimes modeled thinking processes aloud. The researchers concluded that high achievers were not disadvantaged or exploited by working with the low students and that significant achievement gains were made by the low-achieving students who worked with high-achievers. They pointed out that merely to place high and low students together may not be advantageous if the task is not designed for growth at different levels.

Gauvain and Rogoff (1989) examined the effect of sharing responsibility during joint planning. The participants were five- and nine-year-old children. They were divided into three groups to do the task: individual, five-year-old dyad, and nine-year-old dyad. The major conclusion was that having a partner was not related to plan effectiveness on later solitary trials. Rather, sharing task responsibility in the collaboration appeared to be related to planning performance on the posttest trials for both mother-child and peer teams. In other words, an important influence on independent planning appeared to be the prior experience of sharing task responsibility.

Tudge (1992) examined the consequences of collaborating with a partner and the collaborative processes that engendered by the dyads featuring different levels of competence. Children aged from five to nine from kindergarten, first-, second-, third-, and fourth-grade classes participated in the study. A mathematical balance beam was used with 14 different problem configurations. The major findings were that unequal partners were more likely to change rules than equal partners and that arriving at shared meaning or intersubjective understanding in the course of discussion was a highly effective means of bringing about changes in thinking. Attaining intersubjectivity in the course of discussion was more desirable for children to advance cognitive development than existing already intersubjectivity.

Fuchs et al. (1998) examined high-achieving students' interactions and work quality as a function of homogeneous versus heterogeneous ability group compositions as children worked on mathematics performance assessments. Ten teachers and their third- and fourth-grade classrooms participated in this study. One of the major findings was that high-achieving elementary-aged students worked more productively and effectively on a complex, authentic problem-solving task when paired with fellow high achievers rather than with fellow low-achievers. The homogeneous dyads produced better work on the performance assessments. As the homogeneous pairs worked more collaboratively and with greater cognitive conflict and resolution, focus on interacting, and helpfulness and cooperation, high achievers generated better mathematical performances with their high-achieving classmates than with their low-achieving peers.

In order to gain understanding of factors that contribute to different levels of students' involvement in the cooperative small group, especially student passivity and domination, Mulryan (1992) conducted a study with fifth- and sixth-grade mathematics classes. An observational instrument was created to collect data on the type and nature of target students' on-task behavior and off-task behavior and on the nature and frequency of their interactive behavior during whole-class mathematics and cooperative small group. Findings showed that there existed the differences between high- and low-achievers in their behavior and attitudes toward cooperative small-group instruction and that low-achieving students were relatively passive in cooperative small groups in comparison with their high-achieving peers. Some high achievers indicated that they disliked and even avoided giving help to low-achievers in a group. Many students used speed of task completion as an important criterion of group success. In some situations, more competent group members may have excluded low-achiever s to avoid the need for explanations or to maximize the group's chances of success without unwelcome delays.

In sum, from the five studies discussed above, the general conclusion on group structure in dyad was that, if the low-achieving students were paired with the high-achieving student, then high achievers were not disadvantaged or exploited by the low-achieving students. Significant achievement gains were made by the low-achieving students. However, Fuchs et al. (1998) argued that this is just the case for simple tasks. High-achieving elementary students worked more productively and effectively on a complex, authentic problem-solving task such as mathematics performance assessments when paired with fellow high achievers rather than with fellow low achievers. Similarly, according to Mulryan's study (1992), low-achieving students did not always benefit from high-achieving students in small group. They were passive in group interaction because some high achievers disliked and even avoided giving help to low achievers.

By contrast, two studies from Vygotskian tradition (Gauvain & Rogoff 1989; Tudge 1992) argued that sharing task responsibility and arriving at shared meaning or intersubjective understanding in the course of discussion, not working with a partner with whom there was already intersubjectivity, were a highly effective means of bringing about changes in thinking. In other words, attaining intersubjectivity in the course of discussion was more desirable for children to advance cognitive development than existing already intersubjectivity. Therefore, there is still no definite conclusion on group structure: vertical (equal-ability grouping) or horizontal (unequal-ability grouping).

4. Discussion

The main purpose of this paper was to investigate what studies have been done in the area of social construction of mathematics understanding. Three approaches to social construction of mathematics understanding were identified:

1) Social and mathematical norms in mathematics classroom;
2) Teaching productive communication behaviors for active learning in small group; and
3) Participation roles and communication behaviors in different group structure.

The empirical studies reviewed in this paper can partially answer the direction that recent studies on social construction of mathematics understanding take. Above all, in order to promote social construction of mathematics understanding in classrooms, proper social and mathematical norms should be first established. Once these norms are established, the productive communication behaviors for active learning in small groups should be taught. In this case, it is suggested that a shared goal on a task and responsiveness to the partner should be more emphasized. Another way to facilitate social construction of mathematics understanding is to use grouping including pairs.

In conclusion, the three approaches identified through this paper should be understood with the two aspects proposed here, in order that these approaches are applied to the teaching and learning of mathematics in classroom. The first is the importance of collaborative tasks or problems. Collaboration in small groups would not occur when the task is too easy or requires simple memorization. Peer collaboration is an effective learning environment for tasks that require reasoning, but not for tasks that require rote learning or copying. Good et al. (1990) suggested that simply using more grouping did not necessarily lead to more student verbalization, critical thinking, or collaborative student work on mathematics. One reason explaining this situation was that most of the assigned work in classrooms consisted of problems taken from the textbook and involved only practice on problems designed to be solved by individuals. The second aspect is about teachers' beliefs of the nature of mathematics or the teaching and learning of mathematics. Without considering teachers' such beliefs, it is not likely to understand the events taking place in mathematics classroom. A number of studies in mathematics education have indicated that mathematics teachers' beliefs play a significant role in shaping the teachers' characteristic patterns of instructional behavior (Cooney, Shealy & Arvold 1998; Raymond 1997; Thompson 1984). Therefore, classroom teachers' beliefs about mathematics and the teaching and learning of mathematics and the types of collaborative tasks or problems are important factors impacting the success of social construction of mathematics understanding in small group settings.

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