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An Integrated Ordering and Setup Cost Reduction Model

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■ Abstract ■

A vendor supplies a product to a sole/major buyer on a lot-for-lot basis under deterministic inventory control conditions The basic premise is that the setup cost reduction technologies are available to both the buyer and the vendor, and that the vendor's inventory and setup reduction investment costs differ from the buyer's Therefore, an individually designed ordering and setup cost reduction policy will likely cause mismatches between the vendor's and the buyer's optimal cycle times. For this situation, we show that a joint optimal setup cost reduction and ordering policy, together with an appropriate side payment (quantity discount or premium price) schedule, can be designed in a spirit of coordination to eliminate mismatches in individual optimal cycle times

1. Introduction

The success of the setup cost reduction program in reducing production lot sizes and increasing the flexibility of the production system has led to a substantial literature on how a company should invest in such a program to reduce cycle times (see, for example, Porteus [1985, 1986a,b], Spence and Porteus [1987], Fine and Porteus [1989], Paknejad, Nasri, and Affisco [1992], and Leschke and Weiss [1997]). However, a few research has been done from a multi-echelon standpoint. The purpose of this study is to extend the problem of investing in setup cost reduction from a single-echelon situation to a two-echelon situation. Although the idea may be simple, many questions arise, especially with regard to the

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joint problem of order delivery, setup cost reduction coordination, and cost-sharing.

To gain a better understanding of this issue we model a hypothetical two-echelon, EOQ-like inventory system consisting of a vendor and a sole/major buyer. The basic premises are that (1) the setup cost reduction technologies are available to both the vendor and the buyer, and that (2) the vendor's inventory and setup reduction investment cost structures differ from the buyer's. Therefore, individually designed ordering and setup cost reduction plans will likely cause a mismatch between the vendor's and the buyer's optimal cycle times. Under these premises, a desirable practice should regard two parties as a single unit and involve the design of an integrated order and setup cost reduction policy so as to minimize the joint cost of the distribution channel.

Quantity discount has been presented in the literature as a coordination mechanism in the distribution channel designed to eliminate mismatches in cycle times The basic premise is that the vendor's inventory cost structure differs from the buyer's, leading the buyer's cycle time to differ from the vendor's. With this situation in mind, Monahan (1984, 1988) developed a model from a vendor's perspective for establishing an optimal discount schedule, and showed a price discount schedule with a single break point achieving the desired outcome for the vendor. Monahan's work has been advanced by Banerjee (1986a,b), Lee and Rosenblatt (1986), Goyal (1987), Joglekar (1988), Drezner and Wesolowsky (1989), Rubin and Carter (1990), Kohli and Park (1989, 1994), Weng and Wong (1993) and Weng (1995a,b), among others. The purpose of these models is to show a mutually profitable "Joint Order Quantity" that differs from each individual's optimal order quantity, and which can be obtained in a spirit of cooperation. The coordination mechanism works generally as follows : One party develops a joint policy of ordering and offering a break-even discount or price increase to his counter party with the objective being to entice that party to alter his order quantities to achieve a mutually agreeable delivery schedule. The cost sacrifice by the conceding party will be offset by the break-even quantity discount or price increase. In this study, along a similar vein, we try to develop and introduce a "Joint Optimal Ordering and Setup Cost Reduction Model" aimed at synchronizing individually optimal ordering and setup cost reduction programs. Our model differs from the existing literature in not only considering a monetary side payment (quantity discount or premium price) as a means of synchronization, but also considering synchronizing the setup cost reduction programs for a distribution channel to eliminate mismatches in cycle times.

This paper is structured as follows. In §2, a problem description, assumptions, and notations are presented and the objective functions of the buyer and the vendor are developed ; then, the individual optimal policies and their effects on the counter parties are analyzed. In §3, we discuss the issue of how to eliminate mismatches in individual optimal cycle times. In § 4, we discuss a model in which the vendor follows a produce-to-stock production principle, with produced lot size as an integer multiple of the buyer's order size. Finally, a brief discussion is provided in §5.

2. Model Description

We begin our analysis by briefly introducing the notations.

D = Buyer's yearly demand rate ;

A, a = Setup costs per order for the vendor and the buyer respectively ;

A^0, a^0 = Setup costs before implementing the setup cost reduction programs ;

A^R, a^R = Reduced setup costs after implementing the setup cost reduction programs ;

M = Vendor's yearly production rate($M \geq D$) ;

H, h = Per unit annual inventory holding costs incurred by the vendor and the buyer ;

Q, q = Order size variables for the vendor and the buyer ;

Q^0, q^0 = Individual optimal lot sizes (IOQ) in which setup costs are not reduced ;

Q^R, q^R = Individual optimal lot sizes in which setup costs are reduced ;

J = Jointly agreeable (optimal) order size (JOQ) variable ;

R, r = Fractional per unit time opportunity cost of capitals for the vendor and the buye r ; and

E, ε = Retrievable fractions of investment on the setup cost reduction programs for the vendor and the buyer. In addition, let $\widetilde{H} = HD/M$ and $\beta = \widetilde{H}/h$.

Assuming that the buyer's setup cost reduction investment function is logarithmic as discussed by Porteus (1985), let $br = (a^0/a)$ the setup cost reduction investment cost of changing the setup cost from a^0 to a ; $b > 0$ represents the cost of making about a 63% reduction in the setup cost. Spence and Porteus (1987) pointed out that in practice, the setup cost reduction investment function may not be in this form or even known. Nevertheless, in common with other works (for example, Porteus [1985, 1986a,b]), we employ this function as an approximation. We use exactly the same cost structure with different parameter values for the vendor. In particular, the vendor's investment cost in setup cost reduction program is given by $BR\ln(A^0/A)$ where $B > 0$ represents the cost of making about a 63% reduction in the vendor's setup cost. Finally, let $\alpha = BR/br$.

Our model is restricted to a simple transaction scenario. A buyer periodically orders some quantity from a vendor. The vendor, after receiving an order, produces the required quantity of the product, following an order-for-order principle. That is, his production quantity exactly equals the buyer's order size, and he ships the entire lot to the buyer. In section 4, we have relaxed this assumption to include those occasions in which the vendor follows a produce-to-stock production principle, with produced lot size as an integer multiple of the buyer's order size.

Let $C_B(q, a)$ denote the buyer's annual inventory cost and setup cost reduction investment for a given pair of order quantity and setup cost. Then :

$$C_B(q, a) = \frac{Da}{q} + \frac{qh}{2} + br\ \ln\left(\frac{a^0}{a}\right). \qquad (1)$$

The cost function is the sum of the three components : setup cost (Da/q) + holding cost ($qh/2$) + setup reduction cost ($br\ln(A^0/a)$). Adding setup, holding, and setup cost reduction investment, and letting $C_V(Q, A)$ denote the vendor's annual cost, the vendor's annual cost is given by the following expression (see, for example, Banerjee [1986a])

$$C_V(Q, A) = \frac{DA}{Q} + \frac{QHD}{2M} + BR\ln\left(\frac{A^0}{A}\right). \quad (2)$$

The individual optimal order size (IOQ) and setup cost (individual optimal setup cost, IOS) for the vendor is summarized in <Table 1>I. The buyer's IOQ and IOS can be obtained similarly.

<Table 1>I shows that the vendor's setup cost is designed as a direct proportion of IOQ. We now consider a situation in which the buyer, being the dominant party, refuses to fully cooperate with the vendor, and insists on his own optimal order size so that the vendor is forced adopt the buyer's IOQ. Obviously, now, the vendor will be worse off from this arrangement. Let $C_V(q^*, A|A^*(Q^*))$ denote the vendor's annual inventory-related cost when the buyer forces the vendor to adjust the current order size Q^* to q^*.

$$C_V(q^*, A|A^*(Q^*)) = \frac{DA}{q^*} + \frac{q^*\widetilde{H}}{2} + BR\ln\left(\frac{A^0}{A}\right) + (1-\mathrm{E})BR\ln\left(\frac{A}{A^*(Q^*)}\right), \quad (3)$$

where $\mathrm{E} \in [0,1]$ if $A^*(Q^*) < A$ and $A^*(Q^*) \neq A^0$, and E=1 if $A^*(Q^*) \geq A$ or $A^*(Q^*) = A^0$. The first two terms represent the standard inventory related costs (inventory carrying and setup cost) for the order size q^*. The third term represents the setup reduction investment cost for the new setup cost level, and the last term represents the recover of the setup reduction investment when the adjustment leads to a new setup cost(A) higher than the original one($A^*(Q^*)$). The model specifies that if $A^*(Q^*) \leq A$ then only a portion $E \in [0, 1]$ of the original setup cost reduction investment can be recovered. is included in the formulation to account for the penalties of breaking the "lease," if any, resulting from reducing the investment in the setup cost reduction program. One example of such a penalty is poor quality control resulting from increased production lot size. (See Porteus [1986a] for the relationship between lot sizing and quality control.) Although we have included to account for possible adverse side effects, explicit formulation and in depth analysis of possible adverse side effects such as poor quality control are not given in this paper, and warrant further study. The possibility of adjusting the setup cost reduction policy is based on Porteus (1985), who states that the setup cost reduction investment can be regarded as a lease that can be broken on occasion, and a new setup cost level selected. The new setup

<Table 1> Vendor's Individual Optimal Policy, Optimally Adjusted Policy, and Cost Penalty

I. *Individual Optimal Policy*

$Q^*(A^*) = \sqrt{2DA^*(Q^*)/\widetilde{H}}$, $A^*(Q^*) = \min[A^0, A^R(Q^*)]$, $A^R(Q^*) = BRQ^*/D$, $A^R(Q^R) = 2(BR)^2/D\widetilde{H}$, and $Q^R(A^R) = 2BR/\widetilde{H}$

II. *Inventory Cost and Adjusted Setup Cost*

When $q^* \geq Q^* = Q^R$ $A^*(q^*) = \min[A^0, \max(A^R(q^*), A^R(Q^R))]$, $A^R(q^*) = \mathrm{E}BRq^*/D$

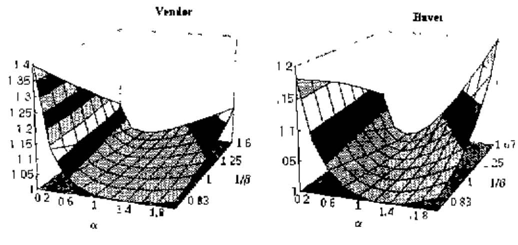
When $q^* \geq Q^* = Q^0$ or $q^* < Q^*$ $A^*(q^*) = \min[A^0, A^R(q^*)]$, $A^R(q^*) = BRq^*/D$.

III. *Penalties for Adopting the Other Party's Optimal Lot Size.(Dropping the superscript for R)**

Sensitivity Value	*Difference in Inventory Costs*
$\frac{C_V(q, A(q)\|A(Q))}{C_V(Q, A(Q))} = \frac{\beta/\alpha + \mathrm{E} + \ln(A^0/A(q)) + (1-\mathrm{E})\ln(A(q)/A(Q))}{2+\ln(A^0/A(Q))}$ ** and	$C_V(q, A(q)\|A(Q)) - C_V(Q, A(Q)) = BR(\beta/\alpha - 2 + \mathrm{E}(1-\ln(\mathrm{E}\beta/\alpha)))$

* We assume both the vendor and the buyer can reduce their setup costs individually ** See Appendix 1

cost (or optimally "adjusted" setup cost) is given in Table 1.II. Table 1.III summarizes the vendor's penalty from adopting the buyer's IOQ when $A^*(q^*) = A^R(q^R)$. It tells us that the penalty increases as the mismatches between the two IOQs-- $q^R/Q^R = \beta/\alpha(Q^R/q^R = \alpha/\beta)$--increase. For example, a high β implies that the vendor's inventory holding cost is relatively more costly compared to the buyer's ; a low α would mean the vendor's setup cost reduction investment cost is relatively less costly compared to the buyer's. Therefore, the vendor's reduced setup cost will likely be lower than the buyer's. As a result, an extremely high β/α will likely lead to a relatively larger IOQ for the buyer and a smaller IOQ for the vendor. We also see that the penalty decreases as retrievable fraction increases. [Figure 1] shows plots of the sensitivity values $C_B(Q, a(Q)|a(q))/C_B(q, a(q))$ and $C_V(q, A(q)|A(Q))/C_V(Q, A(Q))$ against two parameters, $1/\beta$ and α. Here, we assume E, $\varepsilon = 1$



[Figure 1] Sensitivity Values for the Vendor and the Buyer

3. Synchronizing Individual Optimal Policies

Having outlined the drawbacks resulting from adopting individual optimal policies, we now turn to the problem of designing a coordinate means of settlement that focuses on synchronizing mismatches in IOQ and IOS. To address this problem, we need to consider two cases ($q^* \leq Q^*$ and $q^* > Q^*$). Let us concentrate on the case of $q^* \leq Q^*$. Two synchronization arrangements can be considered. In the first arrangement, the vendor initiates the synchronization process. We label this Arrangement 1. (Arrangement 1 without the setup cost reduction option has been discussed in Monahan (1984).) The synchronization arrangement works as follows :

(1) Buyer's Adjustment Stage : The vendor provides a Quantity Discount Pricing Schedule to induce the buyer to adjust his IOQ to be jointly agreeable. Given that the buyer has accepted this request, he then designs a new setup cost according to the jointly agreeable order size, so as to minimize the cost penalty incurred from the synchronization arrangement. Here, the cost penalty incurred by the buyer from making the cooperative adjustment will be compensated by the vendor through a "break-even" quantity discount schedule. Therefore, the buyer is at least indifferent to the synchronization arrangement.
(2) Vendor's Adjustment : The vendor designs an order size (that are agreeable to both parties) and setup cost that minimizes both the buyer's cost penalty ("break-even" discount), and his own cost penalty incurred from the synchronization arrangement.

Next, the buyer initiates the synchronization process (we call this Arrangement 2).

(1) The buyer provides a Premium Pricing Schedule to induce the vendor to simultaneously modify order size and setup cost. Again, the extra cost incurred by the vendor

when making the cooperative adjustment will be compensated by the buyer through a "break-even" premium pricing schedule. Thus, the vendor is at least indifferent to the synchronization arrangement

(2) The buyer modifies his order size and setup cost according to the jointly agreeable order schedule.

Arrangement 1 is given by the following expression.

$$\min_{J,A}\{\Delta C_V(J,A,q^*,A^*(q^*))+\min_{a}\Delta C_B(J,a,q^*,a^*(q^*))\},\ \text{where}$$

$$\Delta C_B(J,a,q^*,a^*(q^*))=C_B(J,a|a^*(q^*))-C_B(q^*,a^*(q^*)),\ \text{and}$$

$$\Delta C_V(J,A,q^*,A^*(q^*))=C_V(J,A|A^*(Q^*))-C_V(q^*,A^*(q^*)|A^*(Q^*)).$$

For example,

$$C_B(J,a|a^*(q^*))=\frac{Da}{J}+\frac{Jh}{2}+br\ln\left(\frac{a^0}{a}\right)+(1-\varepsilon)\,br\ln\left(\frac{a}{a^*(q^*)}\right),$$

where $\varepsilon\in[0,1]$ if $a^*(q^*)<a$ and $a^*(q^*)\neq a^0$, and $\varepsilon=1$ if $a^*(q^*)\geq a$ or $a^*(q^*)=a^0$. Arrangement 1 is identical to the problem $\min_{J,A}\{C_V(J,A|A^*(Q^*))+\min_{a}C_B(J,a|a^*(q^*))\}$ which can be shown to be equivalent to the joint optimization problem

$$\min_{J,A,a}C_J(J,A,a|a^*(q^*),A^*(Q^*))=C_B(J,a|a^*(q^*))+C_V(J,A|A^*(Q^*)).$$

Therefore, in what follows we will label (J^*, A^*, a^*) obtained from Arrangement 1 as *jointly optimal order quantity* (JOQ) and *setup costs* (JOS). It is clear that the sum of the two parties' costs as generated by adopting either one's individual optimal policy cannot be smaller than that generated by adopting a jointly optimal policy that minimizes the two party's joint cost. Although this could mean that one will be worse off from implementing the jointly optimal policy, the other party will certainly benefit from this process (otherwise, the jointly optimal policy would not generate a minimum sum of two costs after all). Therefore, the two parties can design a fair arrangement for dividing the joint cost savings so that neither is worse off from the process. Both Banerjee (1986a) and Rubin and Carter (1990) have used this approach in their work.

We will now provide a detailed analysis of the synchronization Arrangement 1. (Arrangement 2 can be analyzed similarly.) Due to the complexity of the problem, we will focus our analysis to a special case in which $Q^*=\min[Q^R(A^R),Q^0]\geq q^*=q^R(a^R)$. We partition the situation into four cases--*CXY*=(*CRR, CR0, C0R, C00*)--in which we use "R" to represent cases in which the buyer or the vendor can invest in reducing setup cost after the synchronization arrangement, and "0" to represent complementary cases. The first script stands for the buyer and the second script stands for the vendor. We apply this to the superscript system(e.g., J^{R0} represents the JOQ for those occasions where, after the synchronization arrangement, only the buyer can reduce his setup cost). Let $\delta^{XY}=J^{XY}/q^R(a^R)$, $X, Y=R, 0$ represent the ratio between the buyer's IOQ and the JOQ. In what follows, we will label as the buyer's adjustment factors. In order to describe the JOS, we further partition the buyer's case "R" into two mutually exclusive cases--*CX=1* and *2*--in which *C1* applies when $1\leq\varepsilon\delta<a^0/a^R(q^R)$, and *C2*

applies when $\alpha < \delta < 1/\varepsilon$. Combining the two partitioning systems *X=1,2*, and *0*, *Y=R* and *0*, we obtain six cases *CXY*. The JOS and JOQ are provided in Proposition 1.

Proposition 1 (Proof. See Appendix 2)

An optimal ordering and setup cost reduction policy exists for Arrangement 1.

(1.1) The optimal order size, setup costs and adjustment factors are provided in <Table 2-1>.

(1.2) For case *Y=R* and *0* : (i) Case *C1Y* (*C2Y*) implies $\delta^{1Y} \geq (<) \delta^{2Y}$. For case *X=1,2*, and *0* : (ii) Case *CX0* (*CXR*) implies $\delta^{X0} \leq (>) \delta^{XR}$.

(1.3) The optimal solutions provided in <Table 2-1> correspond to six mutually exclusive and collectively exhaustive cases listed in <Table 2-2>

The vendor and the buyer design the jointly optimal order sizes and setup costs by consulting <Table 2-2> first. After determining the corresponding cases *CXY*, they then design the order sizes and setup costs from <Table 2-1>. Assuming $Q^* = Q^R$, and case *C1R* apply, the mismatch in order sizes increases as α/β increases ; therefore, as a counter-measure, δ^{1R} increases, and $\delta^{1R}\beta/\alpha$(the vendor's adjustment factor $J^{1R}/Q^R = (J^{1R}/q^R)(q^R/Q^R) = \delta^{1R}\beta/\alpha$) decrease as α/β increases. Consequently, <Table 2-1> reveals that the buyer's JOS ($a^R(J^{1R})$) increases(more rollback ad-

〈Table 2-1〉 Optimal Order Size, Setup Costs, and Adjustment Factors

$$J^* = \sqrt{2D[A^*(J^*) + a^*(J^*)]/(\widetilde{H} + h)}, \text{ where } A^*(J^*) = \min[A^0, A^R(J^{XR})], \text{ and}$$

$$A^R(J^{XR}) = (\beta/\alpha)\delta^{XR}A^R(Q^R) = \alpha\delta^{XR}a^R(q^R), \text{ X=1, 2, and 0}$$

Jointly Optimal Setup Cost $a^*(J^*)$ and Adjustment Factor δ^* for the Buyer

Case *CXR*, X=1, 2, 0	Case *CX0*, X=1, 2, 0
Case C1R. δ^{1R}: $\frac{\varepsilon + \alpha}{1+\beta}$, $a^R(J^{1R}) = \varepsilon\delta^{1R}a^R(q^R)$	Case C10: δ^{10}: $\frac{\varepsilon + \sqrt{\varepsilon^2 + 4(1+\beta)A^0/a^R(q^R)}}{2(1+\beta)}$, $a^R(J^{10}) = \varepsilon\delta^{10}a^R(q^R)$
Case C2R: δ^{2R}: $\frac{\alpha + \sqrt{\alpha^2 + 4(1+\beta)}}{2(1+\beta)}$, $a^R(J^{2R}) = a^R(q^R)$	Case C20: δ^{20} : $\sqrt{\frac{1 + A^0/a^R(q^R)}{1+\beta}}$, $a^R(J^{20}) = a^R(q^R)$
Case C0R: δ^{0R}: $\frac{\alpha + \sqrt{\alpha^2 + 4(1+\beta)a^0/a^R(q^R)}}{2(1+\beta)}$, $a^0(J^{0R}) = a^0$	Case C00: δ^{00}: $\sqrt{\frac{a^0/a^R(q^R) + A^0/a^R(q^R)}{1+\beta}}$, $a^0(J^{00}) = a^0$

〈Table 2-2〉

$C1R := \{(1+\beta) \leq (\varepsilon+\alpha)\varepsilon < a^0(1+\beta)/a^R,$ $(\varepsilon+\alpha)\alpha < A^0(1+\beta)/a^R\}$ *	$C10 := \{I(1,\varepsilon) \leq A^0(1+\beta)/a^R < I(a^0/a^R, \varepsilon),$ $\alpha(\varepsilon+\alpha) \geq A^0(1+\beta)/a^R\}$
$C2R := \{(\varepsilon+\alpha)\varepsilon < 1+\beta \leq 1+\alpha,\ 1+\beta < I(A^0/a^R, \alpha)\}$**	$C20 := \{I(1,\varepsilon) > A^0(1+\beta)/a^R \geq I(1,1),$ $1+\beta \geq I(A^0/a^R, \alpha)\}$
$C0R := \{a^0(1+\beta)/a^R \leq (\varepsilon+\alpha)\varepsilon,$ $a^0(1+\beta)/a^R < I(A^0/a^R, \alpha)\}$	$C00 := \{I(a^0/a^R, \varepsilon) \leq A^0(1+\beta)/a^R,$ $a^0(1+\beta)/a^R \geq I(A^0/a^R, \alpha)\}$

* $a^R = a^R(q^R)$

** $I(X, Y) := (X(1+\beta)/Y)^2 - X(1+\beta)$

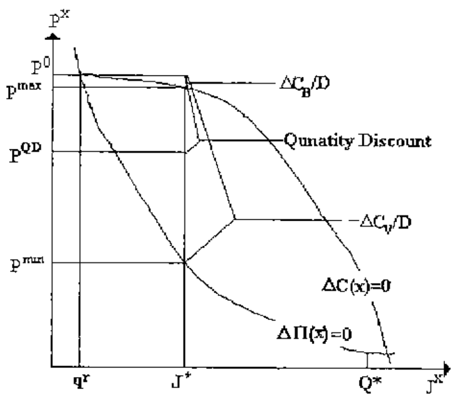
justment) as ε and δ^{1R} increase, and the vendor's JOS ($A^R(J^{1R})$) decreases as α/β increases. <Table 2-1> reveals that the second expression of the vendor's JOS ($a^R(q^R)\alpha\delta^{1R}$)is closely related to the buyer's IOS, the ratio of setup cost reduction investment costs, and the buyer's order size adjustment factor. Therefore, for example, (i) the higher the buyer's IOS, or (ii) the more the buyer's order size adjustment, or (iii) the more expensive the vendor's setup cost reduction investment cost, the fewer efforts are concentrated on the setup cost reduction program. The result reveals that case $C1R$ and $C2R$ apply (both the buyer and the vendor will make investment in setup reduction after the synchronization arrangement) under five circumstances. First, if the vendor's setup cost reduction investment cost is relatively inexpensive so that he can reduce his setup cost in a less costly fashion. Second, if the vendor's carrying cost is relatively expensive, so that he has strong motivations to reduce the setup costs. Notice that the first and the second conditions lead to a smaller value α/β ; this in turn results in a smaller buyer's adjustment factor δ. Third, if the initial unreduced setup costs are relatively expensive for the vendor and the buyer. Fourth, if the retrievable fraction is relatively small for the buyer, rendering the buyer reluctant to rollback adjust his setup cost. Finally, and fifth, if the buyer's IOS is relatively small.

We now proceed to analyze the quantity discount schedule provided by the vendor as a synchronization arrangement. Let $C(x)$ and $C(0)$ denote the buyer's annual inventory costs, including the inventory purchasing costs (see <Table 3>).

Here the unit price for order of size J^X is P^X (q^R is P^0). One may regard P^0 as the original price level, and P^X as a new price level (quantity discount price) designed by the vendor to induce the buyer to increase order size from q^R to J^X. Subtracting $C(x)$ from $C(0)$, and letting $C(0)/D - C(x)/D = \Delta C(x)/D = 0$, then

$$P^X = P^0 - \Delta C_B(J^X, a^*(J^X, a^*(J^X), q^R, a^R(q^R))/D \quad (4)$$

Given $\Delta C(x)$=0, equation (4) specifies an iso-cost curve in the (J^X, P^X) space as illustrated in [Figure 2].



[Figure 2] The Quantity Discount Price

For $\Delta C(x) = 0$ the iso-cost curve passes through (q^R, P^0), and for $\Delta C(x) > 0$ the buyer is on an iso-cost curve that is closer to the origin.

〈Table 3〉 Inventory Cost Functions

$C(x) = C_B(J^X, a^*(J^X)\|a^R(J^R)) + DP^X$ $a^*(J^X) = \min[a^0, \max(a^R(J^X), a^R(q^R))], a^R(J^X) = \varepsilon brJ^X/D$	$C(0) = C_B(q^R, a^R(q^R)) + DP^0$
$\Pi(x) = DP^X - C_V(J^X, A^*(J^X)\|A^*(Q^*))$ $A^*(J^X) = \min[A^0, A^R(J^X)], A^R(J^X) = BRJ^X/D$	$\Pi(0) = DP^0 - C_V(q^R, A^*(q^R)\|A^*(Q^*))$ $A^*(q^R) = \min[A^0, A^R(q^R)], A^R(q^R) = BRq^R/D$

The slope $Da^*(J^X)/(J^X)^2 - h/2$ of the iso-cost curve $\Delta C(x)=0$ is zero at $J^X = q^R$, and strictly negative if $J^X > q^R$. We also see that the iso-cost curve is a strictly concave function of J^X. Similarly, let $\Pi(x)$ and $\Pi(0)$ denote supplier's total profits. Let $(\Pi(x)-\Pi(0))/D = \Delta\Pi(x)/D=0$, then

$$P^X = P^0 + \Delta C_V(J^X, A^*(J^X), q^R, A^*(q^R))/D \quad (5)$$

Given a value of $\Delta\Pi(x)$, equation (5) specifies an iso-profit curve in the (J^X, P^X) space that is a convex function of J^X. For $\Delta\Pi(x)=0$ the iso-profit curve passes through (q^R, P^0), and for $\Delta\Pi(x)>0$, the supplier is on an iso-profit curve that is further away from the origin. Referring to [Figure 2], the slope $\widetilde{H}/2 - A^*(J^X)/(J^X)^2$ of the iso-profit curve is zero at $J^X = Q^*$, and strictly negative if $J^X < Q^*$. We see that, first, the iso-profit curve for $\Delta\Pi(x)=0$ is strictly convex, decreasing if $J^X < Q^*$, and passing through (q^R, P^0). Second, the iso-cost curve for $\Delta C(x)=0$ is strictly concave and strictly decreasing if $J^X > q^R$, and passing through (q^R, P^0). The two curves must intersect at some points, and there always exist a josintly agreed values of $(q^R \le J^X \le Q^*, P^*)$ for which $\Delta\Pi>0$ and $\Delta C>0$ simultaneously. That is, both the vendor and the buyer benefit from the synchronization arrangement. Let $(J^*, A^*(J^*), a^*(J^*))$ be the JOQ and JOS obtained from Arrangement 1, and

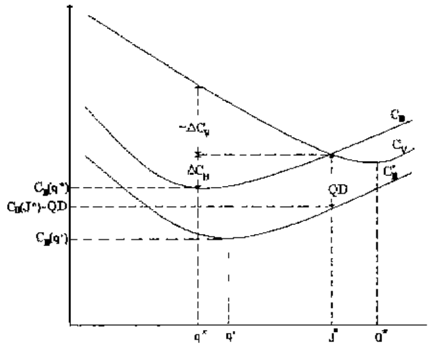
$$P^{\max} = P^0 - \Delta C_B(J^*, a^*(J^*), q^R, a^R(q^R))/D \text{ and}$$
$$P^{\min} = P^0 - \Delta C_V(J^*, A^*(J^*), q^R, A^*(q^R))/D.$$

The annual quantity discount offered by the vendor should at least cover the buyer's yearly cost penalty $D(P^0 - P^{\max}) = \Delta C_B(J^*, a^*(J^*), q^R, a^R(q^R))$ so that the buyer is at least indifferent to the synchronization arrangement, but be no more than the vendor's cost savings $D(P^0 - P^{\min}) = -\Delta C_V(J^*, A^*(J^*), q^R, A^*(q^R))$. The upper and lower bounds for the unit quantity discount price P^{QD} are then set by $P^{\max} \ge P^{QD} \ge P^{\min}$. After providing the "break-even discount" (equals $D(P^0 - P^{\max})$) to the buyer, the vendor's cost savings, net the breakeven discount (amounts to $D(P^{\max} - P^{\min})$), can then be fairly divided by the two parties via a mutually agreeable arrangement. Assume now that case $C1R$ applies, and $\varepsilon = 1/\delta^{1R}$. The "break-even discount" (equals $D(P^0 - P^{\max})$) incurred from the synchronization arrangement after some algebra can be arranged as :

$$\Delta C_B(J^{1R}, a^R(J^{1R}), q^R, a^R(q^R)) = \sqrt{2a^R(q^R)Dh}\,\frac{\delta^{1R} - 2 + 1/\delta^{1R}}{2}$$

The penalty exactly equals the "Break Even Discount" $d_K D$ discussed in Monahan(1984).

Although in this model we have explicitly assumed that the price and the inventory carrying cost are unrelated, in a real-world application a major portion of the inventory carrying cost is commonly the opportunity cost of capital tied to inventory. Therefore, it may be argued that when one party offers a price change (quantity discount or premium price) to induce the other party to adopt the jointly optimal policy, the latter may recompute his IOQ. To address this question, we need to consider four cases involving two arrangements (Arrangements 1 and 2), and two cases ($q^* \le Q^*$ or $q^* \ge Q^*$). [Figure 3] illustrates one of the four cases in which Arrangement 1 and case $q^* \le Q^*$ apply. Let us now assume that some portion of the unit annual holding cost consists of the opportunity cost of capital tied up in inventory. Then, the new IOQ (q'), after receiving the quantity discount may move toward the right. However, regardless of $q^* \neq q'$, the Quantity Discount(QD), so long as it is larger than the buyer's cost penalty $\Delta C_B(J^*, a^*(J^*), q^*, a^*(q^*))$, will always reduce the buyer's total cost from $C_B(q^*, a^*(q^*))$ to $C_B(J^*), a^*(J^*)|a^*(q^*))$ $- QD$, so that the buyer benefits from the process. In the limiting case in which the total quantity discount equals $\Delta C_B(J^*, a^*(J^*), q^*, a^*(q^*))$, the buyer's cost penalty will at least be reimbursed by the vendor. Therefore, a single break point can achieve the desired outcome for the vendor. The other three cases can be illustrated similarly.



ΔC_D<QD = Quantuity Discount <-ΔC_V

C_B:= Buyer's Inventory Cost Curve Without Quantity Discount

C_B^1= Buyer's Inventory Cost Curve With Quantity Discount

C_V = Vendor's Inventory Cost Curve

[Figure 3] Cost Curves for Changing Order quantity from q* to case q*<J* <Q*

Here the numerical example furnished in Monahan[9] is provided to better understand the model. Let, a^0=100, D=10,000, h=2, r=0.2, and M=20,000. All numerical parameters have been kept with a few exceptions to the new parameters. We let a^R=100, b=5000, a^0=500, and E=1. The model experiments with different levels of α=1, 2.5 and 10. A^0=800, 500, and H=1.4, 2 respectively. For the purpose of comparison, in the following analysis, we will restrict our analysis on three relevant cost components, the setup cost, holding cost, and setup reduction investment costs. <Table 4-1> and <Table 4-2> summarize the numerical experiment. For example, Q cost provides the cost of the situation in which the buyer orders and the vendor produces vendor's individual optimal order quantity ; q cost gives the cost of the situation in which the buyer orders and the supplier produces buyer's individual optimal order quantity. J cost exhibits the joint optimal model. For example, R1 and R2 give the supplier's (buyer's) cost before Quantity Discount (Premium Pricing). Quantity Discount(QD) can

〈Table 4-1〉 Results of the Numerical Example

	A^0 = 800				A^0 = 500			
	α=1	α=2.5	α'=2.5	α=10	α=1	α'=1	α=2.5	α=10
CASE	C1R	C1R	C1R	C10	C1R	C1R	C10	C10
δ	1.48	2.59	2.33	2.83	1.48	1.33	2.33	2.33
a^*	148	259	233	283	148	133	233	233
A^*	148	647	583	800	148	133	500	500
J COST								
R1 Supplier	3205	3935		3817	2735		2961	2961
• QD	89	639		790	89		484	484
After QD	3294	4257		4607	2824		3445	3445
R2 Buyer	3698	4248		4399	3698		4093	4093
After QD	3609	3609		3609	3609		3609	3609
(*R1+R2*)	6903	8183		8216	6433		7054	7054
Q COST								
a^*	285.7	500		500	285.7		500	500
A^*	285.7	800		800	285.7		500	500
R3 Supplier	3029	3346		3346	2562		2645	2645
R4 buyer	4417	5826		5826	4417		5102	5102
(*R3+R4*)	7446	9173		9173	6978		7747	7747
q COST								
A^*	100	250		800	100		250	500
R5 Supplier	3429	5758		8350	2959		4582	5350
R6 Buyer	3609	3609		3609	3609		3909	3609
(*R5-R6*)	7038	9367		11959	6568		8189	8959

* The buyer's break-even premium price = max[0, R1-R3]
• The supplier's break-even quantity discount = max[0, R2-R6]
* The α' corresponds to cases in which Supplier's holding cost equals 2

be obtained from R2-R6. Therefore, all together, when the buyer's (supplier's) individual optimal order quantity is adopted, the supplier's (buyer's) costs including QD(PR) can be obtained from R1+QD(R2+PR), and the buyer's (supplier's) cost after QD(PR) can be obtained from R2-QD(R1-PR). In <Table 4-2>, the cost savings generated from implementing the buyer's (supplier's) SR/PR (SR/QD) are presented. For example, when the supplier (buyer) is the dominant party, the buyer's (supplier's) cost savings from adjusting order size from $Q^*(q^*)$ to J^* can be obtained by max[0, $R4$ - $R2$] (max[0, $R5$ - $R1$]).

⟨Table 4-2⟩ Extra Cost and Savings

	$A^0=800$			$A^0=500$		
	$\alpha=1$	$\alpha=2.5$	$\alpha=10$	$\alpha=1$	$\alpha=2.5$	$\alpha=10$
Buyer's SR/PR(Vendor being the dominant party)						
* Premium Price	0	0	0	0	0	0
** Cost Saving	719	1982	1354	719	1009	1009
Net	719	1785	1354	719	1009	1009
Vendor's SR/QD(Buyer being the dominant party)						
• Quantity Discount	89	639	790	89	484	484
•• Cost Saving	224	1823	4533	224	1621	2389
Net	135	1184	3743	135	1137	1905

** The buyer's Cost saving=max[0,R4-R2]

•• The supplier's Cost saving=max[0,R5-R1]

<Table 4-1> verifies (α^*, A^*, δ^*) increasing in B, and decreasing in H. <Table 4-2> shows us that the vendor's net cost savings increases as B increases. This result is apparent, since in the optimal policy, a larger B leads to a smaller reduction in the vendor's setup cost. Hence, comparing to reactively following the buyer's order size and investing in a expensive setup reduction program, more savings are generated by actively inducing the buyer to increase his order size to joint optimal order quantity. The numerical example shows that our model provides a cooperative policy that minimizes the total cost of the supply chain.

4. Extension to Include Produce-to-Stock Policy

In this section, rather than applying a lot-for-lot production principle, the vendor follows a produce-to-stock production principle, and designs an optimal mix policy of quantity discount and production lot size as an integer multiple of the buyer's order size. As in Section 3, we assume that the buyer can individually reduce his setup cost. The synchronization problem analogous to that provided in <Table 2>, can be shown as (see, for example, Joglekar[1988]) :

$$\min_{n,\delta,A}\{\Delta C_V(q^R n\delta, A, q^R, A^*(q^R)) + \min_{a}\Delta C_B(q^R\delta, a, q^R, a^R(q^R))\}, \quad \text{where}$$

$$C_V(q^R n\delta, A|A^*(Q^*)) = \left(\frac{AD}{q^* n\delta}\right) + \frac{q^R\delta((n-1)M/D-(n-2))\widetilde{H}}{2} + BR\ln\left(\frac{A^0}{A}\right) + (1-E)BR\ln\left(\frac{A}{A^*(Q^*)}\right) \tag{6}$$

n in equation (6) represents the integer multiplier of modified order size δq^R. Here, allowing the vendor's production cycle time to be an integer multiple of the buyer's order cycle, a JOQ is initially decided by the vendor and the buyer cooperatively. A production lot size n times the JOQ is then independently decided by the vendor. Lastly, the vendor and the buyer adjust their setup cost according to the modified order schedule, where the buyer's order size is designed according to the JOQ($q^R\delta$), and the vendor's order size is designed according to the integer multiplier of the JOQ($q^R n\delta$). Proposition 2 summarizes the optimal solutions.

Proposition 2 (The Proof is similar to that in Proposition 1 ; hence, it is omitted here.)

An optimal ordering and setup cost reduction policy exists for Arrangement 1.

(2.1) The simultaneously optimal order size, setup costs, and adjustment factors are provided in <Table 5-1>. By allowing fractions, Table 5-1 also provides us with an approximated solution to the integer multiplier by using standard calculus.

(2.2) The optimal solutions provided in <Table 5-1> correspond to six mutually exclusive and collectively exhaustive cases listed in <Table 5-2>.

It is seen that vendor's order size $q^R\delta n = \sqrt{2AD/H(1-D/M)}$ gives a classical Economic Manufacturing Quantity. Proposition 2 reveals that the vendor tends to produce a larger lot size (larger n) for large values of α, $1/\Phi$, and A^0. They lead to a larger (smaller) vendor's (buyer's) IOQ. Proposition 2, cases *C1R* and *C2R*, reveals that both the vendor and the buyer can invest on the setup cost reduction program if $\varepsilon^2 < a^0\Phi/a^R(q^R)$(buyer can invest in setup cost reduction) and $\alpha^2 < \Phi A^0/a^R(q^R)\xi$ (vendor can invest in setup cost reduction) apply. They apply under the conditions identical to those obtained in the previous sections.

6. Conclusion

An EOQ-like inventory system is presented that consists of a vendor and a buyer. Mismatches occur in individual optimal cycle times due to differences in inventory and setup cost reduction investment cost structures. With this situation in mind, we have tried to provide answers regarding when and how to eliminate mismatches in individual optimal cycle times. Our study indicates that when both parties can invest in reducing setup costs, the mismatch in the individual optimal cycle times increases as α/β increases (decreases). For example, a combination of a high α/β will lead to a relatively larger, individual optimal order quantity (IOQ) for the vendor and

〈Table 5-1〉 Optimal Order Size Adjustment

I. *Joint Order Quantity and Setup Cost Reduction :*

$$\delta^*(n^*, A^*, a^*) = \sqrt{(a^* + A^*/n^*)\xi/\Phi a^R(\xi + n^*)},\ n^*(\delta^*, A^*) = \sqrt{A^*\xi/\Phi a^R}/\delta^*, \text{ and } A^*(n^*, \delta^*, a^*) = \min[A^0, a^R a^* n^* \delta^*]$$

where $\xi = (2D + (h-H)M/H)/(M-D)$ and $\Phi = H(1-D/M)\xi/h = 1 + H(2D/M-1)/h$,

II. *Simultaneous Solutions of Adjustment Factor and Integer Multiplier*

Case *CXR*, X=1, 2, 0		Case *CX0*, X=1, 2, 0	
Case C1R	δ^{1R}. ε/Φ, n^{1R}: $\alpha\xi/\varepsilon$	Case C10:	δ^{10}: ε/Φ, n^{10} $\sqrt{\xi\Phi A^0/a^R}/\varepsilon$
Case C2R:	δ^{2R}: $\sqrt{1/\Phi}$ n^{2R} $\xi\alpha\sqrt{1/\Phi}$	Case C20:	δ^{20}. $\sqrt{1/\Phi}$, n^{20}: $\sqrt{\xi A^0/a^R}$
Case C0R:	δ^{0R}: $\sqrt{a^0/a^R\Phi}$, n^{0R}: $\xi\alpha\sqrt{a^R/a^0\Phi}$	Case C00:	δ^{00}. $\sqrt{a^0/a^R\Phi}$, n^{00}: $\sqrt{\xi A^0/a^0}$

〈Table 5-2〉

$C1R$:= $\{\Phi < \varepsilon^2 < a^0\Phi/a^R,\ \alpha^2 < A^0\Phi/a^R\xi\}$;	C10:= $\{\Phi < \varepsilon^2 < a^0\Phi/a^R,\ \alpha^2 \ge A^0\Phi/a^R\xi\}$
$C2R$:= $\{1 \ge \Phi \ge \varepsilon^2, \alpha^2 < A^0\Phi/a^R\xi\}$;	C20:= $\{1 \ge \Phi \ge \varepsilon^2, \alpha^2 \ge A^0\Phi/a^R\xi\}$
$C0R$.= $\{\varepsilon^2 \ge a^0\Phi/a^R,\ \alpha^2 \ge A^0\Phi/a^R\xi\}$	C00:= $\{\varepsilon^2 \ge a^0\Phi/a^R,\ \alpha^2 \ge A^0\Phi/a^R\xi\}$

a relatively smaller IOQ for the buyer. Therefore, it will impose a great cost burden on the vendor or the buyer if the other party insists on adopting his own IOQ. A jointly decided order quantity will, in this case, be most beneficial to both parties.

We would expect synchronization between two individual parties to be helpful, but what is much less readily apparent is how to design a coordinated means of settlement that focuses on synchronizing mismatches between individual optimal policies. We have provided answers to this question by designing a coordinated ordering and setup cost reduction policy and a side payment schedule (quantity discount) to facilitate the fair sharing of joint inventory cost savings. The logic behind the process is intuitively clear : The joint inventory cost generated by adopting either party's individual optimal policy can never be smaller than that generated by adopting a policy that minimizes the joint inventory cost. Therefore, a Pareto-efficient solution can be obtained, and the two parties can design a fair arrangement that divides the cost savings generated from adopting the joint optimal ordering and setup cost reduction policies. This analysis can be carried out from two perspectives : first, the vendor grants a quantity discount to induce the buyer to adjust his IOQ ; and second, the buyer gives a premium price to entice the vendor to adjust his supplying frequency. We have provided a detailed analysis for the first case.

Although the approach guarantees a mutually beneficial outcome for both parties, as Banerjee (1986a) and Rubin and Carter (1990) have pointed out, a number of practical hurdles must be removed before adopting the seemingly "not too" complicated arrangement suggested here. Among these, the greatest hurdle may reside in requiring both sides to release complete information to each other. Rubin and Carter (1990) suggest using a neutral consultant or arbiter to facilitate the negotiation. However, in a real-world application, requiring complete disclosure may not ensure sincere attempts. Therefore, a truly rational and trustworthy relationship has to be developed before this kind of coordination can occur. Perhaps this is one reason recent studies of channel coordination advocate a long-term relationship between a purchaser and a small number of reliable suppliers ; the group works together continuously to remove problems that obscure the effective operation of the supply chains.

The primary limitation of the research is the assumption of complete symmetry between the buyer and the vendor ; hence, no one party dominates another. In many recent studies of channel coordination, the assumption of symmetry is widely applied to avoid the need to view the system in the principal-agent setting, and to be able to take an unbiased approach to the coordination problem under consideration. Future work extending the model to account for the nonsymmetrical relation between the buyer and the vendor would certainly shed further light on the topic.

Appendix 1.

After dropping the superscript R, we derive the sensitivity value for the vendor as follows :

$$\frac{C_V(q, A(q) \mid A(Q))}{C_V(Q, A(Q))}$$

$$= \frac{\widetilde{H}q/2 + A(q)D/q + BR\ln(A^0/A(q)) + (1-E)BR\ln(A(q)/A(Q))}{\sqrt{2A(Q)D\widetilde{H}} + BR\ln(A^0/A(Q))}$$

$$\frac{\widetilde{H}q/2}{\sqrt{2A(Q)D\widetilde{H}} + BR\ln(A^0/A(Q))} =$$

$$\frac{\widetilde{H}q/2}{\widetilde{H}Q+\widetilde{H}Q\ln(A^0/A(Q))/2}=\frac{q/Q}{2+\ln(A^0/A(Q))}.$$

Since $\widetilde{H}Q=2BR$

The remaining terms can be obtained similarly. The sensitivity value can be obtained from adding the four terms. Let $\Delta C_V= C_V(q, A(q)|A(Q))-C_V(Q, A(Q))$. Here, we show ΔC_V increasing in α/β for $\alpha>\beta$, and increasing in β/α for $\alpha\le\beta$. Let $x=\beta/\alpha$ and $\Omega(x)= x-C-\ln(Ex)$. We see that $\partial\Omega(x)/\partial x|_{x<1}=(1-E/x)<0$; therefore, ΔC_V increasing in α/β when $\alpha>\beta$. Similarly, $\partial\Omega(x)/\partial x|_{x>1}=(1-1/x)>0$; therefore, ΔC_V increasing in β/α when $\alpha\le\beta$.

Appendix 2. Proof of Proposition 1

(1.1) We see that $\partial\Delta C_B/\partial a = D/J-\varepsilon br/a$ ($\partial\Delta C_V/\partial A = D/J-EBR/A$), which has the same sign as $Da/J-\varepsilon br$($DA/J-EBR$). We consider the latter function to be defined for all $a\ge0$ ($A\ge0$). It is negative when $a=0$($A=0$), positive when $a=\infty$($A=\infty$), and strictly increasing in a(A). The necessary conditions can be obtained from the first derivatives

$$D/J-\varepsilon br/a=0\Rightarrow a^*=\varepsilon brJ/D=\varepsilon Ja^R(q^R)/q^R$$
$$=\varepsilon\delta a^R(q^R),\ \text{and}$$

$$D/J-EBR/A=0\Rightarrow a^*=BRJ/D=A^R(Q^R)J/Q^R$$
$$E=1\ \text{if}\ J<Q^*$$

$$=(q^R/Q^R)(J/q^R)A^R(Q^R)$$
$$=(\beta/a)\delta A^R(Q^R)$$
$$\text{or}=(BR/br)(J/q^R)(brq^R/D)$$
$$=\alpha\delta a^R(q^R)$$

These conditions are further partitioned into six cases based on the two partitioning system : CXY, X=*1,2,* and 0, and Y=*R,0.* The six cases provided in <Table A-1> are based on the specific values of δ.

For example, case *C1R* implies both the vendor and the buyer can reduce their setup costs after the synchronization arrangement. That is, $\varepsilon\delta^{1R}a^R(q^R)<a^0$ and $\alpha\delta^{1R}a^R(q^R)<A^0$ (or $A^R(Q^R)J^{1R}/Q^R<A^0$).

C1*R*:= $\{1\le\varepsilon\delta^{1R}<a^0/a^R(q^R), \delta^{1R}<A^0/\alpha a^R(q^R)\}$

If the second condition of *C1R* does not hold (that is, the vendor cannot reduce his setup cost), while the first condition holds(the buyer can reduce his setup cost), then case *C10* will apply. If δ is such that $a^R(q^R)\le\varepsilon\delta^{1R}a^R(q^R)<a^0$(leading to *C1*: $1/\varepsilon\le\delta^{1R}<a^0/\varepsilon a^R(q^R)$), then the roll-back adjustment results in a new setup cost $a^*(J^*)=\varepsilon\delta^{1R}a^R(q^R)$. To the contrary, if $a^R(q^R)>\varepsilon\delta^{1R}a^R(q^R)$, then, by definition, ε=1. Assume now that $\varepsilon a^R(q^R)\le\varepsilon\delta^{2R}a^R(q^R)<a^R(q^R)$(leading to *C2*: $1\le\delta^{2R}<1/\varepsilon$) ; then case *C2* applies and the setup cost is $a^*(J^*)= a^R(q^R)$. The six simultaneously optimal adjustment factors given in <Table 2-1> are obtained from substituting $a^*(J^*)$ and $A^*(J^*)$ as given there. For example,

<Table A-1> Six Collectively Exhaustive and Mutually Exclusive Cases

C1*R*:= $\{1\le\varepsilon\delta^{1R}<a^0/a^R, \delta^{1R}<A^0/\alpha a^R\}$*	C10:= $\{1\le\varepsilon\delta^{10}<a^0/a^R, \delta^{10}\ge A^0/\alpha a^R\}$
C2*R*:= $\{1\le\delta^{2R}, \delta^{1R}<1/\varepsilon, \delta^{2R}<A^0/\alpha a^R\}$	C20:= $\{1\le\delta^{20}, \delta^{10}<1/\varepsilon, \delta^{20}\ge A^0/\alpha a^R\}$
C0*R*:= $\{\varepsilon\delta^{1R}\ge a^0/a^R, \delta^{0R}<A^0/\alpha a^R\}$	C00:= $\{\varepsilon\delta^{10}\ge a^0/a^R, \delta^{00}\ge A^0/\alpha a^R\}$

* $a^R=a^R(q^R)$

$$\delta^{1R} = \sqrt{\frac{[A^R(J^{1R}) + a^R(J^{1R})]h}{a^R(q^R)(\widetilde{H} + h)}} = \frac{\varepsilon br + BR}{br} \frac{h}{h + \widetilde{H}} = \frac{\varepsilon + \alpha}{1 + \beta}$$ □

(1.2) Assume *C1R* applies. Suppose, for the purpose of contradiction, that $\delta^{1R}\varepsilon \geq 1$ and $\delta^{2R}\varepsilon < 1$ hold. δ^{XR}, X=*1,2* can be rearranged to :

$(\delta^{1R})^2(1+\beta) - \varepsilon\delta^{1R} - \delta^{1R}\alpha = 0$........(A.2.1), and

$(\delta^{2R})^2(1+\beta) - 1 - \delta^{2R}\alpha = 0$ ....(A.2.2).

Substituting $\delta^{2R}\varepsilon < 1$ into (A.2.2) yields $(\delta^{2R})^2(1+\beta) - \delta^{2R}\varepsilon - \delta^{2R}\alpha > 0$....(R.1). Result (R.1) and equation (A.2.1) imply $\delta^{1R}(\delta^{1R}(1+\beta) - (\varepsilon + \alpha)) < \delta^{2R}(\delta^{2R}(1+\beta) - (\varepsilon+\alpha))$, which leads to $\delta^{2R} > \delta^{1R}$. This then contradicts $\delta^{1R}\varepsilon \geq 1$ and $\delta^{2R}\varepsilon < 1$. Therefore, it follows that for case *C1R*, $\delta^{2R}\varepsilon \geq 1$ holds ; this leads to $\delta^{2R} \leq \delta^{1R}$ through a similar analysis. Other cases can be verified similarly.

(1.3) The conditions listed in <Table 2-2> are obtained from substituting $\delta^* = \delta^{XY}$ provided in <Table 2-1> into the six cases listed in <Table A-1>. For example, *C2R* satisfies $\delta^{2R} < A^0/\alpha a^R(q^R)$, $\delta^{1R} < 1/\varepsilon$ (see <Table A-1>), and $1 \leq \delta^{2R}$. Now, substituting $\delta^{2R} = (\alpha + \sqrt{\alpha^2 + 4(1+\beta)})/2(1+\beta)$ (see <Table 2-1>) into $\delta^{2R} < A^0/\alpha a^R(q^R)$ leads to $1+\beta < I(A^0/a^R(q^R), \alpha)$. Similarly, $\delta^{1R} < 1/\varepsilon$ and $1 \leq \delta^{2R}$ lead to $(\varepsilon+\alpha)\varepsilon \leq 1+\beta \leq 1+\alpha$. We now show that *C1R* is disjointed from the other cases. Using a similar argument, it is not difficult to show the remaining cases. *C1R, C0R,* and *C2R* are easily seen to be disjointed. To show that *C1R* and *C20* we apply the result of proposition (1.2). It tells us that that case *C1R* implies $\delta^{1R} \geq \delta^{2R}$. This and the condition $\delta^{1R} < A^0/\alpha a^R(q^R)$ in *C1R* imply $\delta^{2R} < A^0/\alpha a^R(q^R)$. Proposition (1.2) (ii) reveals $\delta^{2R} \geq \delta^{20}$ for case *C20*. This and $\delta^{2R} < A^0/\alpha a^R(q^R)$ implies $\delta^{20} < A^0/\alpha a^R(q^R)$, which contradicts the second condition of *C20* (see <Table A-1>). *C1R* and *C0R*, and *C1R* and *C00*, can be verified similarly. □

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