

Exchange Rate and Interest Rate Dynamics in an Equilibrium Framework

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요 약

This paper examines the time series dynamics of spot and forward exchange rates and Eurocurrency deposit rates for four bilateral relationships vis a vis the U.S. dollar using daily data. The equilibrium implied by covered interest parity provides a theoretical foundation from which to estimate and analyze the dynamic properties of each system of exchange rates and interest rates. The structural statistical model is identified by relying on the implied cointegration vectors and long-run neutrality restrictions.

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I . Introduction

Since the advent of the modern floating exchange rate regime in 1973, there has emerged a large theoretical literature devoted to the behavior of the exchange rate, both spot and forward, and interest rates, both domestic and foreign, in an open economy setting. This literature differed from its antecedents by concentrating on the stocks of assets as opposed to the flow approach of the Mundell-Fleming models. The first of this genre were the monetary model of exchange rate determination, built upon three key open economy equilibrium relationships: (1) purchasing power parity (PPP) (2) uncovered interest rate parity (UIP) and (3) stable money demands for the relevant currencies. Over time, the empirical evidence against these models mounted and produced several theoretical extensions that relaxed one or more of the three key assumptions.

Dornbusch's (1976) sticky price model relaxed the assumption of continuous PPP. By recognizing that asset markets clear much more quickly than other markets, this model allows for spot rate overshooting. Although Dornbusch presented some evidence of this behavior of the spot rate, there has been little work done since.

The relaxation of the UIP assumption is the major distinction of the portfolio-balance models. In these models all assets, not just currencies, are considered relevant in determining the equilibrium exchange rate. Thus, relative bond supplies came to be considered an important determinant of the exchange rate. Unfortunately, the empirical evidence has not been very supportive.

McKinnon (1982) suggested that the assumption of stable money demands for individual currencies is not supported by the data. He claims that although there is no stable demand for any single national money, there is a stable demand for world money, defined as the supply of currency of an inner group of industrialized nations. The implications of this theory are that domestic and

foreign interest rates adjust in order to clear world money markets.

This paper offers an analysis of the empirical dynamics of the spot and forward exchange rates and the domestic and foreign interest rates within an equilibrium framework. The results can be compared against the implications of various theoretical models in order to determine which has the most empirical support. The theoretical foundation is the covered interest-parity relation that implies an equilibrium relationship between spot and forward exchange rates and domestic and foreign interest rates.

The econometric procedure is the identification of the structural statistical model by relying on the implied cointegration vectors and long-run neutrality restrictions developed by Crowder, et al. (1998). This methodology is advantageous in that it allows the joint identification of permanent and transitory components with fewer ad hoc restrictions.

By construction of the covered-interest differential relationship for the Germany, U.K., Japan, and Switzerland against the U.S., two permanent and one transitory shock for each countries are identified. The permanent and a transitory shocks are defined as the exchange, interest rate, and money-supply shocks, respectively.

The rest of the paper is organized as follows. Section II describes the covered-interest parity relation and the econometric methodology. Section III presents the empirical results including the impulse response analysis. Section IV offers concluding remarks.

II . The Covered Interest Parity Relation and Econometric Methodology

Covered Interest Parity (CIP) is an arbitrage condition that relates the difference between the k -period forward rate and the current spot rate to the

interest differential on the k-period domestic and foreign bonds. Defining the exchange rates as units of domestic currency per unit of foreign currency, the CIP relation can be written as

$$(1 + I_t^k) = \frac{F_t^k}{S_t} (1 + I_t^{*k}), \quad (1)$$

where F_t is the k-period forward rate observed at time t , S_t is the spot rate observed at time t , I_t^k is the interest rate on a k-period domestic asset observed at period t , and I_t^{*k} is the k-period return on a foreign asset observed in period t . The CIP relation is guaranteed to hold (within a transaction cost band) in the long run, through the arbitrage activities of foreign exchange and capital market traders. Taking logs of equation (1) yields a linear relationship in continuous compound rates that can be characterized as a cointegrating relation.¹⁾ This implies that the vector $Z_t = [f_t \ s_t \ (i_t - i_t^*)]'$ is cointegrated with cointegration vector $\beta = [1 \ -1 \ -1]'$. Since the purpose of this paper is to analyze the dynamic relationship between exchange rates and the interest differential, a suitable econometric methodology must be chosen. The long-run equilibrium relationship of the CIP implies cointegration among the variables. The existence of the cointegration relationship can be exploited to achieve structural identification of the shocks or innovations in the system. The identification of the structural model can be analyzed by specifying the reduced-form autoregressive (AR) representation given by

$$\Phi(L)Z_t = u_t, \quad \text{var}(u_t) = \Sigma_u, \quad (2)$$

where $\Phi(L)$ is a matrix of q -order lag polynomials of $p \times p$ coefficient matrices and u_t is $p \times 1$ vector of reduced errors with covariance matrix Σ_u . The cor-

1) The definition of a cointegrating relation is that the variables are individually integrated of the same order, usually one, but the unique linear combination implied by the equilibrium is not integrated, i.e. it is stationary.

responding structural AR model is given in

$$H(L)Z_t = \varepsilon_t \quad \text{var}(\varepsilon_t) = I, \quad (3)$$

where the relations between structural and reduced form parameters are $H(L)=H(0)\Phi(L)$ and $I=H(0)\Sigma_uH(0)'$. The conventional approach to identify the structural parameters and impulse response functions is to impose $p(p-1)/2$ restrictions on $H(0)$, the contemporaneous correlation matrix (see Bernenke (1986) and Sims (1986)). On the other hand, Shapiro and Watson (1988), and Blanchard and Quah (1989) identify the contemporaneous, structural correlation matrix by placing restrictions on the long-run structural impact matrix. However, these empirical models are subject to criticism by having too many as hoc restrictions on the structural model.

In an effort to overcome these problems, the structural VAR methodology recently developed by Crowder, et al. (1998) which is an extension of the common trends model, is used. The desirable feature of this methodology is that it requires fewer ad hoc restrictions for identification of the structural model by exploiting r cointegrating vectors where $r < p$. Consider the reduced-form error correction representation (Johansen, 1988) given by (4).

$$\Delta Z_t = \mu + \pi Z_{t-1} + \sum_{j=1}^{t-1} \Gamma_j \Delta Z_{t-j} + u_t, \quad (4)$$

where matrix π can be decomposed into two $n \times r$ matrices defined as $\pi = \alpha \beta'$. The linear combinations of Z_t given by β represent the r cointegration vectors and α represents the vector of error correction terms. The dynamics of the CIP system can be analyzed by using the MA representation of (4), written as

$$\Delta Z_t = C(L)u_t, \quad (5)$$

where $C(L)$ is a rational polynomial in the lag operator, assumed to exist. The polynomial $C(L)$ can be always expressed as $C(L)=C(1)+(1-L)C^*(L)$, where

$C(1)$, the long-run total impact matrix of reduced form, is characterized by deficient rank $k=p-r$, when there exist r cointegration vectors. Based on Stock and Watson's (1988) common trends representation, equation (5) can be written as

$$Z_t = C(1)(1-L)^{-1}u_t + C^*(L)u_t, \quad (6)$$

where $C^*(L) = \{C(L) - C(1)\}(1-L)^{-1}$, and $C(1)$ determines k permanent effects to the system. When the cointegration relationship is established with r cointegration vectors, Johansen (1991) demonstrates that $C(1) = \beta_{\perp}(\alpha_{\perp}'\pi^*(1)\beta_{\perp})^{-1}\alpha_{\perp}'$ where $\alpha_{\perp}'\alpha = 0$, $\beta_{\perp}'\beta = 0$, and $\pi^*(1) = \Gamma_1 + \Gamma_2 + \dots + \Gamma_{p-1}$. Johansen (1991) interprets α_{\perp} as the common trends and $\beta_{\perp}(\alpha_{\perp}'\pi^*(1)\beta_{\perp})^{-1}$ as the factor loading that measures the long-run response.

Assuming independence between k permanent and r transitory innovations in Z_t , the permanent and transitory innovations can be identified separately by imposing $k(k-1)/2$ and $r(r-1)/2$ restrictions, respectively (see King, et al. (1991) and Warne (1993)). Note that $S(1)$, the long-run structural total impact matrix can be written as $S(1) = C(1)H(0)^{-1}$. The structural total-impact matrix can then be written as $S(1) = \beta_{\perp}^0(\alpha_{\perp}'\pi^*(1)\beta_{\perp}^0)^{-1}\alpha_{\perp}'$, where $\beta_{\perp}^0 = H(0)\beta_{\perp}$. Hence, the identification of the permanent components can be accomplished by imposing restrictions on α_{\perp} .²⁾ Once α_{\perp} is identified, β^* can be derived from $C(1)\alpha_{\perp}(\alpha_{\perp}'\alpha_{\perp}^{-1})$. The identification of transitory components can be achieved by imposing $r(r-1)/2$ restrictions on $H(0)$. But in our case, we do not have to impose any restrictions to identify the transitory components because only one cointegrating vector exists in the CIP system, discussed in more detail in section III.

Imposing the restriction on α_{\perp} is very closely related to the concept of weak exogeneity in the cointegrating system. In equation (4) of the error correction model, the term $\beta_{\perp}'Z_{t-1}$ has a natural interpretation as the deviation of the system from equilibrium. Thus, it can be said to represent the level of arbitrage

2) Restrictions can be imposed on either $\beta^* = \beta_{\perp}(\alpha_{\perp}'\pi^*(1)\beta_{\perp})^{-1}$ or α_{\perp} . See Crowder (1995).

profits (excluding transactions cost) available in period $t-1$. The existence of cointegration implies that at least one of the α_m is non-zero, where m represents a variable in the system. If a certain row of α is zero, such as $\alpha_m=0$, it implies that m does not respond to past disequilibria and thus is not Granger caused by Z_t . Such a variable can be the source of the common trend and is said to be weakly exogenous to the parameters. Note that the common-trend representation of (6) can be written as,

$$Z_t = C(1)\left(\sum_{s=1}^t u_s\right) + C^*(L)u_t \quad (7)$$

The multivariate source of the common trend, which is $\alpha_{\perp}'(\sum u_s)$ in (7), is characterized by non-reversion to the mean. Hence, the orthogonal complement to α can be constructed to accumulate the common trends in the weakly-exogenous variable. Practically, this is achieved by using a Choleski factor with the ordering of the variables determined by their relative exogeneity properties. Variables that are weakly exogenous should be ordered first, since the existence of weak exogeneity implies no contemporaneous response to endogenous variable innovations. In section III, the impulse response analysis will provide insight into the equilibrium dynamics of the variable Z_t .

III. Empirical Results

The data used in this study are daily observations on spot and forward foreign exchange rates and Eurocurrency deposit interest rates, all taken from the DRI database starting December 23, 1982 and ending May 10, 1994. The spot exchange rates are the Deutsche Mark, U.K. Pound, Japanese Yen, and Swiss Franc all versus the U.S. Dollar, i.e. the U.S. Dollar is treated as the foreign currency in all systems, with the spot rate recorded at London close time, as

are the 30-day EuroMark, EuroPound, Euro Yen, EuroSwissFranc, and EuroDollar deposit rates. The interest-rate differential used in this study is defined as $(I-I^*)$. It is generally accepted that most asset prices, including foreign exchange rates, evolve as integrated processes. This is confirmed for the exchange rates and interest differential used here by applying the Augmented Dickey Fuller test using various lag truncations. In no case can the a null of unit root be rejected for any of the series.³⁾ The number of separate equilibrium relationships (cointegration vectors) is analyzed using the method of Johansen (1988). The lag lengths in the VAR are determined by the Akaike Information Criteria, and are set at $k=11$ for both the German and U.K. systems, $k=15$ for the Japanese system, and $k=10$ for the Swiss system.

<Table 1> Johansen Trace Test for Cointegration

Trace Statistics	Germany	U.K.	Japan	Switzerland
$r = 0$	176.69*	183.59*	187.66*	166.64*
$r \leq 1$	3.87	14.19	10.03	8.83
$r \leq 2$	1.39	3.50	1.93	3.09

Note: Critical values are taken from Osterwald-Lenum (1992), and * denotes statistical significance at the 5% level.

As indicated in <Table 1>, the trace test derived by Johansen suggests the existence of only one cointegration vector among the three variables.⁴⁾ Since there is only one cointegration vector in each system, identification of the permanent components can be achieved by one *a priori* restriction. For the identification of transitory components, it is not necessary to impose any

3) The unit root test derived by Kwiatkowski, et al.(1991)(KPSS) is also conducted and produces the same results obtained with the Augmented Dickey-Fuller test for every system.

4) The maximum eigenvalue test with Johansen procedure produces the same result. In addition, the estimated cointegration vectors are all very close to the theoretically implied vector of $[1 \ -1 \ -1]'$.

additional restrictions. The estimated error-correction coefficients, α , from the VECM are given in <Table 2>. As discussed in section II, when the cointegration rank of r has been established, the impact matrix $C(1)$ may be represented as $C(1) = \beta_{\perp}(\alpha_{\perp}'\pi^*(1)\beta_{\perp})^{-1}\alpha_{\perp}'$. β_{\perp} and α_{\perp} are $3 \times (3-1)$ matrices that are orthogonal complements of β and α , respectively, where β is the 3×1 cointegration vector, α is the error correction coefficient vector, and $\beta^* = \beta_{\perp}(\alpha_{\perp}'\pi^*(1)\beta_{\perp})^{-1}$ is interpreted as the factor loadings. Looking at the impact matrices, the restriction can be imposed on α_{\perp} .

<Table 2> Error Correction Estimates

Variables	Germany	U.K.	Japan	Switzerland
f_t	-0.26(1.05)	-3.09(0.18)	-0.88(0.52)	-1.87(0.83)
s_t	0.28(1.05)	-2.62(0.86)	0.02(0.51)	-1.42(0.84)
$i_t - i_t^*$	-0.04(0.01)	0.06(0.02)	-0.007(0.001)	0.04(0.01)

Note : Standard errors are given in parentheses.

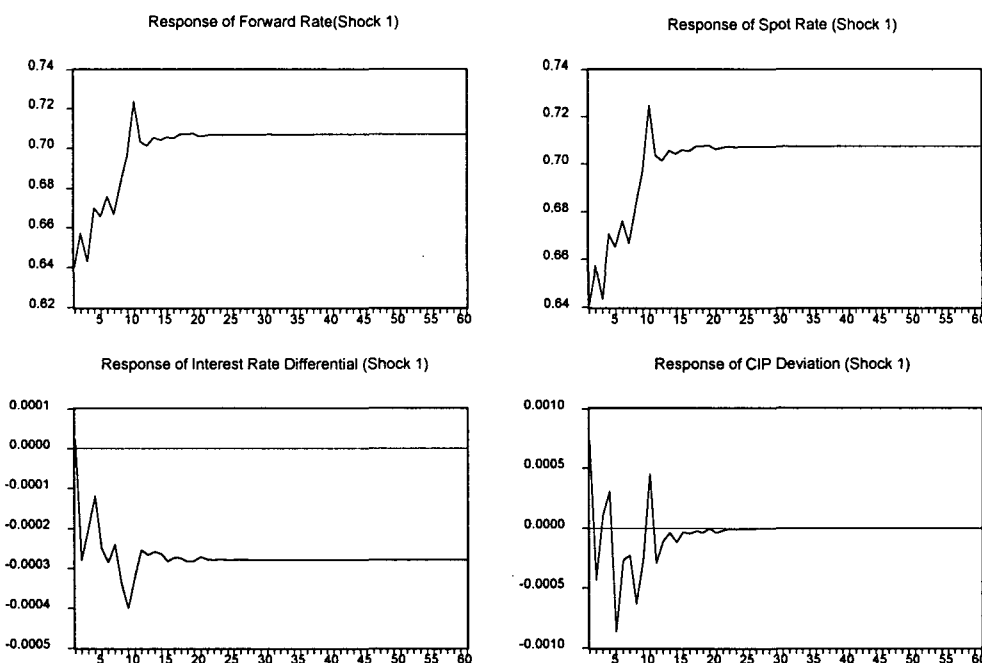
Engle and Granger (1987) have demonstrated that the insignificant error correction coefficients in the vector error correction mechanism (VECM) imply the weak exogeneity of the variables in the system. From table 2, it is evident that there are two insignificant error correction terms in the German and Japanese systems. These variables are the sources of the common trend within the system. In these cases, the identification of the permanent component would be achieved by imposing restrictions on α_{\perp} . This is accomplished by an appropriate ordering as explained earlier.⁵⁾ Figures 1 to 4 depict the impulse response functions (IRFs) including the response of CIP deviations. All systems have two permanent components and one transitory component, and follow very

5) The variables of U.K. and Swiss systems are all determined endogenously. Hence, I tried various orderings of variables for the identification of the permanent component analyzed later in this section and found that different ordering does not affect the qualitative interpretation.

similar patterns. Two permanent shocks (denoted as shock 1 and shock 2) make all variables move toward a new equilibrium (i.e., two innovations are permanent) and one transitory shock (represented as shock 3) affects the movements of all variables only in the short run and eventually dies out. There exist CIP deviations characterized by oscillating patterns.

Figures 1.1 through 1.3 plot the impulse response functions for the German-U.S. system. The permanent shock 1 and the permanent shock 2 affect all variables. [Figure 1.1] shows that the German exchange rates respond 0.707%, while the interest rate differential responds 0.00028% (in an absolute term) to shock 1.⁶⁾ Under shock 1, the interest rate differential does not seem to play a great role determining the exchange rates. Thus, shock 1 can be defined

[Figure 1.1] Impulse Response Functions (Germany, Shock 1)

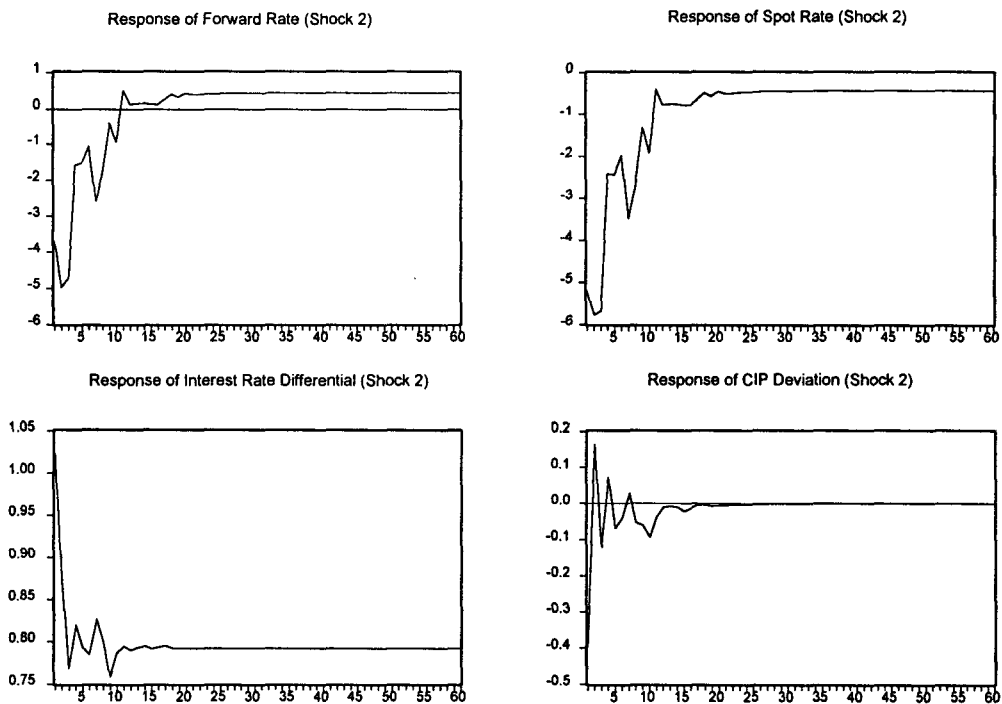


6) Because the terms of IRFs are normalized, the values of IRFs can be interpreted as the percentage terms.

as the exchange-rate shock. Similarly, shock 2 in [Figure 1.2] can be interpreted as the interest rate-shock.

The response of the interest differential should be carefully interpreted. The small response of the interest differential with respect to the exchange rate innovation implies that two interest rates, including the foreign and domestic interest rates, are moving to the same directions.⁷⁾ The exchange-rate innovation affects both the foreign and domestic interest rates to the same direction by offsetting the movement.

[Figure 1.2] Impulse Response Functions (Germany, Shock 2)



7) Preliminary research was conducted to examine the behaviors of the four variables (f , s , I , and I^*) using the same econometric methodology. I found that the two interest rates are moving together.

The large response of interest differentials from the interest rate innovation implies that one interest rate is constant and the other is moving to new equilibrium. The interest innovation affects only one of the foreign and the domestic interest rates. Thus, it can be interpreted as the permanent nominal shock.

It is reasonable for the transitory shock (shock 3) to be defined as a money supply shock such as the domestic money supply shock in this case. These are the common features in the other three systems so that these definitions of the three shocks will be used in all systems in this paper.

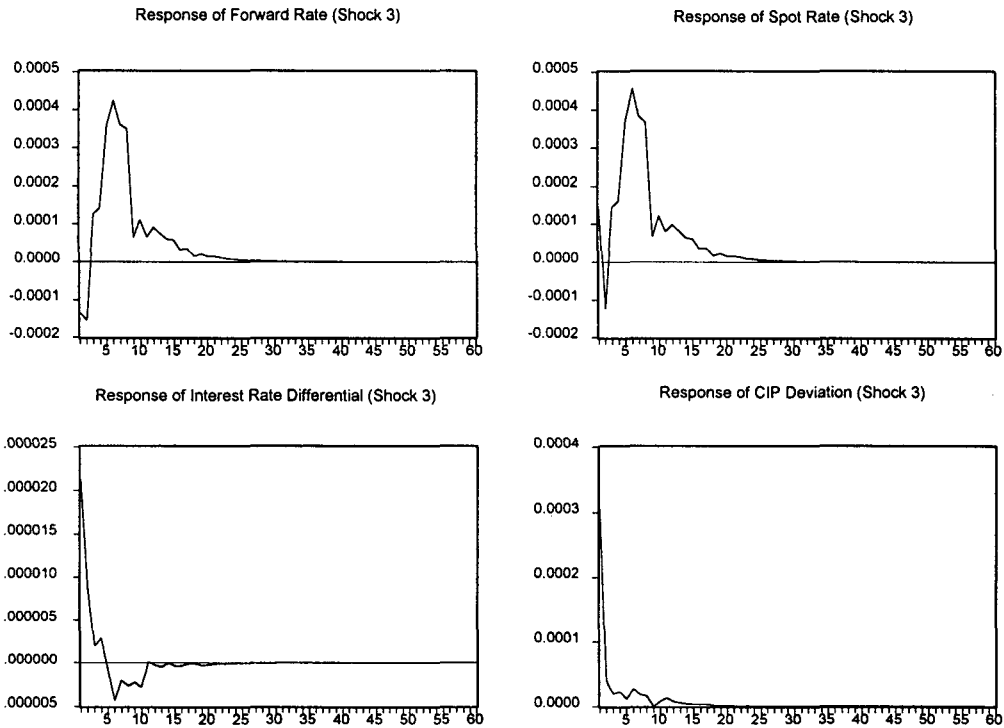
The most striking feature of the exchange-rate shock figured in 1.1 is the large overshooting responses of all 3 variables. No profitable CIP deviation exists.⁸⁾

In [Figure 1.2], the interest-rate shock influences the exchange rates and the interest-rate differential immediately even though the responses of exchange rates are relatively small. Note that in this case there exist profitable CIP deviations.

The IRFs characterized by the transitory innovation are presented in [Figure 1.3]. The transitory innovation in this system could be defined as domestic money supply shock. The analysis of the transitory innovation helps us understand the short-run behaviors of the variables under consideration. Large overshooting responses of the forward and spot rate are observed. Another feature of [Figure 1.3] is the liquidity effect, which is a short-run phenomenon, as opposed to the long-run phenomenon of Fisher effect. The liquidity effect is a short-run theory of the interest rate which states that increases in the money growth lower the interest rate in the short run. IRFs of transitory components capture these liquidity effects.

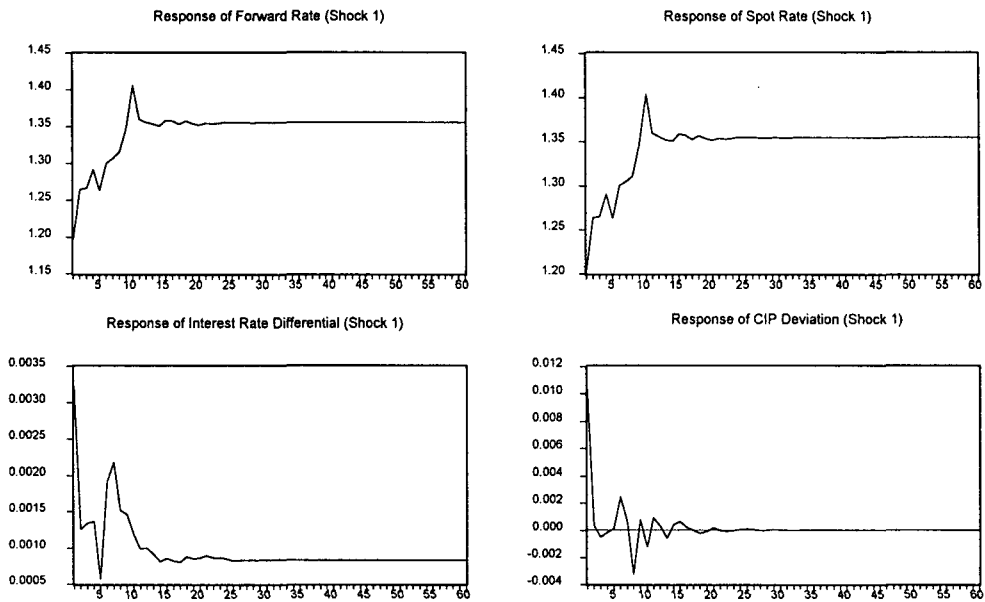
8) The value of 0.06% (positive and negative) will be used as a criteria to identify the CIP deviations. This value came from the transaction cost neutral zone investigated by Clinton (1988).

[Figure 1.3] Impulse Response Functions (Germany, Shock 3)

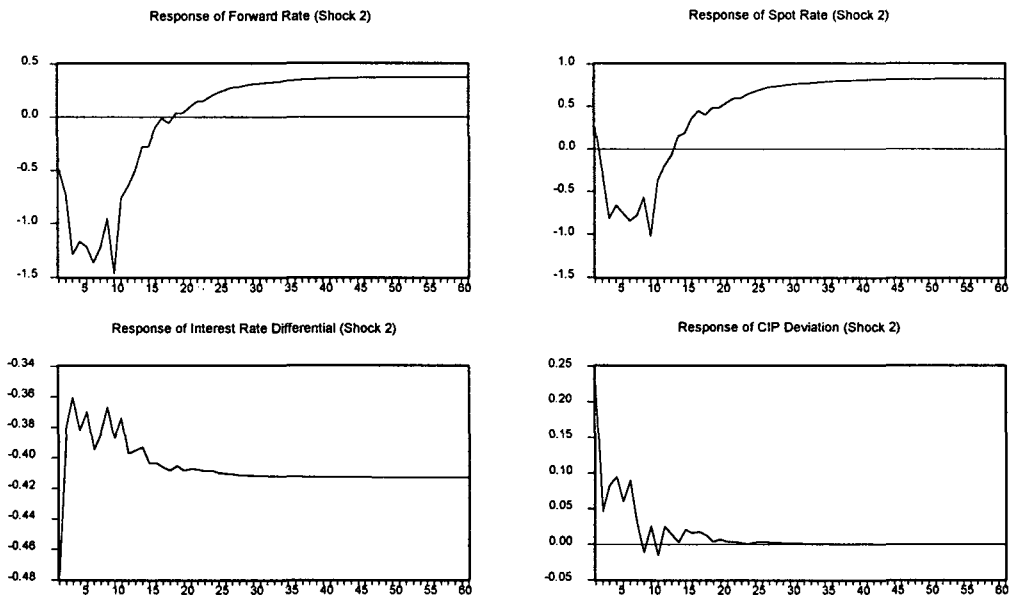


The IRFs for the U.K.-U.S. system are presented in figures 2.1 to 2.3. The three shocks, the shock 1, 2, and 3, can be defined same as in the previous system. The profitable CIP deviations exist in the response of the interest-rate shock, but they are eliminated in 2 days. Overshooting of the spot and forward rates and the liquidity effect are features of this system as well.

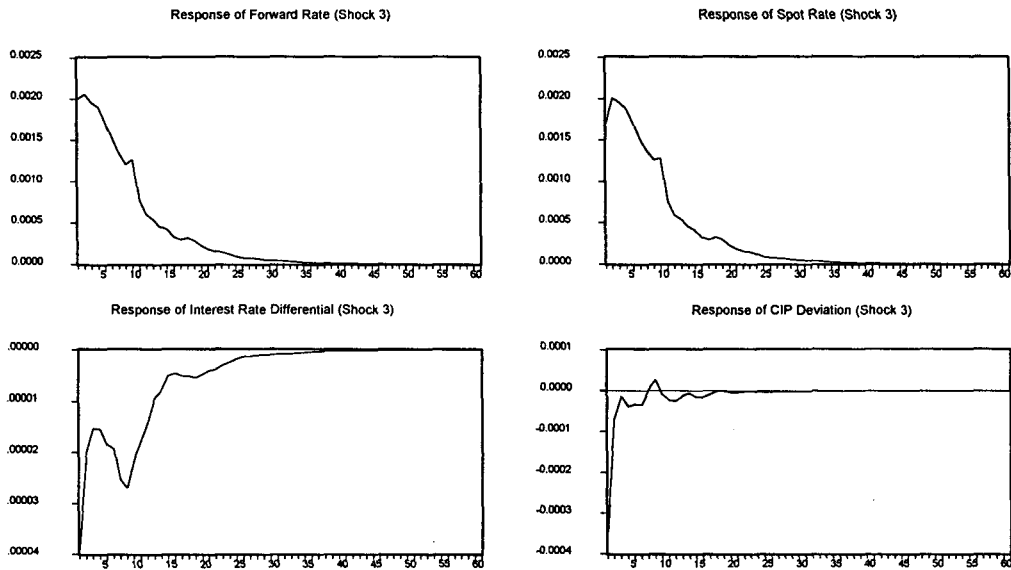
[Figure 2.1] Impulse Response Functions (U.K., Shock 1)



[Figure 2.2] Impulse Response Functions (U.K., Shock 2)

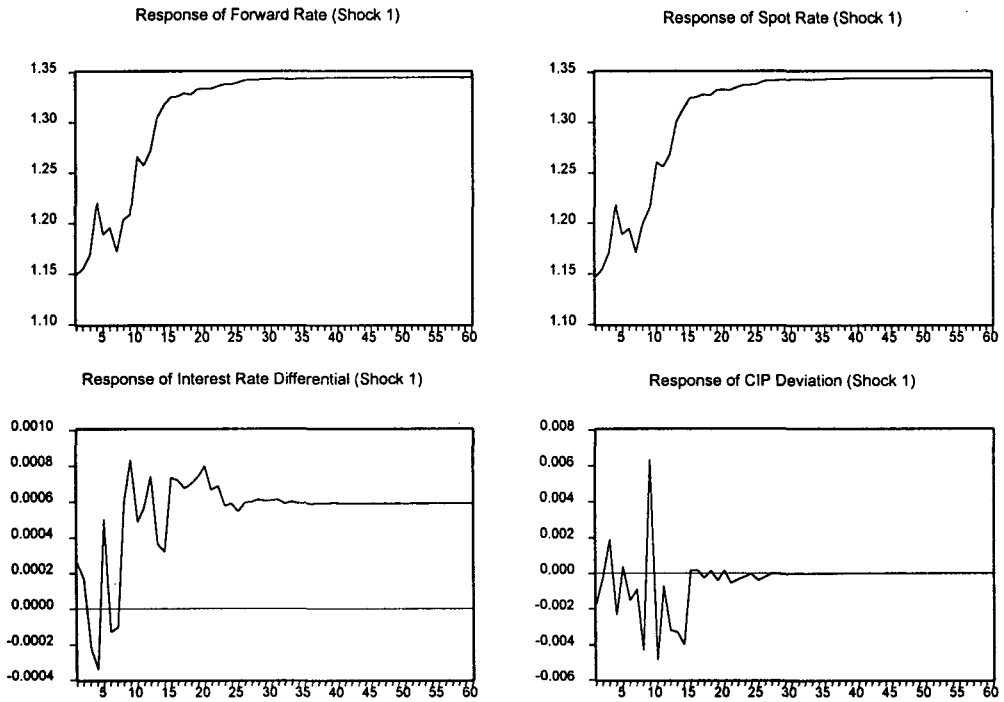


[Figure 2.3] Impulse Response Functions (U.K., Shock 3)

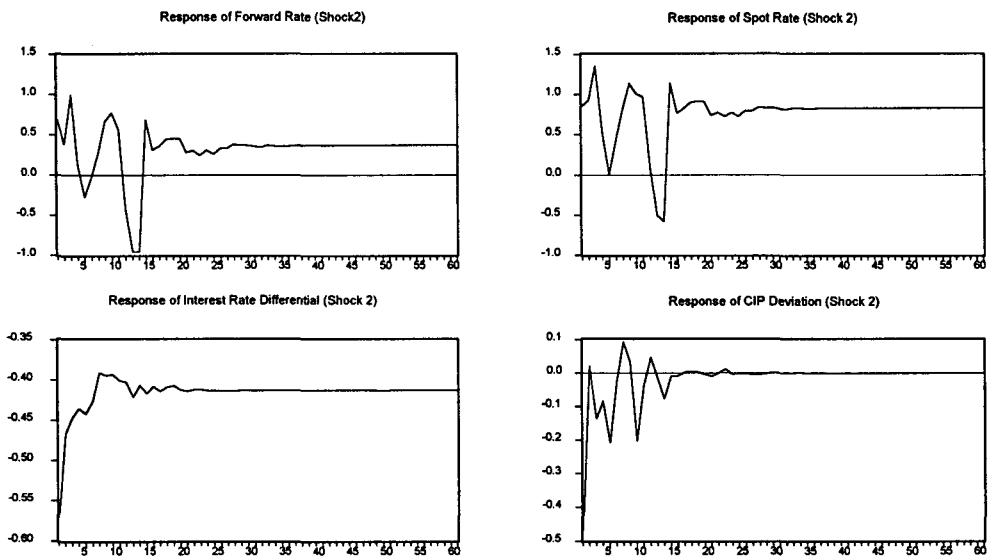


The IRFs for the Japan-U.S. system, presented in figures 3, and the Swiss-U.S. system, presented in figures 4, behave very much like the U.K.-U.S. system. The persistent, profitable deviations for 3 days are observed in Japanese system. In contrast, the Swiss system is relatively well behaved in that profitable CIP deviations are eliminated in one day.

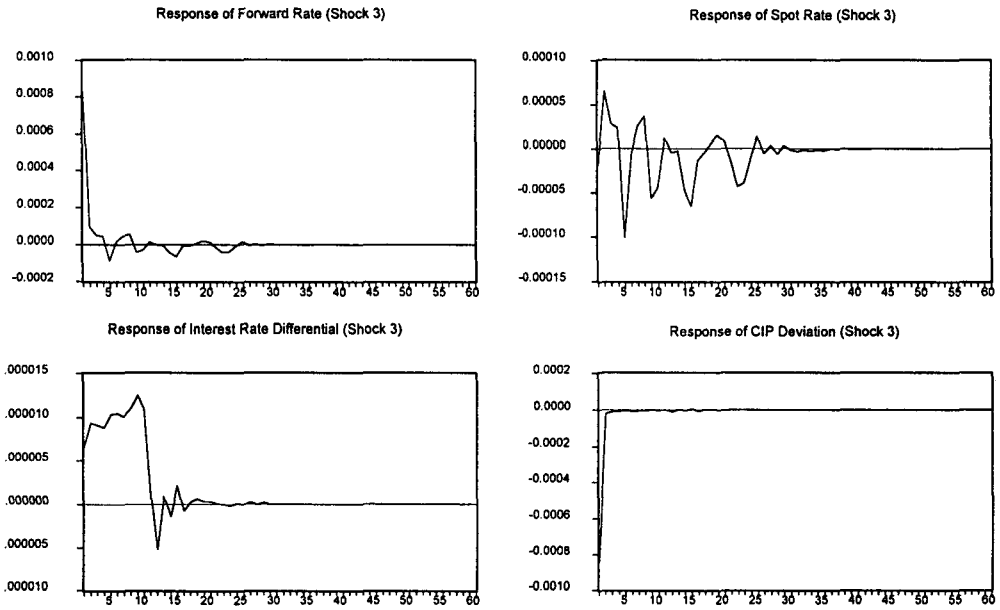
[Figure 3.1] Impulse Response Functions (Japan, Shock 1)



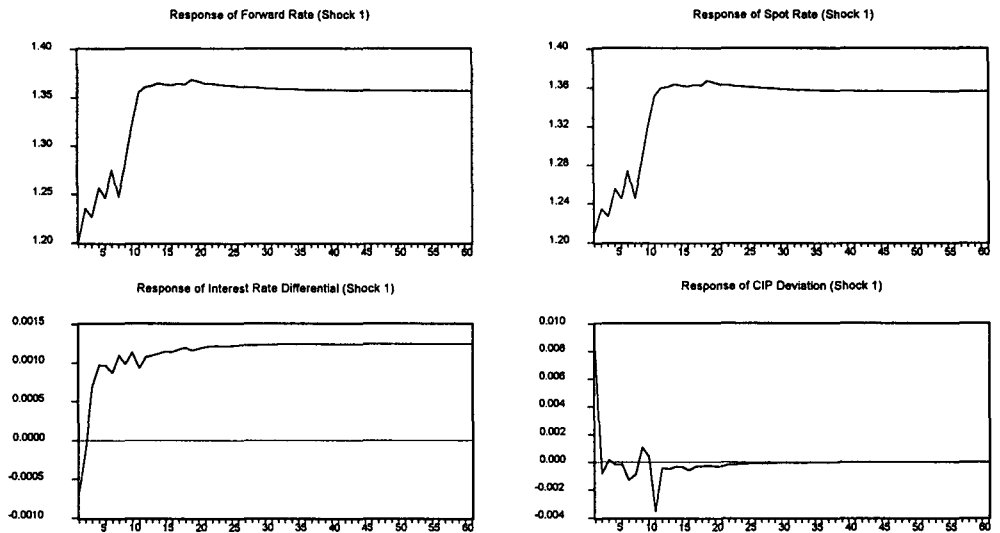
[Figure 3.2] Impulse Response Functions (Japan, Shock 2)



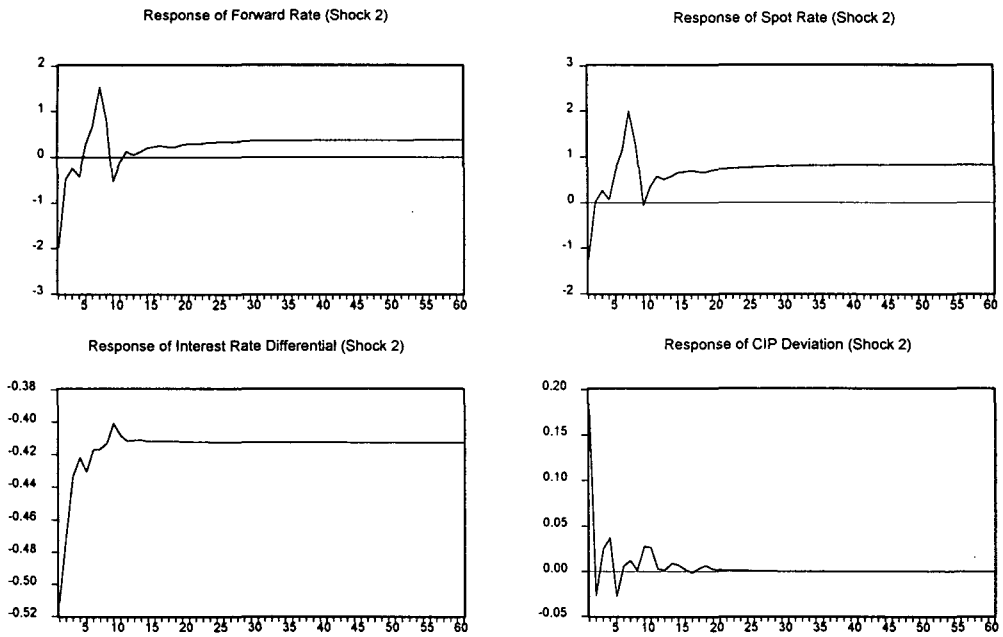
[Figure 3.3] Impulse Response Functions (Japan, Shock 3)



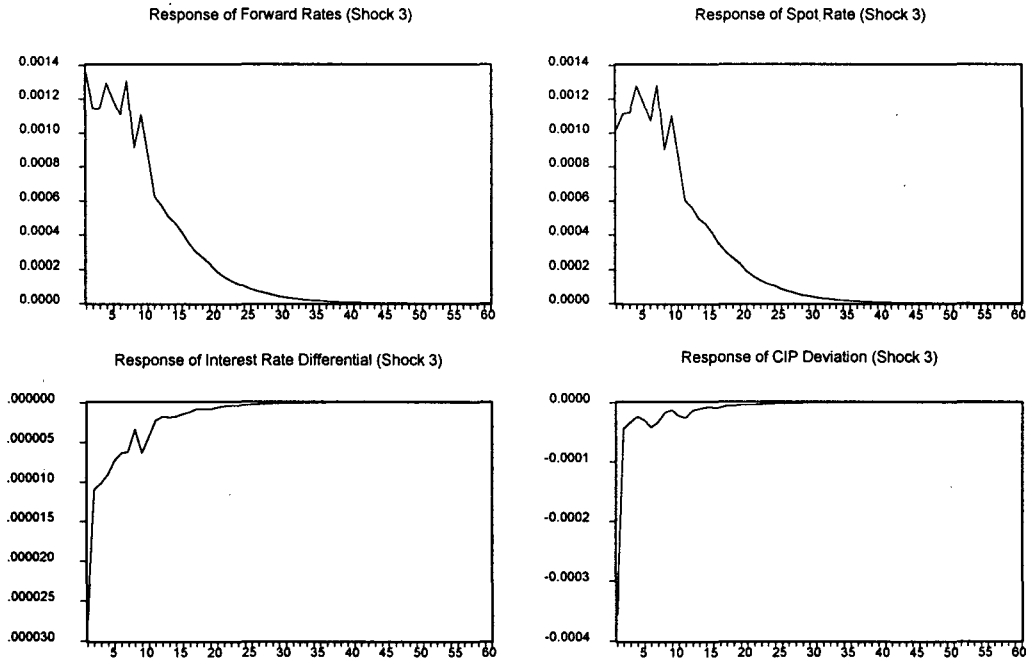
[Figure 4.1] Impulse Response Functions (Switzerland, Shock 1)



[Figure 4.2] Impulse Response Functions (Switzerland, Shock 2)



[Figure 4.3] Impulse Response Functions (Switzerland, Shock 3)



To summarize, the analysis of the impulse response functions reveals that, except for the German system, the exchange-rate innovations affect an immediate depreciation of the domestic exchange rates with a higher domestic interest rate, while the interest-rate shock in each system depreciate the domestic exchange rate accompanied with a lower domestic interest rate (or a higher foreign interest rate). The transitory money-supply shock shows a similar pattern with the interest-rate shock. The interest-rate shock and the money-supply shock share the same patterns. Exchange rates seem to be moving systematically, driven by each individual shock, except for the German system. This can be interpreted by nominal aspects of exchange rate determination. Monetary and portfolio-balance models of exchange rate determination capture these kinds of nominal factors. However, under the two permanent shocks (defined as exchange rate and interest rate shocks, respectively), the interest-rate differential is not the major determinant of exchange rates. This implies that the determinants of exchange rates to the exchange rate shock could be coming from different sources rather than the interest rate differential, which seems to be different from the view of McKinnon (1982). Hence, the monetary and portfolio-balance models, which emphasize nominal factors, may not capture the components of permanent shocks. The behavior of the IRFs, however, support the Dornbusch's (1976) sticky price model. All four industrialized nations under consideration show the short-run overshooting behavior of spot rates to each shock during the floating rate regime.

IV. Conclusion

In this paper, I analyze the empirical dynamics of the spot and forward exchange rates and the domestic and foreign interest-rate differentials with in

the equilibrium framework implied by the covered-interest parity relation. The existence of the cointegrating relationship in the system allows a long-run restriction to identify the structural model. This methodology allows fewer ad hoc restrictions than conventional statistical approaches.

The results show that exchange rates seem to respond to each individual shock. This is clearly interpreted as nominal aspects of exchange rate determination. Actually, monetary and portfolio-balance models of exchange rate determination may capture these kinds of nominal factors. However, the interest rate differential does not seem to be the major determinant of the exchange rate under the two permanent shocks. Specifically, the portfolio-balance models rely heavily on the role of the interest-rate differential to determine the exchange rate. Hence, the monetary and portfolio-balance models which emphasize that nominal factors may not capture the components of the permanent shocks. The results, however, still support Dornbusch's (1976) sticky price model, which relaxed the assumption of continuous PPP. All four industrialized nations under consideration show the short-run overshooting of spot rates to each shock during the floating rate regime. The short-run liquidity effects associated with changes of money supply are found.

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