FUZZY ALMOST PRECONTINUOUS MAPPINGS

G.I. Chae, S.S. Thakur and S. Singh

Abstract. The purpose of this paper is to introduce a new type of weakened fuzzy continuity called fuzzy almost precontinuous and investigate properties of it.

1. Preliminaries

Let $X$ be a set and $I$ be the closed unit interval $[0,1]$. In $[14]$, a fuzzy set $\mu \in X$ is defined to be a mapping $\mu : X \rightarrow I$ and we will denote it by $\mu \in \mathcal{I}^X$. The fuzzy null set 0 and the fuzzy whole set 1 $\in \mathcal{I}^X$ are fuzzy sets such that $0(x) = 0$ and $1(x) = 1$ for all $x \in X$, respectively. For a class $\{\lambda_\alpha \in \mathcal{I}^X : \alpha \in \Lambda\}$, the union $\bigcup_{\alpha \in \Lambda} \lambda_\alpha$ and the intersection $\bigwedge_{\alpha \in \Lambda} \lambda_\alpha$ are, respectively, defined by $\sup_{\alpha \in \Lambda} \{\lambda_\alpha\}$ and $\inf_{\alpha \in \Lambda} \{\lambda_\alpha\}$.

Let $\lambda, \mu \in \mathcal{I}^X$. Then $\lambda$ is said to be contained in $\mu$, denoted by $\lambda \leq \mu$, if $\lambda(x) \leq \mu(x)$ for every $x \in X$. The complement of $\lambda$, denoted by $1 - \lambda$, is defined by $(1 - \lambda)(x) = 1 - \lambda(x)$ for each $x \in X$. A fuzzy point $x_\beta$ of $X$ is a fuzzy set in $X$ which is taking the value 0 for all $y \in X$ except for $x$ and taking $\beta$ at $x$. A fuzzy point $x_\beta$ of $X$ is said to be contained in a fuzzy set $\lambda$, denoted by $x_\beta \in \lambda$, if $\beta \leq \lambda(x)$.

Definition 1.1. [9] Let $\lambda, \mu \in \mathcal{I}^X$. Then

1. A fuzzy point $x_\beta$ is said to be quasi-coincident with $\lambda$, denoted by $x_\beta \approx \lambda$, if $\beta + \lambda(x) > 1$.

2. $\lambda$ is said to be quasi-coincident with $\mu$, denoted by $\lambda \approx \mu$, if there exists a point $x \in X$ such that $\lambda(x) + \mu(x) > 1$.

Received February 12, 2000
Authors wish to acknowledge the financial support of University of Ulsan made in the program year of 1999.
Remark 1.1. It is shown in [9] that for any $\lambda$,

1. $\mu \in \mathcal{I}^X$, $\lambda \leq \mu$ if and only if $\lambda$ and $1 - \mu$ are not quasi-coincident,
2. $x_\beta \in \lambda$ if and only if $x_\beta$ is not quasi-coincident with $1 - \lambda$.

Let $f : X \to Y$ be a mapping and let $\lambda \in \mathcal{I}^X$, $\mu \in \mathcal{I}^Y$. Then $f(\lambda) \in \mathcal{I}^Y$ such that $f(\lambda)(y) = \bigvee_{x \in f^{-1}(y)} \lambda(x)$ if $f^{-1}(y) \neq \emptyset$ and 0, otherwise. And $f^{-1}(\mu) \in \mathcal{I}^X$ such that $f^{-1}(\mu)(x) = \mu(f(x))$ for all $x \in X$.

We use in this paper the definition of fuzzy topology on a set $X$ in the sense of [5], denote it by $\tau(X)$, and the ordered pair $(X, \tau(X))$ is called a fuzzy topological space (fts, for short). $\mu \in \tau(X)$ is called fuzzy open in $X$ and the complement $(1 - \mu)$ is called fuzzy closed in $X$.

Definition 1.2.

1. $\text{Int}(\lambda) = \bigvee \{\mu : \mu \leq \lambda, \mu \in \tau(X)\}$ is called the interior of $\lambda$.
2. $\text{Cl}(\lambda) = \bigwedge \{\mu : \lambda \leq \mu, (1 - \mu) \in \tau(X)\}$ is called the closure of $\lambda$.

Let $(X, \tau(X))$ be an fts. Then $\mu \in \mathcal{I}^X$ is called a Q-neighborhood (shortly, Q-nbd) of a fuzzy point $x_\beta$ [9] (resp. pre Q-nbd [8, 12]) if there exists a $\mu \in \tau(X)$ (resp. $\lambda \in \text{FPO}(X)$) such that $x_\beta \in \text{Q} \lambda \leq \mu$.

Definition 1.3. [1, 2, 10] Let $x$ be an fts. Then $\lambda \in \mathcal{I}^X$ is said to be:

1. fuzzy regular open if $\lambda = \text{Int}(\text{Cl}(\lambda))$,
2. fuzzy feebly open (≡ $\alpha$-open) if $\lambda \leq \text{Int}(\text{Cl}(\text{Int}(\lambda)))$,
3. fuzzy preopen if $\lambda \leq \text{Int}(\text{Cl}(\lambda))$,
4. fuzzy semi open if $\lambda \leq \text{Cl}(\text{Int}(\lambda))$,
5. fuzzy regular closed if $\lambda = \text{Cl}(\text{Int}(\lambda))$,
6. fuzzy feebly closed (≡ $\alpha$-closed) if $\text{Cl}(\text{Int}(\text{Cl}(\lambda))) \leq \lambda$,
7. fuzzy preclosed if $\text{Cl}(\text{Int}(\lambda)) \leq \lambda$,
8. fuzzy semi closed if $\text{Int}(\text{Cl}(\lambda)) \leq \lambda$.

In this paper, we will denote the family of all fuzzy open (resp. fuzzy regular open, fuzzy $\alpha$-open, fuzzy pre-open, fuzzy semi-open and fuzzy regular closed) sets in an fts $X$ by $\tau(X)$ (resp. $\text{FRO}(X)$, $\text{FRO}(X)$, $\text{FPO}(X)$, $\text{FSO}(X)$ and $\text{FRC}(X)$).
REMARK 1.2. For an fts \((X, \tau(X))\), the following holds:
\[
\text{FRO}(X) \subset \tau(X) \subset \text{F}_\alpha \text{O}(X) \subset \text{FPO}(X) \quad \text{(or, FSO}(X))
\]

DEFINITION 1.4. \([1, 2, 5, 10, 12]\) Let \(\lambda \in I^X\). Then
(1) \(\alpha\text{Int}(\lambda) = \bigvee \{\mu : \mu \leq \lambda, \mu \in \text{F}_\alpha \text{O}(X)\}\) is called the \(\alpha\)-interior of \(\lambda\) (feeble interior \(\text{Int}(\lambda)\)),
(2) \(p\text{Int}(\lambda) = \bigvee \{\mu : \mu \leq \lambda, \mu \in \text{FPO}(X)\}\) is called the preinterior of \(\lambda\),
(3) \(s\text{Int}(\lambda) = \bigvee \{\mu : \mu \leq \lambda, \mu \in \text{FSO}(X)\}\) is called the semi interior of \(\lambda\),
(4) \(\alpha\text{Cl}(\lambda) = \bigwedge \{\mu : \mu \leq \lambda, (1 - \mu) \in \text{F}_\alpha \text{O}(X)\}\) is called \(\alpha\)-closure of \(\lambda\) (feeble closure \(\text{Cl}(\lambda)\)),
(5) \(p\text{Cl}(\lambda) = \bigwedge \{\mu : \lambda \leq \mu, (1 - \mu) \in \text{FPO}(X)\}\) is called preclosure of \(\lambda\),
(6) \(s\text{Cl}(\lambda) = \bigwedge \{\mu : \lambda \leq \mu, (1 - \mu) \in \text{FSO}(X)\}\) is called semi closure of \(\lambda\).

REMARK 1.3. Let \(\lambda, \mu \in I^X\). Then
(1) \(\lambda \in \text{FPO}(X)\) if and only if for every fuzzy points \(x_\delta \in \lambda\), there exists \(\delta \in \text{FPO}(X)\) such that \(x_\delta \in \delta \leq \lambda\) \([11]\),
(2) \(\lambda \in \text{FPO}(X)\) if and only if \(\lambda = p\text{Int}(\lambda)\) \([12]\),
(3) \(1 - \lambda \in \text{FPO}(X)\) if and only if \(\lambda = p\text{Cl}(\lambda)\) \([12]\).

DEFINITION 1.5. \([1, 2, 4, 5, 8, 10, 11]\) A mapping \(f : X \rightarrow Y\) is said to be:
(1) fuzzy continuous if \(f^{-1}(\lambda) \in \tau(Y)\) for each \(\lambda \in \tau(Y)\)
(2) fuzzy feebly continuous (fuzzy \(\alpha\)-continuous, fuzzy strongly semi continuous) if \(f^{-1}(\lambda) \in \text{F}_\alpha \text{O}(X)\) for each \(\lambda \in \tau(Y)\).
(3) fuzzy precontinuous if \(f^{-1}(\lambda) \in \text{FPO}(X)\) for each \(\lambda \in \tau(Y)\).
(4) fuzzy M-precontinuous (fuzzy pre-irresolute) if \(f^{-1}(\lambda) \in \text{FPO}(X)\) for each \(\lambda \in \text{FPO}(Y)\),
(5) fuzzy almost continuous if \(f^{-1}(\lambda) \in \tau(X)\) for each \(\lambda \in \text{FRO}(Y)\).

REMARK 1.4. In the above Definition 1.5,
(1) fuzzy M-precontinuity of \([7]\) and fuzzy pre-irresoluteness of \([8]\) are the same mappings on any fts, and
fuzzy feeble continuity of [2] and fuzzy strongly semicontinuity of [10] are also the same mappings and in [4] it was also renamed by fuzzy \( \alpha \)-continuity. So, from now on, we will call it a fuzzy \( \alpha \)-continuous mapping.

2. Fuzzy almost precontinuous mappings

**Definition 2.1.** Let \( X \) and \( Y \) be fts's. A mapping \( f : X \to Y \) is said to be **fuzzy almost precontinuous** (written as \( f.a.p.C. \)) if \( f^{-1}(\lambda) \in \text{FPO}(X) \) for each \( \lambda \in \text{FRO}(Y) \).

**Remark 2.1.** Every fuzzy precontinuous and fuzzy almost continuous mapping are \( f.a.p.C. \). But the converses may not be true, as shown by the following examples.

**Example 2.1.** Let \( X = \{a, b\}, Y = \{x, y\} \) and let \( \lambda \in \mathcal{I}^{X}, \mu \in \mathcal{I}^{Y} \) such that \( \lambda(a) = 0.4, \lambda(b) = 0.3 \), and \( \mu(x) = 0.5, \mu(y) = 0.5 \). and let \( \tau(X) = \{0, \lambda, 1\} \) and \( \tau(Y) = \{0, \mu, 1\} \). Define \( f : (X, \tau(X)) \to (Y, \tau(Y)) \) by \( f(a) = x \) and \( f(b) = y \). Then \( f \) is \( f.a.p.C. \), but not fuzzy precontinuous.

**Example 2.2.** Let \( X = \{a, b\}, Y = \{x, y\} \) and let \( \lambda \in \mathcal{I}^{X}, \mu \in \mathcal{I}^{Y} \) such that \( \lambda(a) = 0.5, \lambda(b) = 0.4 \), and \( \mu(x) = 0.4, \mu(y) = 0.4 \). and let \( \tau(X) = \{0, \lambda, 1\} \) and \( \tau(Y) = \{0, \mu, 1\} \). Define \( g : (X, \tau(X)) \to (Y, \tau(Y)) \) by \( g(a) = x \) and \( g(b) = y \). Then \( g \) is \( f.a.p.C. \), but not fuzzy almost continuous.

In general, the composition of \( f.a.p.C. \) mappings may be not \( f.a.p.C. \), as shown by the following.

**Example 2.3.** Let \( f : (X, \tau(X)) \to (Y, \tau(Y)) \) be the mapping defined in Example 2.1, then \( f \) is \( f.a.p.C. \). Let \( Z = \{v, w\} \) and \( \eta \in \mathcal{I}^{Z} \) such that \( \eta(x) = 0.4, \eta(y) = 0.4 \) and let \( \tau(Z) = \{0, \eta, 1\} \). Define \( g : (Y, \tau(Y)) \to (Z, \tau(Z)) \) by \( g(x) = v \) and \( g(y) = w \). Then \( g \) is also \( f.a.p.C. \). However, the composition \( g \circ f : (X, \tau(X)) \to (Z, \tau(Z)) \) is not \( f.a.p.C. \), because \( \text{Int} \left( \text{Cl} \left( (g \circ f)^{-1}(\eta)) \right) \right) < (g \circ f)^{-1}(\eta) \) and thus \( (g \circ f)^{-1}(\eta) \notin \text{FPO}(X) \) for \( \eta \in \text{FRO}(Z) \). Note that \( f \) is not fuzzy precontinuous.
THEOREM 2.1. Let \( f : X \to Y \) is fuzzy precontinuous and \( g : Y \to Z \) is f.a.p.C., then \( g \circ f : X \to Z \) is f.a.p.C.

PROOF. Let \( \lambda \in \text{FRO}(Z) \). Then \( g^{-1}(\lambda) \in \tau(X) \), because \( g \) is f.a.p.C. Since \( f \) is fuzzy precontinuous, \( f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda) \in \text{FPO}(X) \). Hence \( g \circ f \) is f.a.p.C.

THEOREM 2.2. Let \( f : X \to Y \) is fuzzy \( M \)-precontinuous and \( g : Y \to Z \) is f.a.p.C., then \( g \circ f : X \to Z \) is f.a.p.C.

PROOF. Let \( \lambda \in \text{FRO}(Z) \), then \( f^{-1}(\lambda) \in \text{FPO}(Y) \). Thus \( f^{-1}(g^{-1}(\lambda)) \in \text{FPO}(X) \). Hence \( g \circ f \) is f.a.p.C.

THEOREM 2.3. Let \( f \) be a mapping from an fts \( X \) to an fts \( Y \), then the following are equivalent:

1. \( f \) is f.a.p.C.,
2. \( (1 - f^{-1}(\mu)) \in \text{FPO}(X) \) for each \( \mu \in \text{FRC}(Y) \),
3. \( f^{-1}(\lambda) \leq p\text{Int}(f^{-1}(\text{cl}(\lambda))) \) for each \( \lambda \in \tau(Y) \),
4. \( p\text{cl}(f^{-1}(\text{cl}(\mu))) \leq f^{-1}(\mu) \) for each \( (1 - \mu) \in \tau(Y) \),
5. for each fuzzy point \( x_\beta \) of \( X \) and \( \mu \in \text{FRO}(Y) \) containing \( f(x_\beta) \), there exists \( \lambda \in \text{FRO}(X) \) such that \( x_\beta \in \lambda \) and \( \lambda \leq f^{-1}(\mu) \),
6. for each fuzzy point \( x_\beta \) of \( X \) and \( \mu \in \text{FRO}(Y) \) containing \( f(x_\beta) \), there exists \( \lambda \in \text{FPO}(X) \) such that \( x_\beta \in \lambda \) and \( f(\lambda) \leq \mu \),
7. for each fuzzy point \( x_\beta \) of \( X \) and \( \mu \in \text{FRO}(Y) \) with \( f(x_\beta) \) containing \( \mu \), there exists \( \lambda \in \text{FPO}(X) \) such that \( x_\beta \in \lambda \) and \( f(\lambda) \leq \mu \),
8. for each fuzzy point \( x_\beta \) of \( X \) and \( \mu \in \text{FRO}(Y) \) with \( f(x_\beta) \) containing \( \mu \), there exists \( \lambda \in \text{FPO}(X) \) such that \( x_\beta \in \lambda \) and \( \lambda \leq f^{-1}(\mu) \).

PROOF. (1) \( \Rightarrow \) (2). The proofs are obvious.

(1) \( \Rightarrow \) (3). Let \( \lambda \in \tau(Y) \), then \( \lambda \leq \text{Int}(\text{cl}(\lambda)) \) and hence \( f^{-1}(\lambda) \leq f^{-1}(\text{Int}(\text{cl}(\lambda))) \). From [1, Theorem 5.6-(b)], \( \text{Int}(\text{cl}(\lambda)) \in \text{FRO}(Y) \). Thus \( f^{-1}(\text{Int}(\text{cl}(\lambda))) \in \text{FPO}(X) \) since \( f \) is f.a.p.C. So. \( f^{-1}(\lambda) \leq f^{-1}(\text{Int}(\text{cl}(\lambda))) = \text{pInt}(f^{-1}(\text{Int}(\text{cl}(\lambda)))) \) from Remark 1.3.

(3) \( \Rightarrow \) (1). Let \( \lambda \in \text{FRO}(Y) \), then \( f^{-1}(\lambda) \leq \text{pInt}(f^{-1}(\text{Int}(\text{cl}(\lambda)))) \) = \( \text{pInt}(f^{-1}(\lambda)) \) by (3). So \( f^{-1}(\lambda) = \text{pInt}(f^{-1}(\lambda)) \). Hence \( f^{-1}(\lambda) \in \text{FPO}(X) \).
(2) ⇒ (4): Let $1 - \mu \in \tau(Y)$, then $\text{Cl}(\mu) = \mu$. Thus $\text{Cl}(\text{Int}(\mu)) \leq \mu$ and so $f^{-1}(\text{Cl}(\text{Int}(\mu))) \leq f^{-1}(\mu)$. From [1, Theorem 5.6-(a)], $\text{Cl}(\text{Int}(\mu)) \in \text{FRC}(Y)$. So $(1 - f^{-1}(\text{Cl}(\text{Int}(\mu))) \in \text{FPO}(X)$ by (2). Thus $p\text{Cl}(f^{-1}(\text{Cl}(\text{Int}(\mu))) = f^{-1}(\text{Cl}(\text{Int}(\mu))) \leq f^{-1}(\mu)$.

(4) ⇒ (2): Let $\mu \in \text{FRC}(Y)$, then $p\text{Cl}(f^{-1}(\mu)) = p\text{Cl}(f^{-1}(\text{Cl}(\text{Int}(\mu)))) \leq f^{-1}(\mu)$. Thus $p\text{Cl}(f^{-1}(\mu)) = f^{-1}(\mu)$. So $(1 - f^{-1}(\mu)) \in \text{FPO}(X)$.

(1) ⇒ (5): Let $x_\beta$ be a fuzzy point of $X$ and $\mu \in \text{FRO}(Y)$ with $f(x_\beta) \in \mu$. Putting $\lambda = f^{-1}(\mu)$, then by (1) $\lambda \in \text{FPO}(X)$, $x_\beta \in \lambda$ and $\lambda \leq f^{-1}(\mu)$.

(5) ⇒ (6): Let $x_\beta$ be a fuzzy point of $X$ and $\mu \in \text{FRO}(Y)$ containing $f(x_\beta)$. Then by (5) there exists $\lambda \in \text{FPO}(X)$ such that $x_\beta \in \lambda$ and $\lambda \leq f^{-1}(\mu)$. So $x_\beta \in \lambda$, $f(\lambda) \leq f(f^{-1}(\mu)) \leq \mu$.

(6) ⇒ (1): Let $\mu \in \text{FRO}(Y)$ and let $x_\beta$ be a fuzzy point of $X$ such that $x_\beta \in f^{-1}(\mu)$. Then $f(x_\beta) \in f(f^{-1}(\mu)) \leq \mu$. So by (6) there exists $\lambda \in \text{FPO}(X)$ such that $x_\beta \in \lambda$ and $f(\lambda) \leq \mu$, that is, $x_\beta \in \lambda \leq f^{-1}(\mu)$. Thus by Remark 1.3-(1), $f^{-1}(\mu) \in \text{FPO}(X)$. So $f$ is f.a.p.C.

(1) ⇒ (7): Let $x_\beta$ be a fuzzy point of $X$ and $\mu \in \text{FRO}(Y)$ such that $f(x_\beta) \text{ Q } \mu$. Then $f^{-1}(\mu) \in \text{FPO}(X)$ by (1) and $x_\beta \text{ Q } f^{-1}(\mu)$. Taking $\lambda = f^{-1}(\mu)$, then $\lambda \in \text{FPO}(X)$, $x_\beta \text{ Q } \lambda$ and $f(\lambda) = f(f^{-1}(\mu)) \leq \mu$.

(7) ⇒ (8): Let $x_\beta$ be a fuzzy point of $X$ and $\mu \in \text{FRO}(Y)$ such that $f(x_\beta) \text{ Q } \mu$, then by (7) there exists $\lambda \in \text{FPO}(X)$ such that $x_\beta \text{ Q } \lambda$ and $f(\lambda) \leq \mu$. Thus we have $\lambda \leq f^{-1}(f(\lambda)) \leq f^{-1}(\mu)$.

(8) ⇒ (1): Let $\mu \in \text{FRO}(Y)$. To show $f^{-1}(\mu) \in \text{FPO}(X)$ we use Remark 1.3-(1). Let $x_\beta$ be a fuzzy point of $X$ such that $x_\beta \in f^{-1}(\mu)$. Then $f(x_\beta) \in \mu$. Choosing the fuzzy point $x_{(1-\beta)}$, then $f(x_{(1-\beta)}) \text{ Q } \mu$. So by (8) there exists $\delta \in \text{FPO}(X)$ such that $x_{(1-\beta)} \text{ Q } \delta$ and $f(\delta) \leq \mu$. Now $x_{(1-\beta)} \text{ Q } \delta$ implies that $x_{(1-\beta)}(x) + \lambda(x) = 1 - \beta + \mu(x) > 1$. It follows that $x_\beta \in \lambda \leq f^{-1}(\mu)$. So by Remark 1.3-(1), $f^{-1}(\mu) \in \text{FPO}(X)$. Thus $f$ is f.a.p.C.

**Definition 2.2.** [6] An fts $X$ said to be fuzzy semi regular if for each $\lambda \in \tau(X)$ and for each fuzzy point $x_\beta$ of $X$ with $x_\beta \text{ Q } \lambda$, there exists $\mu \in \tau(X)$ such that $x_\beta \text{ Q } \mu$ and $\mu \leq \text{Int}(\text{Cl}(\mu)) \leq \lambda$.

**Theorem 2.4.** Let $f : X \rightarrow Y$ be a mapping from an fts $X$ to a fuzzy semi regular space $Y$, then $f$ is f.a.p.C.s if and only if $f$ is fuzzy precontinuous.
FUZZY ALMOST PRECONTINUOUS MAPPINGS

**PROOF.** Necessity: Let \( x_\beta \) be a fuzzy point of \( X \) and \( \lambda \in \tau(Y) \) such that \( f(x_\beta) \cap \lambda \). Since \( Y \) is fuzzy semi regular, there exists \( \mu \in \tau(Y) \) such that \( f(x_\beta) \cap \mu \) and \( \mu \leq \text{Int(Cl}(\mu)) \leq \lambda \). Since \( \text{Int(Cl}(\mu)) \in \text{FRO}(Y) \) and \( f \) is \( \text{f.a.p.C.} \), by Theorem 2.3-(7) there exists \( \mu_1 \in \text{FPO}(X) \) such that \( x_\beta \cap \mu_1 \) and \( f(\mu_1) \leq \text{Int(Cl}(\mu)) \). Thus \( \mu_1 \in \text{FPO}(X) \) such that \( x_\beta \cap \mu \) and \( f(\mu_1) \leq \lambda \). So by [11, Theorem 3.4] \( f \) is fuzzy precontinuous.

Sufficiency is obvious and is thus omitted.

**Theorem 2.5.** Let \( f \) be a mapping from an fts \( X \) to an fts \( Y \). If the graph mapping \( G_f : X \to X \times Y \) of \( f \) is \( \text{f.a.p.C.} \), then \( f \) is \( \text{f.a.p.C.} \).

**Proof.** Let \( \mu \in \text{FRO}(Y) \), then \( f^{-1}(\mu) = 1 \cap f^{-1}(\mu) = (G_f)^{-1}(1 \times \mu) \). Since \( 1 \times \mu = 1 \times \text{Int(Cl}(\mu)) = \text{Int}(1 \times \text{Cl}(\mu)) = \text{Int}(\text{Cl}(1 \times \mu)) \), \( 1 \times \mu \in \text{FRO}(X \times Y) \). Since \( G_f \) is \( \text{f.a.p.C.} \), \( f^{-1}(\mu) = (G_f)^{-1}(1 \times \mu) \in \text{FPO}(X) \). Hence \( f \) is \( \text{f.a.p.C.} \).

From the above, we have the following implication diagram:

\[
\begin{array}{ccc}
\text{f.m.p.C.} & \Rightarrow & \text{f.p.C.} \\
\downarrow & & \Rightarrow \\
\text{f.a.C.} & & \text{f.a.p.C.} \\
\uparrow & & \uparrow \\
\text{f.C.} & \Rightarrow & \text{f.a.C.}
\end{array}
\]

where \( \text{f.m.p.C.} \), \( \text{f.a.C.} \), \( \text{f.p.C.} \), \( \text{f.a.p.C.} \), \( \text{f.C.} \) and \( \text{f.a.C.} \) denote fuzzy \( M \)-precontinuous [7, 8], fuzzy \( \alpha \)-continuous [2, 4], fuzzy precontinuous [10], fuzzy almost precontinuous, fuzzy continuous [5] and fuzzy almost continuous [1].

**References**


[12] S S Thakur and S. Singh, Fuzzy M-precontinuous mappings,

G. I Chae
Dept. of Mathematics
University of Ulsan
Ulsan 680-749, Korea
E-mail: gichae@uou.ulsan.ac.kr

S. S. Thakur and S. Singh
Dept. of Applied Math.
Govt Engin. Coll.
Jabalpur 482011, India