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Expected Number of Visits of the Regenerative Process*

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Abstract

Expected number of visits in the steady state of the regenerative process is one of the most useful characteristics The formula for this expected number of visits in the steady state of the regenerative process is presented in this paper. Because this formula is for the general model, it can be applied to many special systems including 2-unit redundant system An example for this formula is also presented

1. Introduction

Many researches [1, 2, 3, 4, 9, 10] have been performed on the reliability analysis of the regenerative process. In these papers 2-unit redundant system has been analyzed because 2-unit redundant system has many applications in the real world. Most of the distribution functions for failure time, repair time, switching-over time are assumed to be arbitrary for achieving the generality of the model. Network topology between these models were different due to the various assumptions Several important characteristics such as the expected number of visits and limiting probability to a certain state, which were expressed in terms of the distributions, were obtained in these papers. These derivations require quite many calculations.

Recently the limiting probability for these regenerative process was analysed and this formula can be applied easily in calculation of the system availability [8]. But the expected number of visits for these regenerative process has not been analysed yet. The formula for this expected

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number of visits is presented in this paper. System evaluation can be also derived using this formula. The results of Sinha [10] are deduced using the formula presented in this paper.

2. Assumptions of the process and notations

2.1 System process

The system is assumed to follow a regenerative process that has at least one regeneration point[5-7, 10]. The system also has a finite state space as in a regular process[5-7, 10]. The initial state epoch of the system is a regeneration point.

2.2 Notations

The following notations are used

(1) $Q_{ij}(\alpha,t)$: CDF (cumulative distribution function) of MRP(Markov renewal process) from the regeneration point #i into #j, following the transition path α, not including any regeneration points in the path α

(2) $H_{ij}(t)$: CDF of the first passage time from the regeneration point #i into #j

(3) $P_{ij}(t)$: probability that the process enters #j, and is still in the state of #j at time t, starting from the regeneration point #i

(4) $Q_{ijk}(\alpha, t)$: CDF of MRP starting from the regeneration point #i into #j through the transition path α , and leaving #j for #k

(5) $M_{ij}(t)$: expected number of visits to #j during (0,t] starting from regeneration point #i

(6) $q_{ij}(\alpha, s)$, $h_{ij}(s)$, $p_{ij}(s)$, $q_{ijk}(\alpha, s)$, $m_{ij}(s)$: Laplace transformation of $Q_{ij}(\alpha,t)$, $H_{ij}(t)$, $P_{ij}(t)$, $Q_{ijk}(\alpha, t)$, $M_{ij}(t)$ respectively.

for example $q_{ij}(\alpha, s) = \int_0^\infty \exp(-st) * dQ_{ij}(\alpha, t)$

(7) $\overline{Q}_{ij}(\alpha)$: mean of the $Q_{ij}(\alpha, t)$

$$= \int_0^\infty t * dQ_{ij}(\alpha, t \lim_{s\to 0})$$

$$= [-\frac{d}{ds} q_{ij}(\alpha, s)]$$

(8) u_{ij} : mean of the first passage time from the regeneration point #i into #j

$$= \int_0^\infty t * dHij(t) = \lim_{s\to 0} [-\frac{d}{ds} hij(s)]$$

(9) P_j : limiting probability of the state #j

$$= \lim_{t\to\infty} P_{ij}(t) = \lim_{s\to 0} p_{ij}(s)$$

(10) $\overline{Q}_{ijk}(\alpha)$: mean of the $d(Q_{ijk}(\alpha, t))$

$$= \int_0^\infty t * dQ_{ijk}(\alpha, t)$$

$$= \lim_{s\to 0} [-\frac{d}{ds} q_{ijk}(\alpha, s)]$$

(11) M_j : expected number of visits to #j in steady state

$$= \lim_{t\to\infty} \frac{Mij(t)}{t} = \lim_{s\to 0} s * m_{ij}(s)$$

(12) SR_j : the set of the regeneration points through which #j may be reached without passing any other regeneration points

(13) L_j : the set of state epochs into which the process may enter after the transition from #j

(14) α_{ij} : the set of transition paths starting from the regeneration point #i, reaching to #j, without passing any regeneration points

3. Expected number of visits

Although the expected number of visits is very important, this had not been yet analysed for the regenerative process. The following theorem gives the expected number of visits to each point in the steady state.

Theorem The expected number of visits to point j in the steady state M_j is

$\sum_i \sum_\alpha Q_{ij}(\alpha, \infty) / u_{ii}$ for the nonregeneration point #j

where $\sum$ for i, α means $i \in SR_j$, $\alpha \in \alpha_{ij}$

$1 / u_{jj}$ for the regeneration point #j

Proof. Hereafter we will omit the summation index set SR_j, α_{ij} for convenience.

Using the renewal theoretic arguments[5, 6, 7], we have for the nonregeneration point #j

$$M_{ij}(t) = \sum_i \{1 + H_{ii}(t) + H_{ii}(t)*H_{ii}(t)+\ldots\} * \{\sum_\alpha Q_{ij}(\alpha, t)\}$$

$$= \sum_i \{1-H_{ii}(t)\}^{-1} * \{\sum_\alpha Q_{ij}(\alpha, t)\}$$

The Laplace transformation of the $M_{ij}(t)$

$$m_{ij}(s) = \sum_i \{1-h_{ii}(s)\}^{-1} * \{\sum_\alpha q_{ij}(\alpha, s)\}$$

$$M_j = \lim_{t\to\infty} M_{ij}(t)/t = \lim_{s\to 0} s*m_{ij}(s)$$

$$= \lim_{s\to 0} \sum_i \{1-h_{ii}(s)\}^{-1} *s* \{\sum_\alpha q_{ij}(\alpha, s)\}$$

Following three properties were proved in previous study[8].

$\lim_{s\to 0} h_{ij}(s) = 1$ for any regeneration point #i and #j

$\lim_{s\to 0} [\sum_\alpha q_{ij}(\alpha, s) - \sum_{\alpha, k} q_{ijk}(\alpha, s)]$

$= 0$ for regeneration point #i such that $i \in SR_j$, $\alpha \in \alpha_{ij}$, $k \in L_j$ and

$\lim_{s\to 0} q_{jk}(\alpha, s) = 1$ for any regeneration point #j such that $\alpha \in \alpha_{jk}$

Using the Lopital's theorem and the above properties

$$M_j = \sum_i \sum_\alpha Q_{ij}(\alpha, \infty) / u_{ii}$$

For the regeneration point #j

$$M_{ij}(t) = 1 + H_{ii}(t) + H_{ii}(t)*H_{ii}(t)+\ldots = \{1-H_{ii}(t)\}^{-1}$$

The Laplace transformation of the $M_{ij}(t)$

$$m_{ij}(s) = \sum_i \{1-h_{ii}(s)\}^{-1}$$

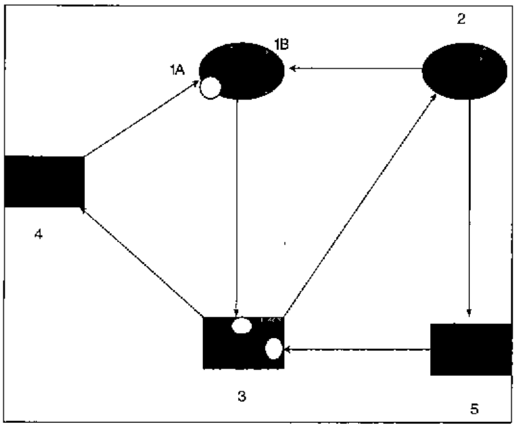
$$M_j = \lim_{t\to\infty} M_{ij}(t)/t = \lim_{s\to 0} s*m_{ij}(s) = \lim_{s\to 0} \sum_i \{1-h_{ii}(s)\}^{-1} *s$$

Using the same procedure as the case of non regeneration point,

$$M_j = 1 / u_{jj}$$

4. An example

We consider the model of Sinha[9] for illustration. This model is the 2-unit cold-standby system, and the distributions for failure, repair, switching times are all arbitrary. The following diagram shows the state transition.



O:Operating S:Standby R:Repair WR:Waiting for repair SW:Switched from standby to operating

[Figure 1] Diagram of 2-unit Redundant System with Slow Switch

In the above diagram, ◯ means the regeneration point. The circular nodes mean the operating state while the rectangular nodes mean the down state. State space is {1, 2, 3, 4, 5}, and state epoch space is {#1A, #1B, #2, #3, #4, #5}. We can derive the limiting probability and the expected number of visits using the formula presented in this paper without tedious calculations as in Sinha[10]. The results are shown in <table 1> for non regeneration points, and <table 2> for regeneration points.

〈table 1〉 properties of the non regeneration point

state epoch	attribute	value
1B	SR_{1B}	{#3}
	L_{1B}	{#3}
	$\alpha_{3\ 1B}$	{(3-2-1B)}
	M_{1B}	$Q_{3\ 1B}(3\text{-}2\text{-}1B.\ \infty)/u_{33}$
2	SR_2	{#3}
	L_2	{#1B, #5}
	$\alpha_{3\ 2}$	{(3-2) }
	M_2	$Q_{3\,2}(3\text{-}2,\ \infty)/u_{33}$
4	SR_4	{#3}
	L_4	{#1A}
	$\alpha_{3\ 4}$	{(3-4)}
	M_4	$Q_{3\,4}(3\text{-}4,\ \infty)/u_{33}$
5	SR_5	{#3}
	L_5	{#3}
	$\alpha_{3\ 5}$	{(3-2-5)}
	M_5	$Q_{3\,5}(3\text{-}2\text{-}5,\ \infty)/u_{33}$

〈table 2〉 properties of the regeneration point

state epoch	attribute	value
1A	SR_{1A}	{#3}
	L_{1A}	{#3}
	$\alpha_{1A\ 3}$	{(1A-3)}
	M1A	$1/u_{1A\ 1A}$
3	SR_2	{#1A}
	L_2	{#2, #4}
	$\alpha_{3\ 2}$ $\alpha_{3\ 4}$	{(3-2)} {(3-4)}
	M_2	$1/u_{33}$

5. Conclusion

Many tedious calculations are required in the system evaluation for stochastic process. If there are regeneration points, the expected number of visits in the steady state can be obtained by using formula presented in this paper. This result can be used in the evaluation of the semi-markov decision process.

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