한국정밀공학회지 제 17 권 제 11 호 (2000 년 11 월)
Journal of the Korean Society of Precision Engineering, Vol. 17, No. 11, November 2000.

로봇 매니퓰레이터의 비선형 H_∞ 제어

임종국*, 박종현**

Nonlinear H_∞ Control of Robot Manipulators

Jongguk Yim* and Jong Hyeon Park**

ABSTRACT

비선형 시스템에 대한 H_∞ 제어 이론은 에너지 소진 (energy dissipation) 개념을 기초로 개발되어 왔다. 에너지 소진을 이용한 비선형 H_∞ 제어기는 외란과 성능 벡터 사이의 L_2 게인의 비를 일정이하로 만드는 방법으로 설계되고, 그 적용을 위해서는 헤밀턴 자코비 부등식의 해를 구하는 것이 필수적이지만, 일반적으로 헤밀턴 자코비 부등식의 해를 구하는 것은 매우 어렵다. 본 논문에서는 로봇 매니퓰레이터의 운동 방정식을 변형하여 헤밀턴 자코비 부등식의 해를 구하기 쉬운 형태, 즉 비선형 행렬 부등식으로 표현하고, 운동 방정식을 구성하는 행렬의 각 항들이 한계가 존재한다는 것을 이용하여 그 부등식의 근사해를 구하였다.

Key Words : H_∞ control(H_∞ 제어), Hamilton Jacobi inequality(헤밀턴 자코비 부등식), Nonlinear matrix inequality(비선형 행렬 부등식), Robot manipulators(로봇 매니퓰레이터)

1. Introduction

H_∞ controllers in linear systems can be obtained in the state space by solving Riccati equation [1,2]. Another approach to H_∞ control in linear systems is a Linear Matrix Inequality (LMI) technique, where the solution is obtained by efficient convex optimization algorithm [10].

Recently H_∞ control problem for nonlinear systems has attracted attention of many researchers. Although the H_∞ control theory in nonlinear systems has been derived by the L_2-gain analysis based on the concept of the energy dissipation [3,5], its applications are not easy due to the solvability of the Hanmilton Jacobi (HJ) inequality. The HJ inequality is a first-order partial differential inequality and it is difficult to obtain its solution in general. H_∞ control problem in nonlinear systems reduces to the existence of solution to HJ inequality. Van der Schaft suggested the successive approximated solution to the HJ inequality [3]. Based on this method, Hu designed a nonlinear H_∞ controller for the inverted pendulum system [7]. Using modified Lyapunov function including a mixed term in link positions and velocities, Astolfi designed a robust PD controller for robot manipulators, whose gain was obtained by solving the associated HJ inequality [4]. As another application to robot manipulators, there was Hamiltonian optimization method using the fact that the approximated solution to HJ equation at equilibrium point is equal to the solution

* 한양대학교 정밀기계공학과 대학원
** 한양대학교 기계공학부

to the Riccati equation [6].

In this paper, the robot dynamics is transformed to the affine nonlinear system about states and input and the associated HJ inequality is derived in the form of a more tractable NLMI. The approximated solution to the NLMI can be obtained from the fact that the terms in matrices that describe the robot manipulators are bounded by trigonometric functions. If the matrices forming the NLMI are bounded, then we only need to solve the finite number of LMIs. For a 2 DOF planar manipulator with mass uncertainty, a H_∞ controller is designed and the robustness is shown through simulation.

This paper is organized as follows. In section Ⅱ, we review the concept of the energy dissipation and an important theorem concerning the nonlinear H_∞ control problem. In section Ⅲ, the nonlinear system is represented in the form of affine nonlinear system about the states and inputs and the associated HJ inequality is transformed to the more tractable NLMI. In section Ⅳ, the dynamics of the robot manipulators is transformed to the affine form using modified error vector for tracking and a possible method is proposed to obtain the approximated solution to the NLMI. In section Ⅴ, simulations are performed to confirm the robust performances of the proposed controller for a robot manipulators under parameter uncertainty. In section Ⅵ, the conclusion is presented.

2. Nonlinear State Feedback H_∞ Control

2.1 Energy Dissipative system

Consider a nonlinear system

$$\begin{aligned} \dot{x} &= f(x,w) \\ z &= h(x,w), \end{aligned} \tag{1}$$

where x is the state, w and z are the input and the output, respectively. With $\gamma > 0$, the system is said to be γ-dissipative if there exists a nonnegative energy storage function E with $E(x(0)) = 0$ such that

$$\gamma^2 \int_0^T \|w\|^2 dt - \int_0^T \|z\|^2 dt \geq E(x(T)). \tag{2}$$

The energy dissipation means that the L_2-gain of the system is less than or equal to γ. Obviously, the system is γ-dissipative if there exists a nonnegative function defined by

$$H = \gamma^2 \|w\|^2 - \|z\|^2 - (\partial E/\partial t) \tag{3}$$

is nonnegative for all x and w in the domain of interest.

2.2 Nonlinear H_∞ Control Problem

Consider a nonlinear system expressed by

$$\begin{aligned} \dot{x} &= f(x) + g_1(x)w + g_2(x)u \\ z &= h(x) + d(x)u, \quad h^T d = 0, d^T d > 0, \end{aligned} \tag{4}$$

where $x \in R^n, u \in R^m$, and $w \in R^w$ are the state, the control input, and disturbances, respectively. $z \in R^z$ represents the performance of the system.

To find a nonlinear state-feedback H_∞ control is to find a stabilizing state feedback control input such that the closed-loop system has a L_2-gain equal to or less than γ in the input-to-output sense. This problem can be solved from the concept of energy dissipation.

A theorem concerning the solution of nonlinear H_∞ control problem described above is introduced without its proof in the following theorem [3].

Theorem 1 Given $\gamma > 0$, suppose there exists a C^1 positive definite function $E(x)$ with $E(x(0)) = 0$ satisfying HJ inequality

$$E_x f - (1/2) E_x \left(g_2 [d^T d]^{-1} g_2^T - (1/\gamma^2) g_1^T g_1 \right) E_x^T + (1/2) h^T h \leq 0, \tag{5}$$

where $E_x = \partial E^T / \partial x$, then the system (4) has the L_2-gain $\leq \gamma$ as well as the closed-loop stability with control input of

$$u = -[d^T d]^{-1} g_2^T E_x^T. \tag{6}$$

The above theorem shows that to construct a nonlinear H_∞ controller, it is essential to find the solution to the associated HJ inequality derived from input-output energy dissipation. If a solution exist, then it will guarantee the stability as well as the disturbance attenuation in the L_2-gain sense. However, the HJ inequality is a first order partial differential inequality and, in general, it is difficult to find its solution.

3. State Feedback H_∞ Controller in Affine Nonlinear System

In this section, it is shown that the associated HJ inequality can be transformed to a NLMI if the nonlinear system is described in suitable form and the fact that the matrices forming it are bounded.

Suppose that the nonlinear system can be described in the form

$$\begin{aligned} \dot{s} &= F(x)s + G_1(x)w + G_2(x)u \\ z &= Hs + Du,\ H^T D = 0, D^T D > 0, \end{aligned} \tag{7}$$

where $F(x)$, $G_1(x)$, $G_2(x)$, H and D are C^0 matrix-valued functions of suitable dimensions, $s(x, x_d) \in R^n$ is a new state in which $x_d \in R^n$ is a desired trajectory vector, $w \in R^w$ is the disturbance vector caused by uncertainties and $u \in R^m$ is the control input. If the nonlinear system can be transformed to Eq. (7), the derived HJ inequality is more tractable than Eq. (5).

The design of H_∞ controller for the nonlinear system in the affine form is summarized in the following theorem.

Theorem 2 Given $\gamma > 0$, suppose there exists a C^0 matrix-valued function P satisfying

$$\begin{aligned} &P^T F(x) + F^T(x)P + (1/\gamma^2)P^T G_1(x)G_1^T(x)P \\ &+H^T H - P^T G_2(x)[D^T D]^{-1} G_2^T(x)P \le 0 \end{aligned} \tag{8}$$

and there exists a positive definite function $E(s)$ such that $\partial E/\partial s = 2s^T P^T$. Then the control input satisfying L_2-gain $\le \gamma$ is

$$u = -[D^T D]^{-1} G_2^T Ps. \tag{9}$$

Proof: Take $E(s)$ as defined in the statement, then

$$\begin{aligned} \dot{E} &= (\partial E/\partial s)\dot{s} \\ &= 2s^T P^T (Fs + G_1 w + G_2 u) \\ &= s^T (P^T F + FP)s + 2s^T P^T (G_1 w + G_2 u) \\ &= \gamma^2 \|w\|^2 - \|z\|^2 + s^T (P^T F + FP)s \\ &\quad + (1/\gamma^2) s^T P^T G_1 G_1^T s - \gamma^2 \left\| w - (1/\gamma^2) G_1^T Ps \right\|^2 \\ &\quad + 2s^T P^T G_2 u + s^T H^T Hs + u^T D^T Du \qquad (H^T D = 0). \end{aligned}$$

When matrix D is not a square matrix, nonsingular matrix R satisfying $D^T D = R^T R$ is defined. Then

$$\begin{aligned} \dot{E} &= \gamma^2 \|w\|^2 - \|z\|^2 + s^T (P^T F + FP)s + s^T H^T Hs \\ &\quad + (1/\gamma^2) s^T P^T G_1 G_1^T s - \gamma^2 \left\| w - (1/\gamma^2) G_1^T Ps \right\|^2 \\ &\quad + 2s^T P^T G_2 R^{-1} Ru + u^T R^T Ru \\ &= \gamma^2 \|w\|^2 - \|z\|^2 - \gamma^2 \left\| w - (1/\gamma^2) G_1^T ps \right\|^2 \\ &\quad + s^T \{ P^T F + FP + (1/\gamma^2) P^T G_1 G_1^T P + H^T H \\ &\quad - P^T G_2 (R^T R)^{-1} G_2^T P \} s + \left\| Ru + R^{-T} G_2^T Ps \right\|^2 \\ &\le \gamma^2 \|w\|^2 - \|z\|^2 \qquad \text{(by Eq. (8) and (9))}. \end{aligned}$$

Integrating the inequality results in

$$E(s(T)) - E(s(0)) \le \gamma^2 \int_0^T \|w\|^2 dt - \int_0^T \|z\|^2 dt.$$

Since $E(s(T)) \ge 0$ and $E(s(0)) = 0$, the closed-loop system becomes dissipative.

To check the asymptotic stability of the closed-loop system with $w = 0$, consider the Lyapunov function candidate

$$L(s) = E(s),$$

where $E(s)$ is positive definite. From the closed loop HJ inequality Eq. (8), $\dot{E}$ satisfies

$$\dot{E} = P^T F(x) + F^T(x)P - P^T G_2(x)[D^T D]^{-1} G_2^T(x)P \le 0$$

and therefore the closed-loop system is stable.

To obtain the solution to the Eq. (8) easily, it is transformed to a NLMI using the Schur complement as

$$\begin{bmatrix} P^T F + F^T P + H^T H + (1/\gamma^2) P^T G_1 G_1^T P & P^T G_2 \\ G_2^T P & D^T D \end{bmatrix} \le 0.$$

Solving the above NLMI yields convex optimization problem. Unlike the linear case, this convex problem is not finite dimensional. However, if the matrices forming the NLMI are bounded, then we only need to solve a finite number of LMIs [8].

4. H_∞ Control for Robot Manipulators

4.1 Dynamic Equation in Affine Form

Consider the dynamics of an n-link robot manipulator

$$M(q)\ddot{q} + N(q,\dot{q})\dot{q} + G(q) = \tau, \tag{10}$$

where $q \in R^n$ is the joint position, $M(q) \in R^{n\times n}$ is the positive definite symmetric inertia matrix, $N(q,\dot{q}) \in R^{n\times n}$ represents the centripetal and coriolis torque, and $G(q) \in R^n$ represents the gravitational torque.

Modified error vector is defined as

$$s = \dot{q} - \{\dot{q}_d - \Lambda(q - q_d)\} = \dot{q} - \dot{q}_r, \tag{11}$$

where q_d and $\dot{q}_d$ are the desired position and velocity, respectively. If the elements of vector s approach to zeros as $t \to \infty$, so does the tracking error of each joint.

Using the modified error vector, we can transform the robot dynamics to affine form.

Proposition 1. Using Eq. (11) and a suitable choice of control input τ, Eq. (10) can be transformed to

$$\dot{s} = F(q,\dot{q})s + G_1(q)w + G_2(q)u, \tag{12}$$

where each matrix is

$$F(q,\dot{q}) = -M^{-1}(q)N(q,\dot{q})$$
$$G_1(q) = G_2(q) = M^{-1}(q).$$

Proof: To transform Eq. (10) to Eq. (12) the following control input is proposed.

$$\tau = \hat{M}(q)\ddot{q}_r + \hat{N}(q,\dot{q})\dot{q}_r + \hat{G}(q) + u, \tag{13}$$

where $\hat{M}(q)$, $\hat{N}(q,\dot{q})$ and $\hat{G}(q)$ are the estimates of $M(q)$, $N(q,\dot{q})$ and $G(q)$, respectively.

Substituting Eq. (13) into Eq. (10),

$$M(q)\dot{s} + N(q,\dot{q})s = \tilde{M}(q)\ddot{q}_r + \tilde{N}(q,\dot{q})\dot{q}_r + \tilde{G}(q) + u, \tag{14}$$

where the model estimate errors are

$$\tilde{M}(q) = \hat{M}(q) - M(q),$$
$$\tilde{V}(q,\dot{q}) = \hat{V}(q,\dot{q}) - V(q,\dot{q})$$
$$\tilde{G}(q) = \hat{G}(q) - G(q).$$

Define a disturbance vector as

$$w = \tilde{M}(q)\ddot{q}_r + \tilde{N}(q,\dot{q})\dot{q}_r + \tilde{G}(q),$$

then Eq. (14) becomes

$$M(q)\dot{s} = -N(q,\dot{q})s + w + u \tag{15}$$

Multiplying the inverse of $M(q)$ to both sides of Eq. (15), we can obtain Eq. (12).

4.2 The Solution to HJ Inequality using LMI

To obtain the HJ inequality for the robot manipulator dynamics transformed to affine form, each matrix term of Eq. (12) is substituted into Eq. (8). Then

$$\begin{aligned} &-(MP^{-T})^{-1}N - N^T(P^{-1}M^T)^{-1} + H^TH \\ &+(1/\gamma^2)(MP^{-T})^{-1}(P^{-1}M^T)^{-1} \\ &-(MP^{-T})^{-1}(D^TD)^{-1}(P^{-1}M^T) \le 0. \end{aligned}$$

Premultiplying and postmultiplying the inequality by positive definite matrices MP^{-T} and $P^{-1}M^T$ respectively, then the HJ inequality becomes

$$\begin{aligned} &-NQM^T - MQ^TN^T + MQ^TH^THQM^T \\ &\left(1/\gamma^2\right)I - (D^TD)^{-1} \le 0, \end{aligned} \tag{16}$$

where $Q = P^{-1}$. Using the Schur complement, Eq. (16) can be described as a NLMI

$$\begin{bmatrix} -NQM^T - MQ^TN^T + \left(1/\gamma^2\right)I - (D^TD)^{-1} & MQ^TH^T \\ HQM^T & -I \end{bmatrix} \le 0. \tag{17}$$

The matrices $M(q)$ and $N(q,\dot{q})$ is the nonlinear function of q and $\dot{q}$ in Eq. (17). However, those matrices include trigonometric functions and can be bounded when each joint velocity ranges between two determined extreme values. Using this fact, we suppose that the matrices forming the above NLMI vary in some bounded sets of the space of matrices, i.e.,

$$[M(q), N(q,\dot{q}), H, D] \in Co\{[M_i, N_i, H, D]|_{i\in\{1,2,\cdots,L\}}\},$$

where Co denote the convex hull.

Therefore, if

$$\begin{bmatrix} -N_iQM_i^T - M_iQ^TN_i^T + (1/\gamma^2)I - (D^TD)^{-1} & M_iQ^TH^T \\ HQM_i^T & -I \end{bmatrix} \le 0$$

have a common nonsingular matrix solution Q for all $i \in \{1,2,\cdots,L\}$, then Q is also a solution to Eq. (17) and the stabilizing control input is determined as

$$u = -(D^TD)^{-1}G_2^TQ^{-1}s.$$

This approach provides a tractable method to get constant solutions to NLMI, which can be used to design the control input.

5. Simulation

A nonlinear H_∞ controller is designed for a 2 DOF planar robot manipulators with uncertainty in its mass. Simulations were performed for two cases according to the size of mass bound to evaluate the proposed H_∞ controller. The objective of the simulation is to show the enhancement of robustness to parameter uncertainty. The set of dynamic parameter is summarized in Table 1. As an extreme disturbance, the mass of link 2 is assumed to vary by 50% at 2 second. The system model matrices forming LMIs are determined by the bound of parameter uncertainty and the trigonometric functions. The LMIs for the matrix Q are solved using an efficient convex algorithm in Matlab toolbox. It should be noted that the easiness of controller tuning can be obtained since the solution of LMIs, if any, is found easily by an optimization algorithm.

The joints of manipulator are commanded to trace trajectories shown in Fig. 1 with some initial errors. The initial errors of the joints are 11.45° and 17.19°, respectively. The estimates of the manipulator model matrices in Eq. (13) are assumed to be $\hat{M} = 0$ and $\hat{N} = 0$. The estimate of the gravity torque G is determined from the equation in the dynamics using the estimates of mass $\hat{m}_1 = 1.8kg$ and $\hat{m}_2 = 0.8kg$.

The performance level can be determined by parameter γ and weighting matrix H and the control input energy can be adjusted by using matrix D. In the simulation, matrix H and matrix D are selected as

$$H = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \end{bmatrix}^T, \; D = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}^T.$$

The value of γ is selected as 7 such that the solution of given LMIs is feasible.

The position error and torque are shown in Fig. 2~3, respectively. The error in each case is similar. However, when the bound of mass uncertainty is large, the control torque increases in initial state. This shows that the large prescribed bound leads to a conservative results. As long as the described control input is within feasible range, the proposed controller shows satisfactory robustness performance even under the large parameter uncertainty.

In Table 2, the maximums of errors of joints are shown when the mass of link 2 is increased by 25% in Case 2. The proposed controller shows robustness to mass variation as though the errors increase.

Table 1 Manipulator parameters used in the simulation

	Real Length	Real Mass	Bound of Mass	
			Case 1	Case 2
Link1	0.5 m	2 kg	[1.5,2.5]	[0.5,4]
Link2	0.3 m	1 kg	[0.5,1.5]	[0.2,3]

Table 2 Maximums of errors of joints in the case of mass variation (after 1.8 sec)

Increase of mass (link 2)	Maximums of errors (joint 1, joint2)
0%	(0.7447°, 0.0118°)
25%	(0.8679°, 0.0121°)
50%	(0.9949°, 0.0945°)
100%	(1.2602°, 0.2618°)

6. Conclusion

We proposed a robust controller for a tracking and disturbance attenuation of robot manipulators. The errors of parameter estimates are considered as disturbance and the robustness to model uncertainties are achieved in the sense of L_2-gain attenuation from the disturbance to performance measure. The associated HJ inequality is transformed to NLMI and its approximated solution is obtained from the fact that the terms in matrices that describe robot manipulators can be bounded by trigonometric functions. The application of the proposed controller is simple since the control gain matrix can be obtained easily by an efficient convex optimization. The proposed controller is applied to a 2 DOF manipulator and its computer simulation shows that the controller sustains its performance under the uncertainty of mass.

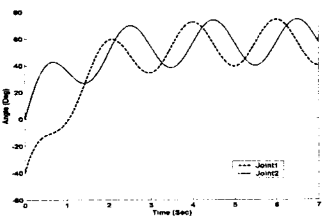

Fig. 1 Desired trajectories of joints

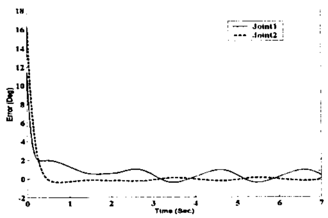

Fig. 2(a) Position errors in Case 1

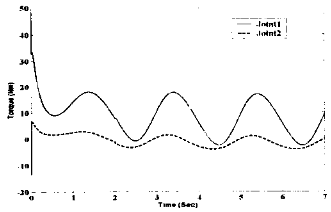

Fig. 2(b) Torque in Case 1

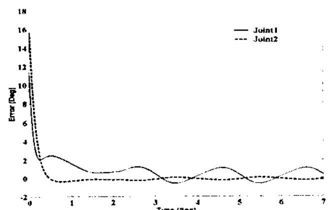

Fig. 3(a) Position errors in Case 2

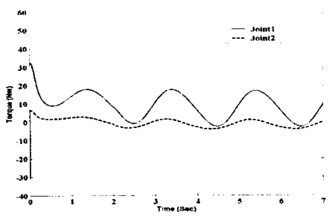

Fig. 3(b) Torque in Case 2

Reference

1. J. C. Doyle, K. Glover, P. Khargonecker and B. A. Francis, "State Space Solution to Standard H_2 and H_∞ Control Problems," IEEE Trans. Automat. Contr., Vol. 34, No. 8, pp. 831-847, 1989.
2. K. Glover and J. C. Doyle, "State-Space Formulae for All Stabilizing Controllers that satisfy an H_∞-norm Bound and Relations to Risk Sensitivity," Sys. & Contr. letters, Vol. 11, No. 2, pp. 167-172, 1988.
3. A. J. van der Schaft, "L_2-Gain Analysis of Nonlinear Systems and Nonlinear State Feedback H_∞ Control," IEEE Trans. Automat. Contr., Vol. 37, No. 6, pp. 770-784, 1992.
4. A. Astolfi and L. Lanari, "H_∞ Control of Rigid Robots," IFAC Robot Control, pp. 171-176, 1994.
5. A. Isidori and A. Astolfi, "Disturbance Attenuation and H_∞-Control Via Measurement Feedback in Nonlinear Systems," IEEE Trans, Automat. Contr., Vol. 37, No. 6, pp. 1283-1293, 1992.
6. Y. J. Choi, W. K. Chung and Y. Youm, "Roust Control of Manipulators using Hamiltonian Optimization," Proc. IEEE Inter. Conf. On Robotics and Automation, pp. 2358-2364, 1997.
7. Shr-Shiung Hu and Bor-Chin Chang, "Design of a Nonlinear H_∞ Controller for the Inverted Pendulum System," Proc. IEEE Inter. Conf. On Control and Application, pp. 699-703, 1998.
8. W. M. Lu and J. C. Doyle, "H_∞-Control of Nonlinear Systems: A Convex Characterization," Caltech CDS Memo, No. CTI-CDS-93-020, 1993.
9. F. L. Lewis, C.T. Abdallab, and D. M. Dawson, Control of Robot Manipulator, Macmillan, 1993.
10. S. Boyd, E. El Ghaoui, E. Feron, and V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, SIAM Books, 1994.