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불확실한 제품 수명주기를 고려한 최적가격결정 모형에 관한 연구*

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Optimal Pricing Policy under Uncertain Product Lifetimes*

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■ Abstract ■

Many studies in marketing and economics have attempted to model price and sales path under the dynamic diffusion process. Most of these models have been based on a fixed product lifetime. The current business climate requiring intensive development of new products, however, affects the diffusion of new products and their lifetime. Many products have not enjoyed the expected life cycle at the launching stage due to intense technical development, competitive reactions, and financial problems. Most diffusion models, however, have not taken account of the lifetime uncertainty of new product. If the products do not last over the planning horizon set by those models, the optimal price derived from them could be futile. Therefore, we had better take such lifetime uncertainty into consideration when developing diffusion models. In this paper, we study the impact of uncertain product lifetime on its optimal pricing path in non-competitive market. We develop an optimal pricing model under uncertain product lifetimes and conduct a simulation study to investigate their effects on the optimal pricing and corresponding sales paths. The simulation study provides some interesting findings on optimal pricing policy under uncertain product lifetime. This study could be a stepping stone for the further extended study of optimal pricing strategy with uncertain product lifetime.

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1. Introduction

Many studies in marketing and economics have attempted to model price and sales path under the dynamic diffusion process [22]. The underlying behavioral theory in the development of these models is that innovation is first adopted by a few innovators, and it spreads like an epidemic, where non-adopters are influenced by adopters as they contact with them via word-of-mouth [25]. Since the Bass model [1], many diffusion models have been introduced to incorporate such marketing variables as optimal price [24, 2, 6, 14], advertisement [7, 21, 27], promotion [20], and also competition [10, 13, 3]. Most of these models have been based on a fixed product lifetime.

The current business climate requiring intensive development of new products affects the diffusion of new products and their lifetime. Many products have not enjoyed the expected life cycle at the launching stage due to intense technical development, competitive reactions, and financial problems. Most diffusion models, however, have not taken account of the lifetime uncertainty of new product. In reality, product diffusion and lifetime are uncertain. If the products do not last over the planning horizon set by those models, the optimal price derived from them could be futile.

The uncertainty of product lifetime is especially high in high-tech products such as computers, software, networking products, and advanced telecommunication services. In the process of their sales development, the products have often been substituted by new ones. In general, the diffusion of most new products is disrupted by the advents of the better products. Therefore, it could be futile to have the optimal price and sales paths of new products without considering such lifetime uncertainty. Uncertainty of new product lifetime may prefer the higher pricing to secure a certain amount of profit at the early stage. In sum, the pricing strategy under uncertain lifetime should be different from the ones with a lifetime fixed.

In this paper, we study the impact of uncertain product lifetime on its optimal pricing path in non-competitive market. We develop an optimal pricing model under uncertain product lifetimes and conduct a simulation study to investigate their effects on the optimal pricing and corresponding sales paths.

The paper is structured as follows. In section 2, we formulate the optimal pricing model with the product lifetime function, in which lifetime uncertainty is expressed by the probability function. In section 3, we conduct a simulation study over the model with a specific sales function and four different lifetime probability distributions. In conclusion, we provide some interesting simulation results and suggest future research extensions.

2. Model Formulation

A typical diffusion model includes three factors: cost reduction by learning curve, demand functions, and interest rates. Suppose a new product diffuse before the market saturates. Let $x(t)$ be the cumulative sales by time t. The incremental sales from t to t+1 is $\Delta x = x(t+1) - x(t)$. For a continuous approximation, it becomes the time derivative of $x(t)$. It is then assumed to be a function of product price $p(t)$ and its cumulative sales $x(t)$: $s(t) = dx(t)/dt = f(p(t), x(t))$. The unit production cost $c(t)$, such as a marginal cost at time t, is assumed to be only a function of cumulated sales x(t). It decreases due to learning effect: $c(t) = c(x(t))$, $dc(x)/dx < 0$ [6, 14].

Sales normally decreases as the price increases, $ds(x, p; t)/dp < 0$. The diffusion effect ds(t)/dx, is the effect of the previous cumulative sales on the current sales. It depends on the product types. It is known to be more distinctive in durable goods and could also vary over during the product lifetime. For example, the diffusion effect of so-called 'good' product is positive at the initial stage, but gets negative as the market saturates.

The problem is to find an optimal pricing strategy p*(t), which maximizes discounted profits over the product planning horizon. Let *r* be the discount rate, and *T* the fixed product lifetime. The sum of discounted profit $\pi(t)$ becomes

$$\pi(T) = \int_0^T e^{-rt}[p(t) - c(x(t))]s(t)dt. \quad (1)$$

Then the optimal pricing problem can be represented as follows:

$$Max_{p(t)}\pi(T) = \int_0^T e^{-rt}[p(t) - c(x(t))]s(t)dt$$
$$s.t. \quad s(t) = f(x(t), p(t)); \quad x(0) = x_0. \quad (2)$$

Thus the solution of equation *p*(t)* is the optimal pricing strategy over a fixed lifetime horizon [0, T]. The product lifetime, however, might not be fixed. We might have some probabilistic information on the product lifetime.

Let G(t) be the cumulative probability function that a product leaves market by time t. Then G'(t) is the probability density function that a product discontinued at time t. Let T be the upper bound of product lifetime. So G(T) = 1. $1 - G(t) = \int_t^T G'(s)ds$ implies the probability that product remains in the market by time t. The total profit by time t will be the function of price, cost and sales along with G'(t). Then we can rewrite the new problem with the time discount effect as follows:

$$\text{Max}_{p(t)}\pi(t) = \text{Max}_{p(t)}\int_0^T G'(t)\Big[\int_0^T e^{-r\tau}[p(\tau) - c(x(\tau))]s(\tau)d\tau\Big]dt \quad (3)$$
$$s.t. \quad s(t) = f(x(t), p(t)); \quad x(0) = x_0.$$

By integration by parts, the above objective function is rephrased as follows:

$$\text{Max}_{p(t)}\pi(t) = \text{Max}_{p(t)}\int_0^T (1 - G(t))\Big[\int_0^T e^{-r\tau}[p(\tau) - c(x(\tau))]s(\tau)d\tau\Big]dt \quad (4)$$
$$s.t. \quad s(t) = f(x(t), p(t)); \quad x(0) = x_0.$$

Accordingly, the optimal price maximizes the integral sum of discounted profits over the product lifetime horizon.

Let $\lambda(t)$ be the shadow price. It shows the net profit increment due to the additional unit sales at time t, which provides the information about future sales effect. For example, additional sales at time t affects the future profits (positively or negatively) according to the sign (+, -) of $\lambda(t)$. We relax the above constraint by adding the discounted shadow price, $e^{-rt}[1 - G(t)]\lambda(t)$ to the objective function. Then we have following Hamiltonian function,

$$H(p, x, \lambda; t) = e^{-rt}[1 - G(t)][(p(t) - c(x(t)) + \lambda(t)]f(x(t), p(t)). \quad (5)$$

In order to maximize the Hamiltonian, we have to satisfy both conditions of (6) and (10), and x(0) = x_0. From the first condition of $\partial H/\partial p = 0$, optimal price p*(t) can be derived as follows:

$$\frac{\partial H}{\partial p} = 0 \tag{6}$$

$$\Rightarrow p*(t) = c(x(t)) - \lambda(t) - \frac{f(x,p;t)}{\partial f(x,p;t)/\partial p(t)} \tag{7}$$

$$= \frac{\eta}{\eta-1}\left[c(x(t)) - \lambda(t)\right], \tag{8}$$

where η is the elasticity of demand

$$\eta_{p=p^*} = -\frac{dx'}{dp}\frac{p}{x'}. \tag{9}$$

$$-\frac{\partial H}{\partial x} = \lambda'(t) \tag{10}$$

The second condition (10) is rephrased as follows:

$$\lambda'(t) = \left[r + \frac{G'(t)}{1-G(t)}\right]\lambda(t) + c'f + \frac{ff_x}{f_p}, \tag{11}$$

$$= \left[r + \frac{G'(t)}{1-G(t)}\right]\lambda(t) + c'f - \frac{f_x p}{\eta}, \tag{12}$$

where $f_x = \partial f/\partial x$, $f_p = \partial f/\partial p$.

The risk premium, r + G'(t)/(1-G(t)), is composed of both the interest rate and so-called '*hazard rate*'.

The solution of the above differential equation is

$$\lambda^*(t) = e^{\int_t^T\left(r+\frac{G'(\nu)}{1-G(\nu)}\right)d\nu}\int_t^T\left(c'f + \frac{ff_x}{f_p}\right) e^{-\int_t^T\left(r+\frac{G'(\nu)}{1-G(\nu)}\right)d\nu}d\tau + Ce\int_t^T\left(r + \frac{G'(\nu)}{1-G(\nu)}\right)d\nu, \tag{13}$$

$$= e^{\int_t^T\left(r+\frac{G'(\nu)}{1-G(\nu)}\right)d\nu}\int_t^T\left(c'f + \frac{f_x p}{\eta}\right) e^{-\int_t^T\left(r+\frac{G'(\nu)}{1-G(\nu)}\right)d\nu}ds + Ce\int_t^T\left(r + \frac{G'(\nu)}{1-G(\nu)}\right)d\nu, \tag{14}$$

where C is the constant determined by x_0. The shadow price $\lambda^*(t)$ becomes a function of cost, diffusion, price, and interest rate. The hazard rate reflecting the uncertainty of product lifetime also influences $\lambda^*(t)$. From equation (8), the optimal price p*(t) becomes

$$p^*(t) = \frac{\eta}{\eta-1}\left[c(x(t)) - e^{\int_t^T\left(r+\frac{G'(\nu)}{1-G(\nu)}\right)d\nu}\int_t^T\left(c'f + \frac{ff_x}{f_p}\right)e^{\int_\tau^T\left(r+\frac{G'(\nu)}{1-G(\nu)}\right)d\nu}d\tau\right] \tag{15}$$

In fact, the above solution p*(t) is compounded with diffusion effect f_x and pricing effect f_p, c(x(t)) reflecting learning effect, and also G(t) constructing risk premium r + G'(t)/(1-G(t)). It is difficult to derive intuitions about the dynamic relationship between optimal price and those factors. Thus we need to conduct a simulation study.

3. Simulation Study

3.1 Simulation Models and Assumptions

To investigate the effect of optimal pricing policy under uncertain product lifetime, we redefine the general equation (15) into a concrete functional form with specific parameters. We first assume the cost function c(t) have only the learning effect. That is, as the unit cost c(t) decreases along with the increase of x(t) from a initial investment, it can be represented as a functional form of c(t) = $k - a(x(t)+c)^b$, where $k >> 0$, $a > 0$, $b > 0$.

As for the sales function s(t), we could classify it into four different types. A sales function could depend either only on the price, s(t) = f(p(t)), or only on diffusion effect like the original Bass model, s(t) = f(x(t)). It could also depend on both, such as either a separable multiplicative form s(t) =

f(p(t))g(x(t)) [6], or a combined one, s(t) = f(p(t), x(t)). We choose a separable multiplicative sales function, such as $s(p, x; t) = \alpha \mathbf{p}(t-1)^{\beta}\mathbf{x}(t-1)^{\gamma}$, $\alpha > 0$. Here, β and γ represent the effects of price and diffusion respectively. The reason why we exploit the multiplicative form of sales function is mostly due to analytic convenience. In fact, a separable sales function of cumulative sales x(t) and price p(t) provide the better intuition of each effect by controlling the other. In similar context, Kalish(1983) also provided additional reasoning for using the separable sales functions (Kalish, Marketing science, 1983, P 142-143)[1]. In general, $\beta < 0$ and γ varies according to the types of diffusion effect (i.e., $\gamma > 0$ when there is a positive diffusion effect and vice versa). It accommodates the first two simple cases by adjusting the parameters.

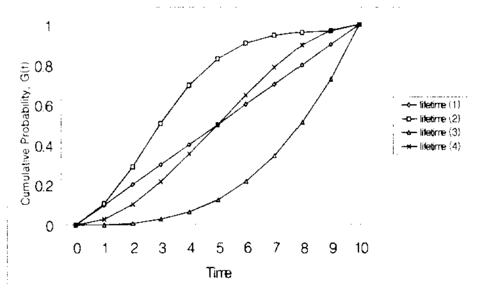
The uncertainty of product lifetime could be captured into four different cumulative probability functions (G(t)) of product lifetime. First, they are all the probability density function which are positive through their range, and also the sums of their values are 1's. For the comparison purpose, three consistent probability density functions and one transitional function are chosen for distinctive representations of product lifetime.

Based upon the characteristics of probability density function (pdf, the first derivative of G(t)), their specific mathematical functions are exemplified in <Table 1>. Type (1) function represents the case of uniform pdf function, i.e. the pdf is constant. The slopes of type (2) and type (3) are rather consistently decreasing and increasing respectively throughout the product lifetime period. However, type (4) typifies the case that the pdf dynamically varies from increasing to decreasing. The different features of the product lifetime cdf G(t) are shown in [Figure 1].

〈Table 1〉 Four Functional Types of G(t)

	G(t)	G'(t)	α-parameter
Type 1	t/T	1/T	
Type 2*	$\sum_{t=0}^{T} \frac{e^{-\lambda}\lambda^t}{t!}$	$\frac{e^{-\lambda}\lambda^t}{t!}$	3.5
Type 3	$\left(\frac{t}{T}\right)^{\alpha}$	$\alpha * \frac{t^{\alpha-1}}{T^{\alpha}}$	3
Type 4	$\frac{\alpha * t^2\left(\left(1+\frac{1}{\alpha}\right) * T - t\right)}{T^3}$	$\frac{2*(\alpha+1)*T*t-3*\alpha*t^2)}{T^3}$	2

• Note: Discrete Poisson function is chosen for Type 2 representing the continuously decreasing pdf.. In order to adjust the discrete probability function to the continuous case, we set that G(0) = 0, G(T) = 1. For the purpose of simulation, its difference (discrete or continuous) does not matter.



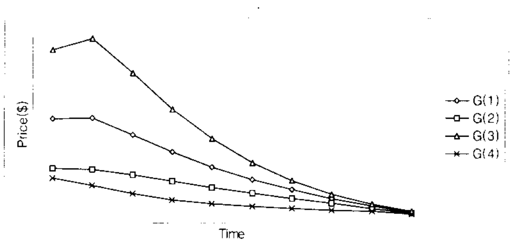
[Figure 1] Cumulative Probability Functions of Product Lifetime

3.2 Results and Analyses

Our simulation study compares the optimal pricing strategies under different product lifetime probabilities. Let's first look at the positive diffusion situation as representatively shown in [Figure 2]. We summarize some interesting results

1) Kalish(1983) said that separable sales functions were naturally simple way to model the interaction between price and experience, and several previous studies of pricing over the life cycle had used such function.

as follows. First, the optimal price varies over the planned horizon depending on the probability distribution of product lifetime. When the pdf's of product lifetime are fixed or consistent (increasing or decreasing) over a planned horizon (i.e., the cases (1), (2), (3)), their prices are higher than that of the case (4). The pdf of case (4) varies from increasing to decreasing during the product lifetime period. In other words, not only the knowledge of product lifetime T is relevant, but the probability function of product lifetime also matters.

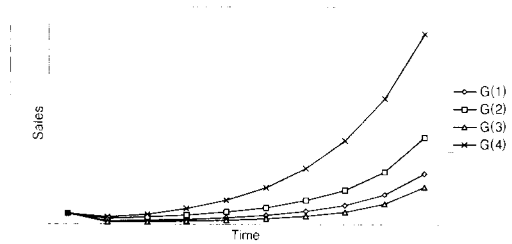


[Figure 2] Optimal Pricing Policy under Positive Diffusion Effect

Second, the more likely the product survives at the initial stage, the higher optimal pricing for the product could be preferable. It means that if you believe your product is more likely to survive at the launching stage, you had better set the higher price to maximize your dynamic profit. This can be applied, however, only to the consistent pdf cases. When pdf is not consistent throughout the lifetime as in case (4), it could be more difficult to expect the future. In that case, you could not set such a high price as in the consistent case.

Third, the optimal price decreases over the planning horizon. The higher the initial price, the more rapidly the price decreases. At the final stage, all the prices converge on the price which is the lower than the launching prices. It implies that the uncertainty of production lifetime have the significant effect rather at the early stage.

Fourth, the trend of corresponding sales for optimal pricing consistently depends on the price level as shown in [Figure 3]. That is, the higher pricing in the case of positive diffusion consistently affects the decrease of sales.

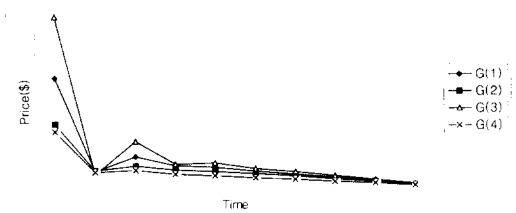


[Figure 3] Corresponding Incremental Sales under Positive Diffusion Effect

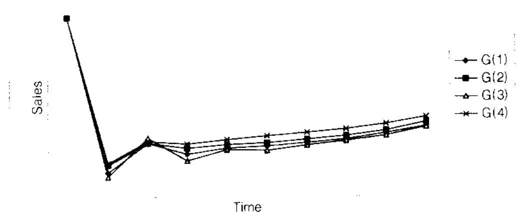
Now, we see the effect of product lifetime probability under the negative diffusion. The probability distribution of product lifetime also matters as shown in [Figure 4]. We summarize the findings as follows. First, the optimal prices get bounced at the earlier stage. The launching prices are determined by both pdf's type and its degree. The presence of negative diffusion (such as 'bad mouth') imposes a drastic decrease of the price.

Second, the effect of uncertain product lifetime is smaller compared to that of positive diffusion. In fact, the bad word-of-mouth reduces the uncertainty effect of product lifetime on price because negative diffusion affects the sales in a decreasing way. However, the effect of product lifetime probability still remains in a consistent manner.

Third, the corresponding sales show rather stagnant as shown [Figure 5] in accordance with the less fluctuating prices. Its degree depends on the functional balance between pricing effect and diffusion one. Our case shows the diffusion effect is relatively dominant.



[Figure 4] Optimal Pricing Policy under Negative Diffusion Effect



[Figure 5] Corresponding Incremental Sales under Negative Diffusion Effect

In our model for simulation study, the cost function is assumed only for learning effect. The cost thus depends only on the sales amount; the more sales, the less cost. That is, the dynamic trend of the cost does not include other information. The corresponding profit directly depends on both prices and sales.

4. Conclusion

In the previous studies, the lifetime of a certain product is assumed fixed whether it is known or not. In this paper, we discuss the case that the product lifetime is a probability function over the lifetime horizon. Since mathematical approach has a limit to get a closed form solution, however, we conduct a simulation study.

The simulations provide some interesting findings on optimal pricing policy under uncertain product lifetime. First, the uncertainty of product lifetime is a significant factor in optimal pricing. With uncertainty of product lifetime, the traditional optimal pricing strategy might be no longer appropriate. The higher pricing strategy is recommended as the product is more likely to survive at the initial stage. However, if the product tends to die away at the beginning stage, we had better keep the price lower. It would prolong the product lifetime to maximize the profit. The lower pricing policy might not attract the new entries and withhold additional investment due to the low profit. Accordingly, the company would extend its product life by sustaining the low competition.

Second, the functional form of pdf is significant. The functional type of pdf reflecting its transitionality is an another important factor to affect optimal pricing strategy. When the probability distribution varies in a monotonic manner (i.e., increasing or decreasing over the planning horizon), we had better set the higher price at the initial stage. Otherwise, the relatively lower price can be chosen.

Third, the effect of uncertain product lifetime on the price is more distinctive under the positive diffusion. In the negative diffusion, the effect of lifetime uncertainty on pricing gets considerably reduced due to the shrinking diffusion itself. Thus, when we expect the good word-of-mouth of a product, we should take the uncertainty of product lifetime into more serious considerations.

In sum, this study investigates the possible implication of uncertainty on product lifetime. The optimal pricing strategy should be more considerately decided especially on the high-tech products that are increasingly threatened by new technology and high competition. We hope our study could be a contribution to a series of interesting research on optimal pricing strategy. Since our study is far from the completion, future extensive research under diverse situations should be necessary. They could either change our assumptions embedded into our model, or extend our model to include other exogenous effects.

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