Journal of the Optical Society of Korea, Vol. 4, No. 1, March 2000, pp. 30-36

Optimization of Diffractive Optical Elements by Genetic Algorithm

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(Received December 17, 1999)

In this paper, a method based on the Genetic Algorithm(GA) for phase optimization is proposed. The programmable hybrid optical interconnection system implemented by the spatial light modulator is tested for a near-real-time optical processing. Designed diffractive grating has a 74.7[%] high diffraction efficiency and a 1.73×10^{-1} uniformity quantitatively. The dependence of characteristics on several parameters in the grating design are analyzed. Also, as a result of the geometrical transformation to obtain quantitative data for 3×3 spot beams, an objective optical experiment using CCD array detector produces the mean of beam intensity as a gray level of 202, the maximum value is 225, the minimum value is 186, and uniformity is quantitatively 1.93×10^{-1}, similar to the simulation result.

I. INTRODUCTION

Spatial light modulators(SLMs) are electro-optic devices that can modulate properties of an optical wavefront, for example, the amplitude, phase, intensity, and polarization. [1,2] The programmability in signal processing is primarily due to the advances in spatial light modulators that allow construction of various types of near-real time hybrid optical processing. Thus, advances could be made to overcome pure optical processing's difficulties in programming, increasing accuracy, and decision-making.

Because of high diffraction efficiency and flexibility of design, phase diffractive optical elements [1,2] can be used in many applications, including optical signal processing, optical computing, optical interconnection, and spatial filtering. Since reconstruction noise is a serious problem in many applications, it is necessary to optimize the phase distribution in order to decrease the noise. Several methods, for example, DBS(direct binary search), [3] SA(simulated annealing), [4-6] ANN(artificial neural network), [7] have been adopted for phase optimization.

Optimization techniques identify three main types (calculus-based, enumerative, and random). Calculus-based searches, such as steepest-descent and hill-climbing lack robustness. Enumerative searches, for example, dynamic programming and linear programming lack efficiency. But, random searches, such as GA [4,8,9] and SA are empirically proven to provide a robust and efficient search. [8] We propose a method based on GA for phase optimization. A real-time optical interconnection system is tested by diffractive gratings using GA. The characteristics several parameters in the desired diffractive gratings design are analyzed.

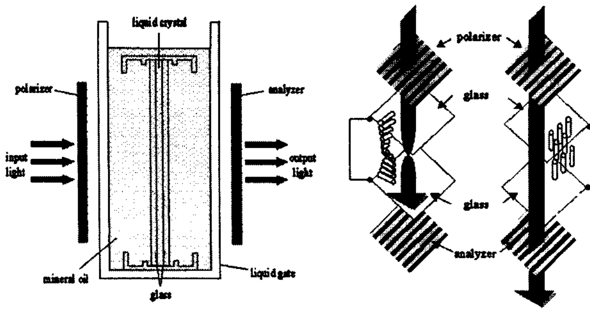
II. BASIC CONCEPT OF SLM AND DIFFRACTIVE GRATING

II. A. SLM

SLMs [1,2] are very primary components in many electro-optic systems. These devices are classified into two major classes. An electrically addressed SLM is usually constructed of a pixelated structure and its advantage is an ability to interface with electronic units and its disadvantages are low efficiency due to a dead zone between the electrodes and multiple diffraction patterns which are caused by the pixelated structure. An optically addressed SLM consists of a continuous structure and its advantage is a capability to modulate a light beam by another light beam and its disadvantage is high fabrication cost.

The LCTV [1,2] was originally produced as a pocket television set. A commercially available LCTV with proper modification could be used as an electrically addressed SLM. That is useful mainly for rudimentary demonstrations and optical quality is not outstanding. The most important attribute is extremely low cost, as compared with other SLM technologies.

The basic structure and operation of the LCTV is shown in Fig. 1. Each liquid crystal cell of the LCTV is a nematic liquid crystal twisted by 90° and individu-



(a) Structure (b) Basic operation

FIG. 1. A LCTV as an SLM, (a) Structure, (b) Basic operation.

ally controlled by electronic signals. When no electric field is applied, the plane of polarization for linearly polarized light is rotated by 90° by the twisted liquid crystal molecules and so, no light can be transmitted through the analyzer, as shown in the left part of Fig. 1(b). However, when an electric field is applied, the twist and tilt of the molecules are altered, and the liquid crystal molecules attempt to align parallel with the electric field, which results in a partial transmission of light through the analyzer. As the electric field increases further, all the liquid crystal molecules align in the direction of the applied field and so all the light passes through the analyzer, as illustrated in the right part of Fig. 1(b). As a consequence varying the applied voltage at each liquid crystal cell means light transmission will be varied.

In this paper, diffractive spots from displaying the diffractive gratings on the LCTV could be obtained at the back focal plane of the Fourier transform lens. In this optical configuration, a programmable hybrid optical processing can be realized if an SLM with sufficient space bandwidth product(SBWP) and resolution is available for the display of the computer-generated complex spatial filter.

II. B. Design of diffractive grating

The diffractive grating [1,2] for a set of opaques, or slits spaced at a constant distance, is a useful optical device to generate a regular array in the Fourier plane. There are phase types for which optical thickness or refraction index changes regularly, and amplitude types, for which amplitude transmittance varies. Especially, the phase diffractive grating composed of multi-phase levels to provide continuous phase distribution produces a high diffractive efficiency.

In this paper, to model the binary diffractive grating which can be applied usefully in the fields of free-space optical interconnection and optical computing, the manufacture of pixel type(cell-based) is applied. After dividing one period of grating into $K \times L$ cells, each cell is given the phase of 0 or π. According to the state of grating presented with two gray values of 0 or 255, the electrical signal makes each cell of the programmable LCTV turn on or off.

When a cell-based phase diffractive grating designed by microcomputer is illuminated by a plane wave, the reconstructed spots are derived in the focal plane of the Fourier lens and the wavefront from the grating is given by $\exp(i\phi)$, where ϕ is the phase distribution designed to have a value between 0 and 2π. In most cases, the phase ϕ can be quantized because of the phase-modulation property of the recording device.

The intensity distribution incident on the focal plane in which the coordinates of the output plane are denoted by (m, n) can be written as

$$I(m,n) = |\frac{1}{KL}\sum_{k=1}^{K}\sum_{l=1}^{L}\exp(i\phi_{kl}) \times \exp(-2\pi i[mk/K + nl/L])|^2 \qquad (1)$$

where K and L are a cell number of the periodic diffractive grating, k and l are each cell position of the periodic diffractive grating, m and n are a relative position of diffracted beams in the output plane.

III. GENETIC LEARNING ALGORITHM

GA based on the mechanism of natural genetics is a search algorithm developed by J. Holland and his colleagues in 1975. It has been focused on research for performance improvement, optimization problems, and machine learning using a classifier system. This paper is an example of research for optimization that uses GA as a tool.

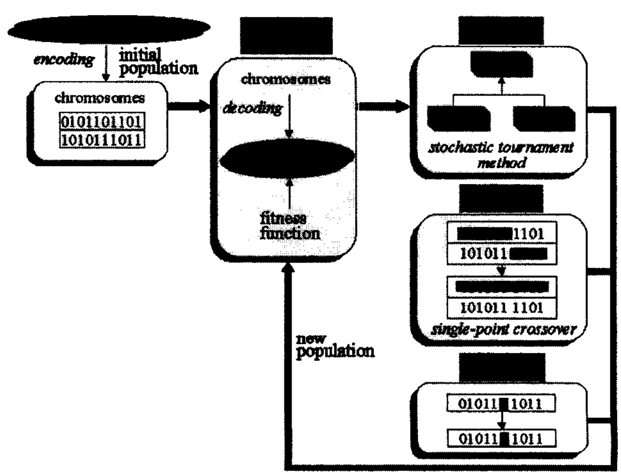


FIG. 2. Block diagram of genetic algorithm.

GA is different from traditional methods in the following four ways. Firstly, it works with a coding of the parameter set, not the parameters themselves. Secondly, it searches from a database of points simultaneously, not a point. Thirdly, it uses only objective function information, not all auxiliary information. Fourthly, it uses a probabilistic transition rule, not deterministic. Thus, GA is theoretically and empirically proven to provide a robust search in complex spaces.

GA is composed of selection, crossover, and mutation operators. To adapt GA for optimizing gratings, a chromosome is coded as a binary integer of length 32 × 32, a selection with the stochastic tournament selection method for decreasing the stochastic sampling error is performed, and a single-point crossover having 16 × 16 block size is used. A flow diagram of the learning procedure is illustrated in Fig. 2.

III. A. Coding and initialization of the population

A binary vector is used as a chromosome to represent real values of the variable. The length of the vector depends on the required precision. To create a population of chromosomes, all chromosomes are initialized at random. The coding of the problem often moves GA to operate in a different space. Such failures are caused by a premature convergence [10] to a local optimum. The premature convergence is a common problem of GA and other optimization algorithms. If convergence occurs too rapidly, then valuable information developed in part of the population is often lost.

In this paper, a pixelated binary phase grating having 32 × 32 size constructed of 0 and π is designed. The size of the initial population is 100, 200, 300, 400, 500, 600..

III. B. Mapping to fitness function

In many problems, the objective is naturally stated as the minimization of some cost function $g(x)$ rather than the maximization of some utility function. Even if the problem is stated in maximization form, the utility function will be nonnegative for all variables. Conclusively, it is often necessary to map the underlying natural objective function to a fitness function form through one or more mappings.

In order to consider simultaneously diffraction efficiency and uniformity between diffraction spots in the output plane, $g(x)$ can be written as

$$g(x) = \sum_{m=1}^{M}\sum_{n=1}^{N}(T_{mn}/spots - I_{mn})^2 + W\sum_{m=1}^{M}\sum_{n=1}^{N}(eff/spots - I_{mn})^2 \qquad (2)$$

where m and n is a cell position of the grating, T_{mn} is a target value, I_{mn} is an intensity of diffraction beams obtained, W is a weight factor, *spots* is the number of diffraction beams, eff is a calculated efficiency and $eff = \sum I_{mn}$. The first term of Eq. (2) considers diffraction efficiency and the second term considers uniformity. In this paper, W=10, T_{mn}=1.0, m=32, n=32.

To transform a minimization problem into a maximization problem, the cost function is multiplied by minus one. Also, to be nonnegative, the cost function is added to C_{max} taken as an input constant, as the largest $g(x)$ value observed in the current population. The following cost-to-fitness transformation is used.

$$f(x) = C_{max} - g(x) \qquad (3)$$

III. C. Genetic operators

Selection is a process in which chromosomes are copied by their fitness function values as some measure of profit that we want to maximize. Copying chromosomes to their fitness function values means that chromosomes with a higher value have a higher probability of surfacing in the next generation.

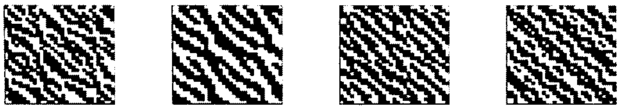
Tools to copy chromosomes are a roulette wheel selection method, tournament selection method, proportional selection method, stochastic tournament selection method and so forth. In this paper, a stochastic tournament selection method is applied. The selection probabilities are calculated normally and pairs of chromosomes are elected using a roulette wheel selection. After electing a pair, the chromosome with higher fitness is the winner, inserted in the new population, and another pair is elected. This process continues until the mating pool is full.

After reproduction, crossover may proceed. Members of the newly selected strings are mated at random, and then each pair of strings undergoes crossing over using a cross site chosen at random. Namely, the partial exchange of information is performed. In this paper, a single-point crossover having 16 × 16 block size is used.

Mutation is needed because selection and crossover may become overzealous occasionally and lose some potentially useful genetic information, even though they search effectively. Mutation is a random walk and the occasional random alteration of the value of a string position occurs with small probability. This means changing 1 to 0 and inversely.

IV. EXPERIMENTAL RESULTS

The optical interconnection based on designed beam



(a) 5×5 (b) 3×3 (c) '+' (d) '×'

FIG. 3. Gratings for generating various spot beams, (a) 5×5, (b) 3×3, (c) '+', (d) '×'

TABLE 1. Experimental condition

Optical source	He-Ne laser (633nm)
LC-SLM	Epson TFT LCD Resolution : 640×480 Frame speed : 56 Hz Contrast ratio : 200:1
CCD array detector	Panasonic : 768×494
Image board	Kasan : WinX perfect IV

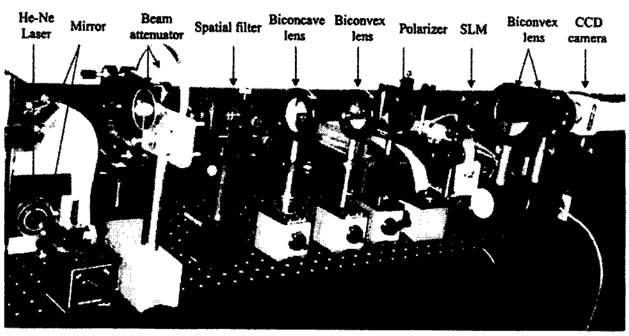
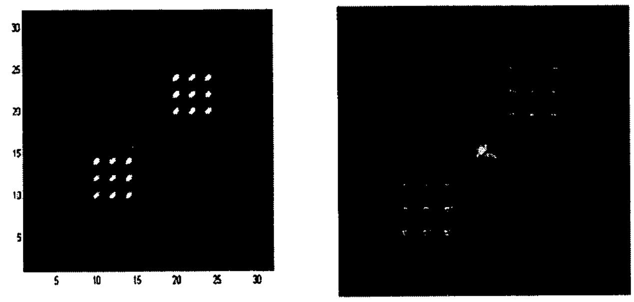


FIG. 4. System architecture for the optical interconnection using LC-SLM.

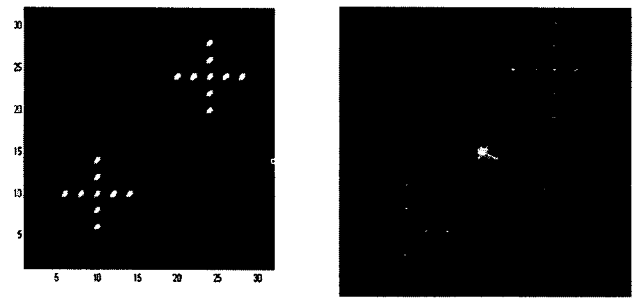
patterns projected on an output plane can be of various types, with gratings for (a) 5 × 5 (b) 3 × 3 (c) '+' (d) '×' shaped beams, as shown in Fig. 3. For the grating in Fig. 3(a), the optimum occurs when the mutation probability is 0.001, the crossover probability is 0.75, and the population size is 300. The optimum of Fig. 3(b) occurs when the mutation probability is 0.001, the crossover probability is 0.35, and the population size is 400. The optimum of Fig. 3(c) occurs when the mutation probability is 0.001, the crossover probability is 0.75, and the population size is 400. The optimum of Fig. 3(d) occurs when the mutation probability is 0.001, the crossover probability is 0.65, and the population size is 300.

Table 1 illustrats the experimental setup condition. Fig. 4 is the system architecture for the real-time optical interconnection using LC-SLM, CCD camera, IBM-PC, He-Ne laser, and Fourier transform lens. A He-Ne laser of wavelength $\lambda = 633$nm is used as a light source. A light through a beam expander and spatial filter is illustrated by collimating to the LC-SLM(TFT LCD). A designed grating is displayed on the TFT LCD with resolution of 640 × 480 SVGA using a microcomputer.



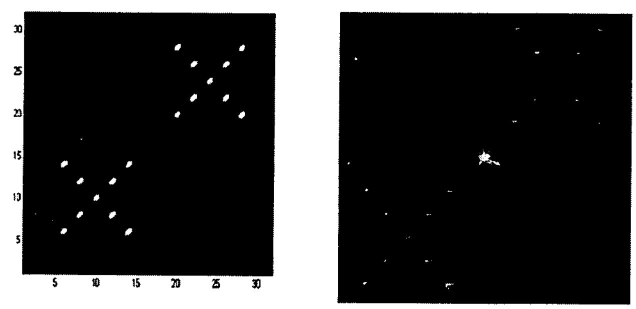
(a) Pseudocolor (b) Result of experiment

FIG. 5. 5×5 spot beams, (a) Pseudocolor, (b) Result of experiment.



(a) Pseudocolor (b) Result of experiment

FIG. 6. 3×3 spot beams, (a) Pseudocolor, (b) Result of experiment.



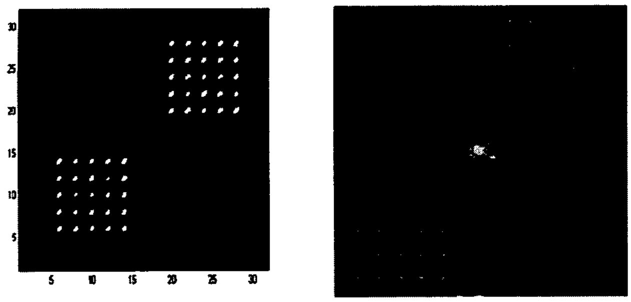
(a) Pseudocolor (b) Result of experiment

FIG. 7. '+' spot beams, (a) Pseudocolor, (b) Result of experiment.

The intensity distribution across the back focal plane of the biconvex lens is measured with the CCD array detector.

Part (a) of Figs. 5, 6, 7 and 8 are peudocolor results taken by computer simulation using gratings shown in Fig. 3. Part (b) of Figs. 5, 6, 7 and 8 are experimental results in the output plane obtained by the system architecture illustrated in Fig. 4.

In order to measure the quantitative data for the spot beams obtained at the CCD array detector by the optical experiment truthfully, a geometrical transformation is processed. The data obtained at the CCD



(a) Pseudocolor (b) Result of experiment

FIG. 8. 'x' spot beams, (a) Pseudocolor, (b) Result of experiment.

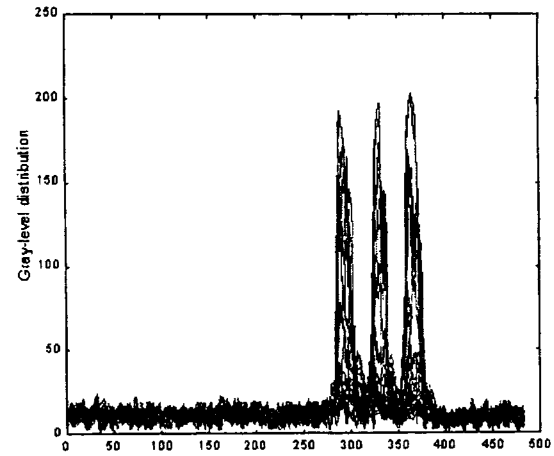


FIG. 10. 1×3 spot beams.

array detector is transformed into 0 ~ 255 gray level, then saved to raw file form. These results are shown in Fig. 9. For 3 × 3 spot beams, the mean of beam intensity as a gray level is 202, the maximum value is 225, the minimum value is 186, and a uniformity is quantitatively 1.93×10^{-1} similar to the simulation result. Here, the uniformity is a deviation between the output spots and is calculated by $(I_{max} - I_{min})/I_{avg}$.

To show clearly a deviation between the output spots, only 1 × 3 spot beams out of 3 × 3 spot beams are illustrated in Fig. 10 and are seen to be very alike without deviation. This paper's focus is to measure the quantitative data for spots captured by the CCD array detector and to analyze uniformity between the output spots obtained.

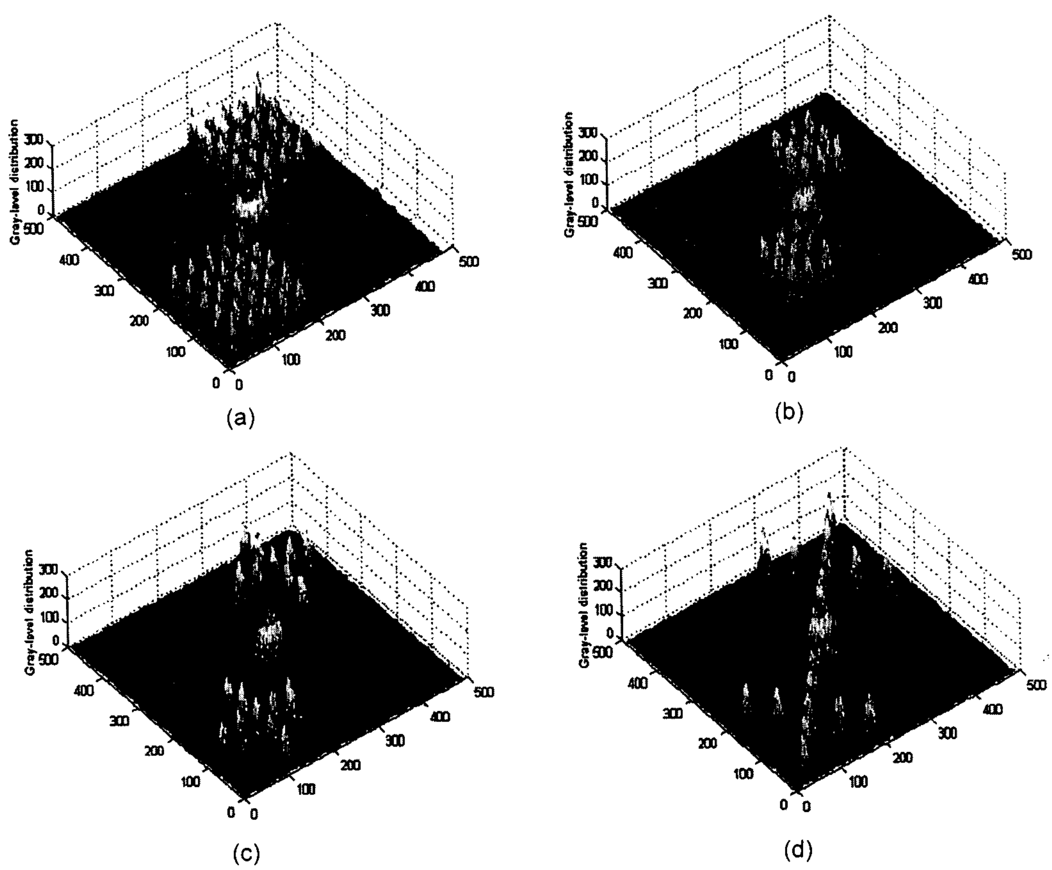


FIG. 9. Quantitative data for the result of experiment, (a) Mesh of 5×5 spot beams, (b) Mesh of 3×3 spot beams, (c) Mesh of '+' spot beams, (d) Mesh of 'x' spot beams,

TABLE 2. Optimal diffraction efficiency and uniformity for different crossover probabilities when p_m is 0.001 and *pop_size* is 300

p_c [%]	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.45	0.55	0.65	0.75	0.85	0.95
generation	800	800	800	800	800	800	800	700	600	700	800	600	700
diffraction efficiency[%]	32.8	53.3	60.8	65.2	67.0	70.9	75.6	68.5	72.4	74.2	74.7	72.2	73.0
uniformity	0.932	0.443	0.304	0.492	0.298	0.220	0.250	0.283	0.274	0.212	0.173	0.207	0.238

TABLE 3. Optimal diffraction efficiency and uniformity for different mutation probabilities when p_c is 1.0 and *pop_size* is 100, 200, 300, 400, 500, 600, respectively

p_m [%]	0.0005	0.001	0.0015	0.002	0.0025	0.003	0.0035	0.004
pop_size	500	300	600	600	600	600	600	500
diffraction efficiency[%]	72.8	74.4	73.3	71.1	73.9	72.1	65.5	64.4
uniformity	0.168	0.161	0.274	0.237	0.340	0.339	0.624	0.487

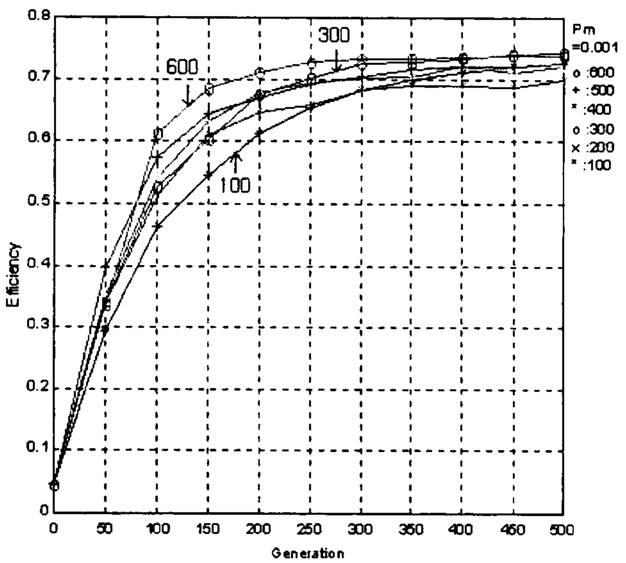


FIG. 11. Diffraction efficiency on the generation when p_m is 0.001, p_c is 1.0, and *pop-size* is 100, 200, 300, 400, 500, 600, respectively.

V. THE DEPENDENCE OF THE CHARACTERISTICS ON THE GENETIC PARAMETERS

In the grating design for 5 × 5 spot beams, the dependence of the characteristics on several parameters, that is, the crossover probability, the mutation probability, and the population size are analyzed. Firstly, for the analysis of the effect on the crossover probability(p_c) where the mutation probability(p_m) is 0.001, the population size(*pop_size*) is 300, and the crossover probability is 0.05 ~ 0.95, we observe that a designed grating with crossover probability of 0.75, has a 74.7[%] diffraction efficiency and a 1.73×10^{-1} uniformity as shown in Table 2.

Secondly, for analysis of the effect on the mutation probability where the crossover probability is 1.0, the population size is 100, 200, 300, 400, 500, 600, and the mutation probability is 0.0005 ~ 0.004, we observe that a designed grating with the mutation probability of 0.001 has a 74.4[%] efficiency and a 1.61×10^{-1} uniformity as shown in Table 3.

Thirdly, for the analysis of the effect on the population size where the mutation probability is 0.001, the crossover probability is 1.0, and the population size is 100, 200, 300, 400, 500, 600, we observe that a designed grating with population size of 300 and the generation of 400 has above 74[%] diffraction efficiency as illustrated in Fig. 11.

VI. CONCLUSIONS

GA is applied to minimize the error between the Fourier transform result and the desired pattern. Gratings to generate spots freely are designed using GA for a phase change of each pixelated grating, and a real-time optical interconnection system architecture using LC-SLM, CCD array detector, IBM-PC, He-Ne laser, Fourier transform lens is tested. A pixelated binary phase grating is displayed on LC-SLM and could interconnect incoming beams to desired output spots freely in real-time. To measure quantitatively the data obtained at CCD array detector by optical experiment truthfully, geometrical transformation is applied. As a consequence, for 3 × 3 spot beams, the mean of beam intensity as a gray level is 202, the maximum value is 225, the minimum value is 186, and uniformity is quantitatively 1.93×10^{-1} similar to the simulation result.

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