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Real-Time Digital Fuzzy Control Systems considering Computing Time-Delay

Chang-Woo Park, Hyun-Seok Shin and Mignon Park

Department of Electronic Eng., Yonsei University

ABSTRACT

In this paper, the effect of computing time-delay in the real-time digital fuzzy control systems is investigated and the design methodology of a real-time digital fuzzy controller(DFC) to overcome the problems caused by it is presented. We propose the fuzzy feedback controller whose output is delayed with unit sampling period. The analysis and the design problem considering computing time-delay is very easy because the proposed controller is syncronized with the sampling time. The stabilization problem of the digital fuzzy control system is solved by the linear matrix inequality(LMI) theory. Convex optimization techniques are utilized to find the stable feedback gains and a common positive definite matrix $\boldsymbol{P}$ for the designed fuzzy control system. Furthermore, we develop a real-time fuzzy control system for backing up a computer-simulated truck-trailer with the consideration of the computing time-delay. By using the proposed method, we design a DFC which guarantees the stability of the real time digital fuzzy control system in the presence of computing time-delay.

1. Introduction

A real-time digital fuzzy control can be thought of as a three-stage process : data acquisition from sensors, data processing to generate control command, and outputting the results to actuators. Although each of the three stages will take time to complete, this paper is concerned only with the time taken by the most complicated stage, data processing, since the other two are much simpler and more static. More precisely, the time taken to execute programs that implement control algorithms, computing time-delay is the subject of this paper. For a given fixed sampling interval the effect of computing time-delay are classified into the delay and loss problems. The delay problem occur when the computing time-delay is nonzero but smaller than the sampling interval, while the loss problem occur when the computing time-delay is greater than, or equal to, the sampling interval, i.e., loss of the control output. We will focus on the delay problem on the performance of the fuzzy control systems since the computing time-delay is a piecewise continuous, random variable which is usually smaller than the corresponding sampling interval. Note that due to its randomness, the computing time-delay is totally different from system delay, which is not the subject of this paper.

The control problems for the real time digital fuzzy control systems with the time-delay in computing the control output have been paid attention over a few decades since the computing time-delay is frequently a source of instability and encountered in the implementations of the various engineering systems. Extensive research has already been done in the conventional control to find the solutions [1,2]. However, for fuzzy control systems, there are few studies on the stabilization problem especially for real-time control systems considering computing time-delay [3,4]. A linear controller like PID controllers has a short computing time-delay in calculating the output since its algorithm is so simple. However, in the case of a complex algorithm like fuzzy or neural networks, a considerable time-delay in computing the output can occur because so many calculations are needed to get the output. Nevertheless, the most conventional discrete time fuzzy controllers are the ideal controllers not considering the problems in the digital implementation, computing time-delay. Recently, to deal with the time-delay, the design methods of the fuzzy control systems with higher order have been proposed in [5]. However the structure of the control system is very complex because the design of higher order fuzzy rule-base is very difficult.

In this paper, we raise the computing time-delay problems when control algorithms are implemented on a digital computer and to remedy these problems, the real-time digital fuzzy control system considering computing time-delay is developed and its stability analysis and design method are proposed. We use the discrete Takagi-Sugeno(TS) fuzzy model and parallel distributed compensation(PDC) conception for the controller[6-9]. And we follow the linear matrix inequality(LMI) approach to formulate and solve the problem of stabilization for the fuzzy controlled systems. The analysis and the design of the discrete time fuzzy control systems by LMI theory are considered in [10-12].

If the considerable computing time-delay exists the analysis and the design of the controller are very difficult since the time-delay makes the output of the controller not synchronized with the sampling time. We propose the PDC-type fuzzy feedback controller whose output is delayed with unit sampling period and predicted using current states and the control input to the plant at previous sampling time. In this scheme, the computing time-delay is approximated to be one sampling period. The analysis and the design of the controller are very easy because the output of the proposed controller is synchronized with the sampling time. Therefore, the proposed control system can be designed using the conventional methods for stabilizing the discrete time fuzzy systems and the feedback gains of the controller can be obtained using the concept of the LMI feasibility problem.

The proposed real-time DFC is applied to backing up control of a computer-simulated truck-trailer considering the computing time-delay in the digital implementation of the system to verify the validity and the effectiveness of the control scheme. Note that the term Digital fuzzy control system is used corresponding to the existing "Discrete time fuzzy system" to emphasize the proposed aspect that the problem in the real-time implementation of the controller, computing time-delay is considered.

2. Discrete TS Model Based Fuzzy Control

In the discrete time TS fuzzy systems without control input, the dynamic properties of each subspace can be expressed as the following fuzzy IF-THEN rules[6].

$$\begin{aligned} &\text{Rule } i: \text{If } x_1(k) \text{ is } M_{i1} \ldots \text{and } x_n(k) \text{ is } M_{in} \\ &\qquad \text{THEN } \boldsymbol{x}(k+1) = \boldsymbol{G}_i\boldsymbol{x}(k) \end{aligned} \quad i = 1,2,\ldots,3 \tag{1}$$

where $\boldsymbol{x}(k) = [x_1(k)\ x_2(k)\ \ldots\ x_n(k)]^T \in \Re^n$ denotes the state vector of the fuzzy system, r is the number of the IF-THEN rules, and M_{ij} is fuzzy set.

If the state $\boldsymbol{x}(k)$ is given, the output of the fuzzy system expressed as the fuzzy rules of Eq. (1) can be inferred as follows.

$$\boldsymbol{x}(k+1) = \frac{\sum_{i=1}^{r} w_i(k)\boldsymbol{G}_i\boldsymbol{x}(k)}{\sum_{i=1}^{r} w_i(k)} = \sum_{i=1}^{r} h_i(k)\boldsymbol{G}_i\boldsymbol{x}(k) \tag{2}$$

where $w_i(k) = \prod_{j=1}^{n} M_{ij}(x_j(k))$, $\quad h_i(k) = \dfrac{w_i(k)}{\sum_{i=1}^{r} w_i(k)}$

A sufficient condition for ensuring the stability of the fuzzy system(2) is given in **Theorem 1**.

Theorem 1 : The equilibrium point for the discrete time fuzzy system (2) is asymptotically stable in the large if there exists a common positive definite matrix $\boldsymbol{P}$ satisfying the following inequalities.

$$\boldsymbol{G}_i^T\boldsymbol{P}\boldsymbol{G}_i - \boldsymbol{P} < \boldsymbol{0},\ i = 1,2,\ldots,r \tag{3}$$

Proof : The proof can be given in [7].

In the discrete time fuzzy system with control input to the plant, the dynamic properties of each subspace can be expressed as the following fuzzy IF-THEN rules.

$$\begin{aligned} &\text{Rule } i: \text{If } x_1(k) \text{ is } M_{i1} \ldots \text{and } x_n(k) \text{ is } M_{in} \\ &\qquad \text{THEN } x(k+1) = \boldsymbol{A}_i\boldsymbol{x}(B) + \boldsymbol{B}_i\boldsymbol{u}(k) \end{aligned} \quad i = 1,2,\ldots,r \tag{4}$$

where

$\boldsymbol{x}(k) = [x_1(k)\ x_2(k)\ \cdots\ x_n(k)]^T \in \Re^n$ denotes the state vector of the fuzzy system.

$\boldsymbol{u}(k) = [u_1(k)\ u_2(k)\ \cdots\ u_m(k)]^T \in \Re^m$ denotes the input of the fuzzy system.

r is the number of the fuzzy IF-THEN rules, and M_{ij} is the fuzzy set.

If the set of $(\boldsymbol{x}(k), \boldsymbol{u}(k))$ is given the output of the fuzzy system (4) can be obtained as follows.

$$\begin{aligned} x(k+1) &= \frac{\sum_{i=1}^{r} w_i(k)\{A_i\boldsymbol{x}(k) + \boldsymbol{B}_i\boldsymbol{u}(k)\}}{\sum_{i=1}^{r} w_i(k)} \\ &= \sum_{i=1}^{r} h_i(k)\{A_i\boldsymbol{x}(k) + \boldsymbol{B}_i\boldsymbol{u}(k)\} \end{aligned} \tag{5}$$

where

$$w_i(k) = \prod_{j=1}^{n} M_{ij}(x_j(k)),\ \text{and}\ h_i(k) = \frac{w_i(k)}{\sum_{i=1}^{r} w_i(k)}.$$

In PDC, the fuzzy controller is designed distributively according to the corresponding rule of the plant[9]. Therefore, the PDC for the plant (4) can be expressed as follows.

$$\begin{aligned} &\text{Rule } j: \text{If } x_1(k) \text{ is } M_{j1} \ldots \text{and } x_n(k) \text{ is } M_{jn} \\ &\qquad \text{THEN } \boldsymbol{u} = -\boldsymbol{F}_j\boldsymbol{x}(k) \end{aligned} \quad j = 1,2,\ldots,r \tag{6}$$

The fuzzy controller output of Eq. (6) can be inferred as follows.

$$\boldsymbol{u}(k) = \frac{\sum_{i=1}^{r} w_i(k)\boldsymbol{F}_i\boldsymbol{x}(k)}{\sum_{i=1}^{r} w_i(k)} = -\sum_{j=1}^{r} h_j(k)\boldsymbol{F}_j\boldsymbol{x}(k) \tag{7}$$

where $h_j(k)$ is the same function in Eq. (5).

Substituting Eq. (7) into Eq. (5) gives the following closed loop discrete time fuzzy system.

$$(k+1) = \sum_{i=1}^{r} h_i(k)\left\{A_i x(k) - B_i \sum_{j=1}^{r} h_j(k) F_j x(k)\right\}$$
$$= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(k)h_j(k)\{A_i - B_i F_j\}x(k) \qquad (8)$$

Defining $G_{ij} = A_i - B_i F_j$, the following equation is obtained.

$$x(k+1) = \sum_{i=1}^{r} h_i(k)h_i(k)G_{ii}x(k) + 2\sum_{i<j}^{r} h_i(k)h_j(k)\left\{\frac{G_{ij}+G_{ji}}{2}\right\}x(k) \qquad (9)$$

Applying **Theorem 1** to analyze the stability of the discrete time fuzzy system (9), the stability condition of **Theorem 2** can be obtained.

Theorem 2 : The equilibrium point of the closed loop discrete time fuzzy system (9) is asymptotically stable in the large if there exists a common positive definite matrix P which satisfies the following inequalities for all i and j except the set (i, j) satisfying $h_i(k)\cdot h_j(k)=0$.

$$G_{ii}^T P G_{ii} - P < 0 \quad i = 1, 2, \ldots, r \qquad (10a)$$

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^T P\left(\frac{G_{ij}+G_{ji}}{2}\right) - P \le 0, \quad 1 \le i \le j \le r \qquad (10b)$$

Proof : The proof can be given in [7].

If $B = B_1 = B_2 = \cdots = B_r$ in the plant (5) is satisfied, the closed loop system (8) can be obtained as follows.

$$x(k+1) = \sum_{i=1}^{r} h_i(k)\left\{A_i x(k) - B\sum_{j=1}^{r} h_j(k)F_j x(k)\right\}$$
$$= \sum_{i=1}^{r} h_i(k)\{A_i - BF_i\}x(k) = \sum_{i=1}^{r} h_i(k)G_i x(k) \qquad (11)$$

where $G_i = A_i - BF_i$

Hence, **Theorem 1** can be applied to the stability analysis of the closed loop system (11).

3. LMI Approach for Fuzzy System Design

To prove the stability of the discrete time fuzzy control system by **Theorem 1** and **Theorem 2**, the common positive definite matrix P must be solved. LMI theory can be applied to solving P [13]. LMI theory is one of the numerical optimization techniques. Many of the control problems can be transformed into the LMI problems and the recently developed Interior-point method can be applied to solving numerically the optimal solution of these LMI problems[14].

Definition 1: linear matrix inequility can be defined as follows.

$$F(x) = F_0 + \sum_{i=1}^{m} x_i F_i > 0 \qquad (12)$$

where $x = x[x_1 \ x_2 \ \cdots \ x_m]^T$ is the parameter, the symmetric matrices $F_i = F_i^T \in \Re^{n\times n}$, $i = 0, \cdots, m$ are given, and the inequality symbol "> 0" means that $F(x)$ is the positive definite matrix.

LMI of Eq. (12) means the convex constraints for x. Convex constraint problems for the various x can be expressed as LMI of Eq. (12). LMI feasibility problem can be described as follows.

LMI feasibility problem : The problem of finding x^{feasp} which satisfies $F(x^{feasp}) > 0$ or proving the unfeasibility in the case that LMI $F(x) > 0$ is given.

And the stability condition of **Theorem 1** can be transformed into the LMI feasibility problem as follows.

LMI feasibility problem about the stability condition of Theorem 1 : The problem of finding P which satisfies the LMIs, $P > 0$ and $G_i^T P G_i - P < 0$, $i = 1, 2, \cdots, r$ or proving the unfeasibility in the case that $A_i \in \Re^{n\times n}$, $i = 1, 2, \cdots, r$ are given.

If the design object of a controller is to guarantee the stability of the closed loop system (5), the design of the PDC fuzzy controller(7) is equivalent to solving the following LMI feasibility problem using Schur complements[13].

LMI feasibility problem equivalent to the PDC design problem (Case I) : The problem of finding $X > 0$ and $M_1, M_2, \cdots, M_r$ which satisfy the following inequalities.

$$\begin{bmatrix} X & \{A_i X - B_i M_i\}^T \\ A_i X - B_i M_i & X \end{bmatrix} > 0, \quad i = 1, 2, \ldots, r$$

$$\begin{bmatrix} X & \frac{1}{2}\{A_i X + A_j X - B_i M_j - B_j M_i\}^T \\ \frac{1}{2}\{A_i X + A_j X - B_i M_j - B_j M_i\} & X \end{bmatrix} > 0,$$

where $X = P^{-1}$, $M_1 = F_1 X$, $M_2 = F_2 X$, $\cdots$, and $M_r = F_r X$.

The feedback gain matrices $F_1, F_2, \cdots, F_r$ and the common positive definite matrix P can be given by the LMI solutions, X and $M_1, M_2, \cdots, M_r$ as follows.

$$P = X^{-1}, F_1 = M_1 X^{-1}, F_2 = M_2 X^{-1}, \ldots, \text{ and } F_r = M_r X^{-1}$$

If $B = B_1 = B_2 = \cdots = B_r$ is satisfied, the design of the PDC fuzzy controller(7) is equivalent to solving the following LMI feasibility problem.

LMI feasibility problem equivalent to the PDC design problem (Case II) : The problem of finding $X > 0$ and $M_1, M_2, \cdots, M_r$ which satisfy the following equations.

$$\begin{bmatrix} X & \{A_i X - BM_i\}^T \\ A_i X - BM_i & X \end{bmatrix} > 0 \quad i = 1, 2, \ldots, r$$

where $X = P^{-1}$, $M_1 = F_1 X$, $M_2 = F_2 X$, $\cdots$, and $M_r = F_r X$.

The feedback gain matrices F_1, F_2, $\cdots$, F_r and the common positive definite matrix P can be given by the LMI solutions, X and M_1, M_2, $\cdots$, M_r, as follows.

$$P=X^{-1}, F_1=M_1X^{-1}, F_2=M_2X^{-1}, \ldots, \text{and } F_r=M_rX^{-1}$$

4. Real-Time Digital Fuzzy Control System considering Computing Time-Delay

As mentioned earlier, we are interested in analyzing the effect of the computing time-delay that results from the implementation of a control algorithm on a digital computer. The presence of the computing time-delay in a control system can be represented by a delay element after the D/A converter and hold circuit, as shown in Fig. 1. Hence the analysis of the effect of the computing time-delay must be done in a continuous-time domain.

When control algorithms are implemented on a digital computer, a considerable computing time-delay can occur due to the complex data processing. Let τ be defined as this computing time-delay. In the real-time digital fuzzy control systems, the output of the discrete fuzzy controller is applied to the plant after the computing time-delay τ.

Because the time-delay makes the output of the controller not synchronized with the sampling time, the analysis and the design of the controller are very difficult. In this paper, DFC which has the following fuzzy rules is proposed to consider the computing time-delay.

$$\begin{aligned}&\text{Rule } j\text{: If } x_1(k) \text{ is } M_{j1} \ldots \text{and } x_n(k) \text{ is } M_{jn} \quad i=1,2,\ldots,r\\ &\quad \text{THEN } \boldsymbol{u}(k+1)=\boldsymbol{D}_j\boldsymbol{u}(k)+\boldsymbol{E}_j\boldsymbol{x}(k)\end{aligned} \tag{13}$$

The output of DFC (13) can be inferred as follows.

$$\begin{aligned}\boldsymbol{u}(k+1) &= \frac{\sum_{j=1}^{r} w_j(k)\{\boldsymbol{D}_j\boldsymbol{u}(k)+\boldsymbol{E}_j\boldsymbol{x}(k)\}}{\sum_{j=1}^{r} w_j(k)}\\ &= \sum_{j=1}^{r} h_j(k)\{\boldsymbol{D}_j\boldsymbol{u}(k)+\boldsymbol{E}_j\boldsymbol{x}(k)\}\end{aligned} \tag{14}$$

The general timing diagram of fuzzy control loop in a continuous-time domain is shown in Fig. 2. T is the sampling period of the control loop, τ_v and τ_c are the delay made by the data processing from a sensor system

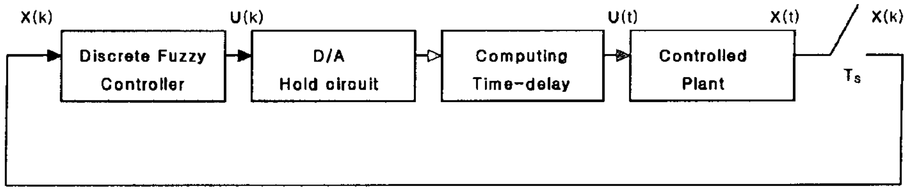


Fig. 1. A real-time digital fuzzy control system in the presence of computing time-delay

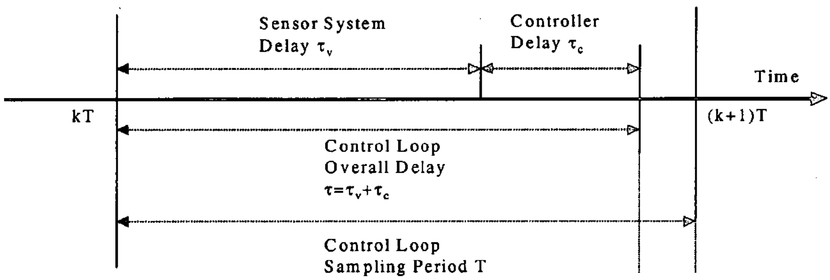


Fig. 2. Timing Diagram of the Fuzzy Control Loop

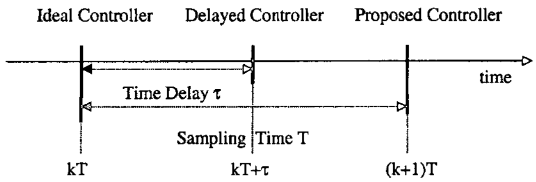


Fig. 3. Output Timing of the Controllers (three cases)

and a fuzzy controller respectively. Therefore the output of the controller is applied to the plant after overall dealy $\tau = \tau_v + \tau_c$.

The output timing of a ideal controller, a delayed controller, and the proposed controller is shown in the Fig. 3. In the ideal controller, it is assumed that there is no computing time-delay. When this controller is implemented in real-time digital control systems, the problem of the computing time-delay τ cannot be avoidable. However, the analysis and the design of this system with delayed controller are very difficult since the output of controller is not syncronized with the sampling time.

On the other hand, the analysis and the design of the proposed controller are very easy because the controller output is syncronized with the sampling time delayed with unit sampling period. Using this proposed controller, we can realize a control algorithm during the time interval $T-\tau_v$ in Fig. 2. In this time interval, a complex algorithm such as not only fuzzy algorithm but also nonlinear control algorithm can be sufficiently implemented in real time.

Defining the new state vector as $w(k) = \begin{bmatrix} x(k) \\ u(k) \end{bmatrix}$, the design problem of the DFC is transformed into the design problem of the PDC fuzzy controller.

***PDC design problem equivalent to DFC design problem* :**

The problem of designing the PDC fuzzy controller $v(k) = -\sum_{j=1}^{r} h_j(k)\bar{F}_j w(k)$ in the case that the fuzzy plant $w(k+1) = \sum_{i=1}^{r} h_i(k)\{\bar{A}_i w(k) + \bar{B}v(k)\}$ is given.

where $\bar{A}_i = \begin{bmatrix} A_i & B_i \\ 0 & 0 \end{bmatrix}$, $\bar{B} = \begin{bmatrix} 0 \\ I \end{bmatrix}$, and $\bar{F}_j = -\begin{bmatrix} E_j & D_j \end{bmatrix}$

Then, we combining the fuzzy plant with the PDC fuzzy controller the closed loop system can be given as

$$w(k+1) = \sum_{i=1}^{r} h_i(k) G_i w(k) \tag{15}$$

where $G_i = \begin{bmatrix} A_i & B_i \\ E_i & D_i \end{bmatrix}$

The stability condition of the closed loop system (15) becomes the same as the sufficient condition of **Theorem 1** and the stability can be determined by solving ***LMI feasibility problem about the stability condition of Theorem 1***. Also, the design problem of the DFC guaranteeing the stability of the closed loop system can be transformed into ***LMI feasibility problem***.

Therefore, using the same notation in section 3, the design problem of the DFC can be equivalent to the following LMI feasibility problem.

***LMI feasibility problem equivalent to DFC design problem* :**

The problem of finding $X > 0$ and $M_1, M_2, \cdots, M_r$ which satisfy the following equation.

$$\begin{bmatrix} X & \{\bar{A}_i X - \bar{B}M_i\}^T \\ \bar{A}_i X - \bar{B}M_i & X \end{bmatrix} > 0, \quad i = 1, 2, \ldots, r$$

where $X = P^{-1}$, $M_1 = \bar{F}_1 X$, $M_2 = \bar{F}_2 X$, $\cdots$, and $M_r = \bar{F}_r X$

The feedback gain matrices $\bar{F}_1, \bar{F}_2, \cdots, \bar{F}_r$ and the common positive definite matrix P can be given by the LMI solutions, X and $M_1, M_2, \cdots, M_r$ as follows.

$$P = X^{-1},\ \bar{F}_1 = M_1 X^{-1},\ \bar{F}_2 = M_2 X^{-1},\ \ldots,\ \bar{F}_r = M_r X^{-1} \tag{16}$$

Therefore, the control gain matrices $D_1, \cdots, D_r, E_1, \cdots, E_r$ of the proposed DFC can be obtained from the feedback gain matrices $\bar{F}_1, \bar{F}_2, \cdots, \bar{F}_r$.

5. Real-Time Backing up Control of a Computer-Simulated Truck-Trailer

We have shown an analysis technique of the proposed DFC under the condition that the computing time-delay exists in section 4. Some papers have reported that backing up control of a computer-simulated truck-trailer could be realized by fuzzy control[9,11,15,16]. However, these studies have not analyzed the computing time-delay effect on control system performance in digital implementation of the fuzzy controller. In this section, we consider the computing time-delay in real-time digital fuzzy control systems and apply the controller with unit time-delay to the real-time backing up control of a truck-trailer system.

5.1 Models of a Truck-Trailer

M. Tokunaga derived the following model about the truck-trailer system [16]. Fig. 4 shows the schematic diagram of this system.

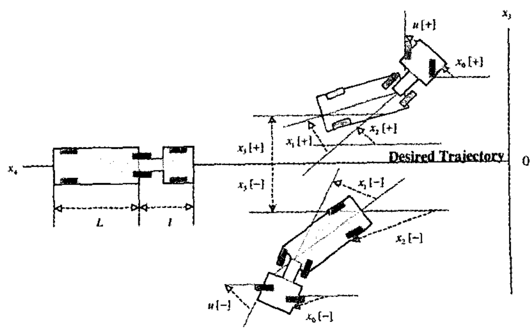


Fig. 4. Truck Trailer Model and Its Coordinate System

$$x_0(k+1) = x_0(k)+vT/l\ \tan[u(k)]$$
$$x_1(k) = x_0(k)-x_2(k)$$
$$x_2(k+1) = x_2(k)+vT/L\sin[x_1(k)]$$
$$x_3(k+1)=x_3(k)+vT\cos[x_1(k)]\sin[\{x_2(k+1)+x_2(k)\}/2]$$
$$x_4(k+1)=x_4(k)+vT\cos[x_1(k)]\cos[\{x_2(k+1)+x_2(k)\}/2] \tag{17}$$

where, $x_0(k)$: The angle of the truck referenced to the desired trajectory
$x_1(k)$: The angle difference between the truck and the trailer
$x_2(k)$: The angle of the trailer referenced to the desired trajectory
$x_3(k)$: The vertical position of the trailer tail end
$x_4(k)$: The horizontal position of the trailer tail end
$u(k)$: The steering angle of the truck
l: The length of the truck, L: The length of the trailer
T: Sampling time, v: The constant backward speed

K. Tanaka defined the state vector as $x(k)=[x_1(k)\ x_2(k)\ x_3(k)]^T$ in the truck-trailer model (17) and expressed the plant as two following fuzzy rules[9]. Fig. 5 shows the membership functions in the premise part in the fuzzy system (18).

$$\begin{aligned}&\text{Rule 1: If } x_2(k)+vT/\{2L\}x_1(k) \text{ is } M_1\\ &\quad \text{THEN } \boldsymbol{x}(k+1)=\boldsymbol{A}_1\boldsymbol{x}(k)+\boldsymbol{B}_1u(k)\\ &\text{Rule 2: If } x_2(k)+vT/\{2L\}x_1(k) \text{ is } M_2\\ &\quad \text{THEN } \boldsymbol{x}(k+1)=\boldsymbol{A}_2\boldsymbol{x}(k)+\boldsymbol{B}_2u(k)\end{aligned} \tag{18}$$

where, $A_1=\begin{bmatrix}1-\frac{vT}{L} & 0 & 0\\ \frac{vT}{L} & 1 & 0\\ \frac{v^2T^2}{2L} & vT & 1\end{bmatrix}, A_2=\begin{bmatrix}1-\frac{vT}{L} & 0 & 0\\ \frac{vT}{L} & 1 & 0\\ \frac{dv^2T^2}{2L} & dvT & 1\end{bmatrix},$

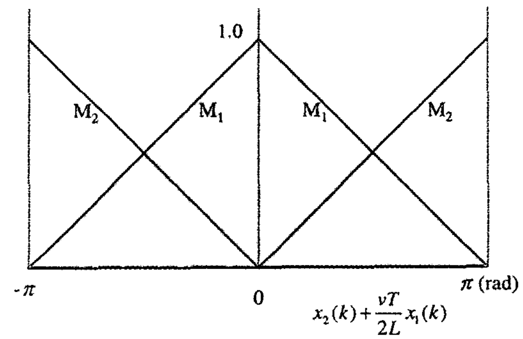


Fig. 5. Membership functions

$$\boldsymbol{B}=\boldsymbol{B}_1=\boldsymbol{B}_2=\begin{bmatrix}\frac{vT}{l}\\ 0\\ 0\end{bmatrix}$$

$l=2.8$[m], $L=-1.0$[m/s], $v=-1.0$[m/s], $T=2.0$[s], $d=10^{-2}/\pi$

5.2 Discrete Time Fuzzy Controller without considering the computing time-delay

In this subsection, the backing up control of a truck-trailer is simulated by the conventional discrete time fuzzy controller based on the discrete TS fuzzy model of the system. To solve the backward parking problem of Eq. (18), the PDC fuzzy controller can be designed as follows.

$$\begin{aligned}&\text{Rule 1: If } x_2(k)+vT/\{2L\}\cdot x_1(k) \text{ is } M_1\\ &\quad \text{THEN } u(k)=\boldsymbol{F}_1^T\boldsymbol{x}(k)\\ &\text{Rule 2: If } x_2(k)+vT/\{2L\}\cdot x_1(k) \text{ is } M_2\\ &\quad \text{THEN } u(k)=\boldsymbol{F}_2^T\boldsymbol{x}(k)\end{aligned} \tag{19}$$

where $\boldsymbol{F}_1=\begin{bmatrix}1.2837\\ -0.4139\\ 0.0201\end{bmatrix}$ and $\boldsymbol{F}_2=\begin{bmatrix}0.9773\\ -0.0709\\ 0.0005\end{bmatrix}$.

Ricatti equation for linear discrete systems was used to determine these feedback gains. The detailed derivation of these feedback gains was given in [9].

Substituting Eq. (19) into Eq. (18) yields the following closed loop system due to $\boldsymbol{B}=\boldsymbol{B}_1=\boldsymbol{B}_2$.

$$\boldsymbol{x}(k+1)=\sum_{i=1}^{2}h_i(k)\boldsymbol{G}_i\boldsymbol{x}(k) \tag{20}$$

where, $\boldsymbol{G}_1=\begin{bmatrix}0.448 & 0.296 & -0.014\\ -0.364 & 1 & 0\\ 0.364 & -2 & 1\end{bmatrix}$ and

$$\boldsymbol{G}_2=\begin{bmatrix}0.448 & 0.296 & -0.014\\ -0.364 & 1 & 0\\ 0.116\times10^{-2} & -0.637\times10^{-2} & 1\end{bmatrix}.$$

Since there exists a common positive definite matrix $\boldsymbol{P}$ which satisfies the stability sufficient condition (3), the closed loop system is asymptotically stable in the large. That is, the backward parking can be accomplished for all initial contitions.

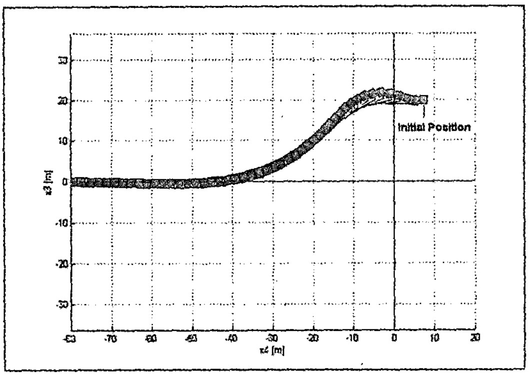
Common positive definite matrix :

$$P=\begin{bmatrix}113.9 & -92.61 & 2.540\\ -92.61 & 110.7 & -3.038\\ 2.540 & -3.038 & 0.5503\end{bmatrix}$$

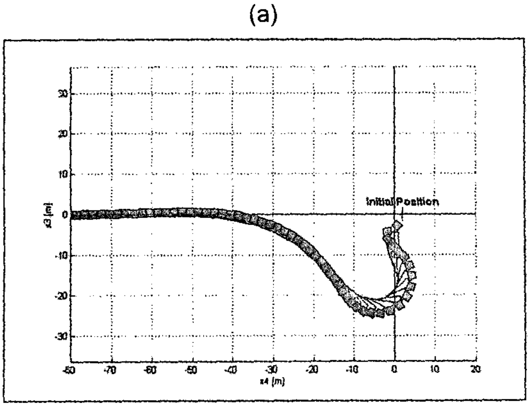
Two initial conditions used for the simulations of the

Table 1. The initial conditions of the truck-trailer system

CASE	$x_1(0)$[deg]	$x_2(0)$[deg]	$x_3(0)$[deg]
CASE I	0	0	20
CASE II	-90	135	-10



(a)



(b)

Fig. 6. (a): Simulation result for CASE I, (b): Simulation result for CASE II

truck-trailer system are given in Table 1.

Fig. 6(a) and (b) show the simulation results for CASE I and CASE II. As can be seen in these Figures, the backing up control for each initial condition is accomplished effectively.

5.3 Computing Time-Delay Effect in Real Time Digital Fuzzy Control System

In many real-time implementations of the digital controller, a vision sensor is generally needed to measure the state $x(k)$ of the truck-trailer system[17]. The computing time-delay can be made by the data processing from a vision sensor. Also, it can be made by the calculation of the fuzzy algorithm and by the actutor in adjusting the steering angle. Let τ be defined as the sum of all this time-delay. Then the output of the designed fuzzy controller is delayed with computing time-delay τ when the fuzzy control algorithms are implemented on real time digital computer. In this practical case, the discrete time fuzzy controller in section 5.2 cannot guarantee the stability of the system due to the computing time-delay.

5.4 Proposed DFC applied to the Real-Time Digital Fuzzy Control System considering Computing Time-Delay

In this subsection, we design the real-time DFC considering computing time-delay. Under the same simulation condition, we apply the proposed DFC to the system. Following the design technique of DFC in section 4, we can construct the DFC for the backing up control problem as follows.

$$\begin{aligned}&\text{Rule 1: If } x_2(k)+vT/\{2L\}\cdot x_1(k) \text{ is } M_1\\&\quad\text{THEN } u(k+1)=\boldsymbol{D}_1u(k)+\boldsymbol{E}_1x(k)\\&\text{Rule 2: If } x_2(k)+vT/\{2L\}\cdot x_1(k) \text{ is } M_2\\&\quad\text{THEN } \boldsymbol{u}(k+1)=\boldsymbol{D}_2u(k)+\boldsymbol{E}_2\boldsymbol{x}(k)\end{aligned} \tag{21}$$

Combining Eq. (18) with Eq. (21), the augmented closed loop system is given as follows.

$$w(k+1)=\sum_{i=1}^{2}h_i(k)\boldsymbol{G}_i w(k) \tag{22}$$

where, $\boldsymbol{G}_1=\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{B}_1\\ \boldsymbol{E}_1 & \boldsymbol{D}_1\end{bmatrix}$, $\boldsymbol{G}_2=\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{B}_2\\ \boldsymbol{E}_2 & \boldsymbol{D}_2\end{bmatrix}$,

To obtain the control gain matrices $\boldsymbol{D}_1$, $\boldsymbol{D}_2$, $\boldsymbol{E}_1$, $\boldsymbol{E}_2$ guaranteeing the stability of the closed loop system (22), we solve the ***LMI feasibility problem equivalent to DFC design problem*** as follows.

The problem of finding $X>0$ and M_1, M_2 which satisfy the following inequalities :

$$\begin{bmatrix} X & \{\bar{A}_iX-\bar{B}M_i\}^T\\ \bar{A}_iX-\bar{B}M_i & X\end{bmatrix}>0$$

$$\text{where } \bar{A}_i=\begin{bmatrix}A_i & B_i\\ 0 & 0\end{bmatrix} \text{ and } \bar{B}=\begin{bmatrix}0\\ I\end{bmatrix},\ i=1,2 \tag{23}$$

The matrices X and M_1, M_2 in LMIs are determined using a convex optimization technique offered by [19].

$$X=\begin{bmatrix}157.0056 & 61.9680 & -1.6565 & 220.727\\ 61.9680 & 50.4822 & 69.8423 & 53.4329\\ -1.6565 & 69.8423 & 489.4416 & -2.3866\\ 220.7727 & 53.4329 & -2.3866 & 442.6866\end{bmatrix},$$

$$M_1 = \left[-96.3672\ -43.1521\ 41.8056\ -5.8356\right],$$

$$M_2 = \left[-116.3143\ -66.0021\ 1.3065\ -22.9842\right]$$

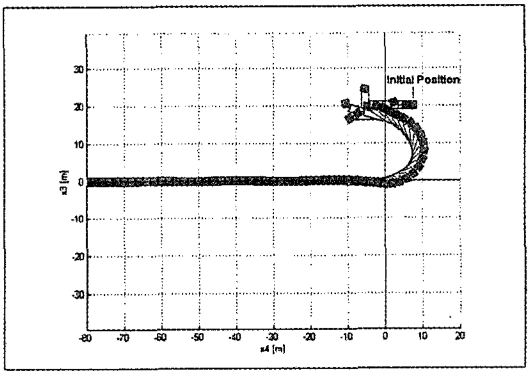
The feedback gains and a common positive definite matrix, P are determined by the relationship (16) as follows.

$$P = X^{-1} = \begin{bmatrix} 0.0995 & -0.1036 & 0.0149 & -0.0370 \\ -0.1036 & 0.1373 & -0.0198 & 0.0350 \\ 0.0149 & -0.0198 & 0.0049 & -0.0050 \\ -0.0370 & 0.0350 & -0.0050 & 0.0165 \end{bmatrix},$$

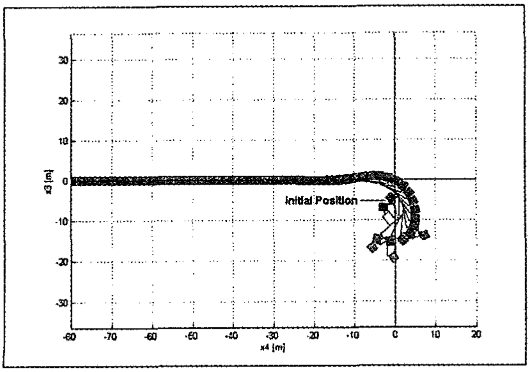
$$\bar{F}_1 = M_1X^{-1} = -\left[E_1\ D_1\right] = \left[-3.9047\ 2.6765\ -0.3020\ 1.5869\right],$$

$$\bar{F}_2 = M_2X^{-1} = -\left[E_2\ D_2\right] = \left[-3.8624\ 2.1564\ -0.3102\ 1.6123\right] \quad (24)$$

Therefore, the closed loop system is asymptotically stable in the large considering computing time-delay and the control gain matrices are given as follows by ***PDC design problem equivalent to DFC design problem.***



(a)



(b)

Fig. 7. (a): Simulation result by DFC for CASE I, (b): Simulation result by DFC for CASE II

$$D_1 = -1.5869,\ D_2 = -1.6123,$$

$$E_1 = \left[3.9047\ -2.6765\ 0.3020\right],\ E_2 = \left[3.8624\ -2.1564\ 0.3102\right]$$

Fig. 7 (a) and (b) show the simulation results of the designed real-time DFC. As can be seen in these figures, the backward parking is accomplished successfully for CASE I and CASE II. And the stability of the system can be guaranteed in the presence of a computing time-delay because the controller can be designed syncronized with the sampling time.

6. Conclusions

In this paper, the effect of computing time-delay in the implementation of the fuzzy control algorithm on a digital computer was investigated and a real-time DFC framework was developed to remedy the problems of computing time-delay. Because the proposed controller was syncronized with the sampling time delayed with unit sampling period, the analysis and the design problem considering computing time-delay could be very easy. Convex optimization technique based on LMI has been utilized to solve the problem of finding stable feedback gains and a common positive definite matrix P. Therefore, the stability of the real-time fuzzy control system was guaranteed in the presence of the computing time-delay. The analysis of the real-time fuzzy controlled truck-trailer system has demonstrated how a given system could be stabilized in the presence of the computing time-delay using the proposed fuzzy controller.

Acknowledgments

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References

[1] I. Horowitz, "Some properties of delayed controls," *Int. J. Contr.*, Vol. 38, No. 5, pp. 977-990, 1983.

[2] A. Leva, C. Maffezzoni and R. Scattolini, "Self-tuning PI-PID regulators for stable systems with varying delay," *Automatica*, Vol. 30, No. 7, pp. 1171-1183, 1994.

[3] J. H. Kim, J. H. Park and S. W. Lee, "A two-layered fuzzy logic controller for systems with deadzones," IFSA93 World Congress, Seoul, Korea, pp. 826-829, July 1993.

[4] M. J. Jang and C. L. Chen, "Fuzzy successive modelling and control for time-delay systems," *International Journal of Systems Science*, Vol. 27, No. 12, pp. 1483-1486, 1996.

[5] H. X. Li and S. K. Tso, "Higher order fuzzy control

structure for higher order or time-delay systems," *IEEE Trans. Fuzzy Systems*, Vol. 7, No. 5, pp. 540-542, 1999.
[6] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Systems, Man, and Cybernetics*, Vol. 15, No. 1, pp. 116-132, 1985.
[7] K. Tanaka and M. Sugeno, "Stability analysis and design of fuzzy control systems," *Fuzzy Sets and Systems*, Vol. 45, No. 2, pp. 135-156, 1992.
[8] C. S. Ting, T. H. S. Li and F. C. Kung, "An approach to systematic design of fuzzy control system," *Fuzzy Sets and Systems*, Vol. 77, pp. 151-166, 1996.
[9] K. Tanaka and M. Sano, "A robust stabilization problem of fuzzy control systems and its application to backing up control of a truck-trailer," *IEEE Trans. Fuzzy Systems*, Vol. 2 No. 2, pp. 119-133, 1994.
[10] H. O. Wang, K. Tanaka and M. F. Griffin, "An approach to fuzzy control of nonlinear systems: Stability and design issues," *IEEE Trans. Fuzzy Systems*, Vol. 4, No. 1, pp. 14-23, 1996.
[11] K. Tanaka, T. Kosaki and H. O. Wang, "Backing control problem of a mobile robot with multiple trailers: fuzzy modeling and LMI-based design," *IEEE Trans. Fuzzy Systems*, Vol. 28, No. 3, pp. 329-337, 1998.
[12] K. Kiriakidis, "Non-linear control system design via fuzzy modelling and LMIs," *Int. J. Control*, Vol. 72, No. 7/8, pp. 676-685, 1999.
[13] S. Boyd, L. E. Ghaoui, E. Feron and V. Balakrishnan, Linear matrix inequalities in systems and control theory, SIAM, Philadelphia, 1994.
[14] Y. Nesterov and A. Nemirovsky, Interior-point polynomial methods in convex programming, SIAM, Philadelphia, 1994.
[15] S. G. Kong and B. Kosko, "Adaptive fuzzy systems for backing up a truck-and-trailer," *IEEE Trans. Neural Net.*, Vol. 3, pp. 211-223, 1992.
[16] M. Tokunaga and H. Ichihashi, "Backer-upper control of a trailer truck by neuro-fuzzy optimal control," 8th Fuzzy System Symp., Japan, pp. 49-52, May 1992.
[17] K. Tanaka and T. Kosaki, "Design of a stable fuzzy controller for an articulated vehicle," *IEEE Trans. Systems, Man, and Cybernetics*, Vol. 27, No. 3, pp. 552-558, 1997.
[18] M. Sampel, T. Tamura, T. Itoh and M. Nakamichi, "Path tracking control of trailer-like mobile robot," Proc. of IEEE/RSJ International Workshop on Intelligent Robots and Systems, Osaka, pp. 193-198, 1991.
[19] P. Gahinet, A. Nemirovski, A. Laub, and M. Chilali, LMI Control Toolbox, The MathWorks, Inc., Natick, 1995.
[20] R. Palm, D. Driankov and H. Hellendoorn, Model Based Fuzzy Control: Fuzzy Schedulers and Sliding Mode Fuzzy Controllers, Springer-Verlag, Berlin, 1997.
[21] L. X. Wang, A Course in Fuzzy systems and Control, Prentice-Hall International, Inc., U.S.A., 1997.

박 창 우 (Chang-Woo Park)

was born in Seoul, Korea, in 1973. He received the B.S. degrees in electronics from Korea University and M.S. degrees in electronics from Yonsei University, Seoul, Korea, in 1997 and 1999, respectively. He is currently working toward the Ph.D. degree in electrical and computer engineering at Yonsei University.

His current research interests include fuzzy control theory, nonlinear control theory and robot system.

신 현 석 (Hyun-Seok Shin)

was born in Seoul, Korea, in 1972. He received the B.S. and M.S. degrees in electronics from Yonsei University, Seoul, Korea, in 1996 and 1998, respectively. He is currently working toward the Ph.D. degree in electrical and computer engineering at Yonsei University.

His current research interests include robot manipulator control, fuzzy control theory, nonlinear control theory.

박 민 용 (Mignon Park)

received the B.S. and M.S. degrees in electronics from Yonsei University, Seoul, Korea, in 1973 and 1977, respectively, and the Ph.D degree from the University of Tokyo, Tokyo, Japan, in 1982. From 1977 to 1982, he was a Researcher at the Institute Biomedical Engineering, University of Tokyo, Japan.

In 1982 he was a researcher at Massachusetts Institute of Technology, Cambridge, MA, and at the University of California, Berkely. He was a Visiting Researcher in Robotics Division, Mechanical Engineering Laboratory (MEL) and Ministry of International Trade and Industry (MITI), Tsukuba, Japan, from 1986 to 1987. Since 1982, he has been a Professor in the Electrical and Computer Department of Yonsei University. His research interests include fuzzy control and application engineering, robotics, fuzzy biomedical system, etc. Dr. Park was a Program Chairman of the Korea-Japan Joint Conference on Fuzzy Systems and Engineering in 1992 and is the 1999 Vice-Chair of IEEE-FUZZ99. He is a member of the Japanese Society for Fuzzy Theory and Systems and the Japanese Society for Biomedical Fuzzy Systems, among others.