Analysis of Dynamic Characteristics of Structural Joints Using Stiffness Influence Coefficients

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This paper proposes a new modeling method for joints in mechanical structures in order to reduce the errors in eigenvalue analysis due to joint modeling. The new modeling method uses both a stiffness influence method and a condensation method to obtain the dynamic characteristic matrix of the joint region. It also employs the displacement and reaction of finely modeled finite element analysis in the calculation of stiffness influence coefficients. In order to check the validity of the proposed method, natural frequencies and mode shapes of a simple structure with a bolted joint are investigated by the proposed method and by experiments. The eigenvalue analysis using the proposed method shows more accurate results than that using rigid joints modeling, when the natural frequencies are compared with the experimental results. In addition, the differences between the natural frequencies obtained by the proposed method and those by the rigid joints modeling are notable in the modes where the joint has elastic deformation.

Key Words : Dynamic Characteristics, Structural Joints, Stiffness Influence Coefficients, Condensation, Finite Element Analysis

1. Introduction

When we design highly precise or high speed structures such as machine tools, airplanes and rotating machines, it is required to predict their static and dynamic characteristics for the estimation of their stability or performances. Particularly it is important to analyze precisely the dynamic characteristics of the structures such as natural frequencies and mode shapes in order to control vibration or positions of the structures for their desired functions. But the accurate analysis is difficult to solve in case that joints exist in the structure, since the characteristic parameters used in the equations of motion are inaccurate.

The dynamic characteristics of complex structures can be analyzed by two approaches, namely experimental approach using modal testing and numerical approach using finite element analysis. There may exist however some differences between the results by experimental approach and those by numerical approach. The errors of numerical approach may be brought by the limitation in number of finite elements or the simplification of complex boundary conditions. Particularly joint area or sliding surfaces gives large error to dynamic analysis. Experimental approach does not have those errors brought by the foregoing incompleteness of model, but it may have errors due to signal noise during data processing. As experimental approach requires the objective structures to be equipped, it is

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difficult to obtain the dynamic characteristics of structures in a design stage and it is troublesome to analyze configuration changes by experimental approaches. It is therefore important to model joints simply and accurately in order to analyze large complex structures in a design stage.

The researches on joint characteristics have been performed on the joints of machine tools, and they are focused on the identification of the static characteristics. On the other hand, for the dynamic characteristics of complex structures, methods which use both finite element analysis and experimental modal analysis have been studied mainly after 1980s. Namely, semi-experimental and semi-analytical methods have been investigated to identify the joint parameters through experiments after having modeled joints with simple numerical models.

As a numerical method, Tanaka et al.(1981) analyzed a screw joint by modeling with axisymmetric finite elements and treated preloads with applying tension. Grosse and Mitchell(1990) obtained nonlinear stiffness by modeling bolted joints as axisymmetric, used heat displacements for preloads, and considered the friction between contact surfaces. Lee and Lee(1990) analyzed structures with nonlinear joints by using substructure synthesis method and describing function method in order to reduce the total degrees of freedom. Moon et al. (1999) analyzed the nonlinear vibration of mechanical structures by using substructure synthesis method and perturbation method and they obtained the approximated solutions in the nonlinear component and assembling region by applying the perturbation method. Yoon et al. (1999) analyzed the three dimensional flow for the compression molding of unidirectional polymeric composites with the slip between a mold and a material by treating the composite as an incompressible Newtonian fluid and by a formulation technique of finite element analysis.

As an experimental method, Tsai and Chou (1988) proposed a method obtaining the joint characteristics of single bolt joint from the frequency response functions of total structure and the frequency response functions of each substructure, where receptance was used as frequency response function, and the mathematical model of mass, damping, and stiffness matrices was not used. Wang and Liou(1991) proposed a method identifying the joint characteristics from the frequency response functions of given structures. Also they researched to overcome the problems of noise inevitable in frequency response functions obtained by experiments.

A general dynamic modeling technique for joints is required in order to predict the natural frequencies and the natural modes of complex structures before their production for the purpose of motion control in design stage.

This paper aims to propose a new modeling method for joints in a structure which can reduce the errors of natural frequencies brought by joint modeling. The new modeling method uses both a stiffness influence coefficients(Lee et al., 1995 and Ko, 1996) and a condensation concept(Guyan, 1965) to obtain the dynamic characteristic matrix of the joint region. First the joint area is discretized finely with finite elements and some selected reactions are calculated for assumed displacements. Second the forces are employed for the calculation of stiffness influence coefficients. Next the condensed stiffness matrix of joint are calculated by using the stiffness influence matrix and the condensation technique. Then a finite element model of total structure which is easily solvable by practical numerical analysis is obtained.

A beam type structure with bolted joint is selected as a benchmark example in order to verify the foregoing method. The stiffness influence coefficients are investigated under various condition of the stiffness of the contact elements, the friction coefficients of the contact region, the magnitude of assumed displacement. Then dynamic characteristics of the overall systems are analyzed and compared with results of other methods.

2. Condensation Using Stiffness Influence Coefficients

2.1 Static condensation

Generally the finite element model of a given structure has many elements and nodes when the structure is complex or large. Its eigenvalue problem becomes complicated and the efficiency of the dynamic analysis may be deteriorative. So a method is required to reduce the number of degrees of freedom without changing the structural properties of the original structure and it is called condensation (Guyan, 1965).

In order to analyze the behavior of a structure, we first obtain the following Eq. (1) from the static relation between the force vector $\{F\}$ and the displacement vector $\{u\}$

$$[K]\{u\}=\{F\} \tag{1}$$

where $[K]$ is a stiffness matrix. In order to apply static condensation method, let the degrees of freedom which are remained after condensation process be the primary degrees of freedom (subscript p) and those eliminated temporarily during condensation be the secondary degrees of freedom (subscript s). Assuming that external loads are applied only to the primary degrees of freedom, then Eq. (1) can be partitioned in the form

$$\begin{bmatrix} K_{ss} & K_{sp} \\ K_{ps} & K_{pp} \end{bmatrix}\begin{Bmatrix} u_s \\ u_p \end{Bmatrix}=\begin{Bmatrix} 0 \\ F_p \end{Bmatrix} \tag{2}$$

The stiffness matrix in Eq. (2) can be simplified by the Gauss-Jordan elimination method,

$$\begin{bmatrix} I & -\bar{T} \\ 0 & \bar{K} \end{bmatrix}\begin{Bmatrix} u_s \\ u_p \end{Bmatrix}=\begin{Bmatrix} 0 \\ F_p \end{Bmatrix} \tag{3}$$

yielding the relationships

$$\{u_s\}=[\bar{T}]\{u_p\} \tag{4}$$

$$[\bar{K}]\{u_p\}=\{F_p\} \tag{5}$$

where the matrix $[\bar{T}]$ is the transformation matrix that represents the relation between the displacement vector of the secondary degrees of freedom $\{u_s\}$ and that of the primary degrees of freedom $\{u_p\}$; and the matrix $[\bar{K}]$ is the condensed stiffness matrix in $\{u_p\}$. The transformation matrix and the condensed stiffness matrix are respectively

$$[\bar{T}]=-[K_{ss}]^{-1}[K_{sp}] \tag{6}$$

$$[\bar{K}]=[K_{pp}]-[K_{ps}][K_{ss}]^{-1}[K_{sp}] \tag{7}$$

The relation between the condensed stiffness matrix $[\bar{K}]$ and the overall stiffness matrix $[K]$ is

$$[\bar{K}]=[T]^T[K][T] \tag{8}$$

where the transformation matrix $[T]$ is obtained from the relation between the primary degrees of freedom and the overall degrees of freedom,

$$\{u\}=[T]\{u_p\} \tag{9}$$

2.2 Stiffness influence coefficients and static condensation

Equations (5)～(8) are the static relations between the forces and the displacements for the primary degrees of freedom. And the overall stiffness matrix $[K]$ is obtained when total structure can be modeled with linear elements. If mechanical joints are included in the structure which is analyzed statically or dynamically by finite element method, the joints should be finely modeled in order to obtain accurate solutions. In this case, the modeling work becomes complex and the number of degrees of freedom becomes large because the geometric shape is complex and the joint interface has nonlinearity. As joints have contacts of faces with faces, faces with lines, etc., they show different phenomena under compression and under tension as well. The joints are usually modeled with gap elements or contact elements, but in case that such nonlinear elements should be included, the degrees of freedom cannot be reduced by a conventional static condensation. By using the concept of stiffness influence coefficients (Meirovitch, 1967), a condensed stiffness matrix $[\bar{K}]$ can be obtained. A stiffness influence coefficient S_{ij} is defined as the force given at $x=x_i$ in order to generate unit displacement $u_j=1$ only at $x=x_j$.

In order to apply such concept of stiffness influence coefficients, the primary degrees of freedom $\{u_p\}$ is selected in the overall degrees of freedom $\{u\}$ of total finite element model which includes nonlinear elements. Let the displacement of the k-th degree of freedom in the selected primary degree of freedom be unity, then the elements of $\{u_p\}$ are

$$u_{pk}=1$$

$$u_{pj}=0,\ \text{for } j\neq k,\ j=1,2,\cdots,n \tag{10}$$

The loads corresponding to this displacement constraints become the stiffness influence coefficients vector for k-th degree of freedom

$$\{S\}_k=[F_{p1}\ F_{p2}\ \cdots\ F_{pn}]^T \tag{11}$$

Executing the process for each degree of freedom gives the condensed stiffness matrix of total structure

$$[S]=[\{S\}_1\ \{S\}_2\ \cdots\ \{S\}_n] \tag{12}$$

A static condensation method and the proposed method using stiffness influence coefficients are applied respectively to a mass-spring system shown in Fig. 1. At first, the condensed stiffness matrix is obtained by a common static condensation method. The overall stiffness matrix for the 5 degree-of-freedom system is

$$[K]=\begin{bmatrix} K_1+K_2 & -K_2 & 0 & 0 & 0 \\ -K_2 & K_2+K_3 & -K_3 & 0 & 0 \\ 0 & -K_3 & K_3+K_4 & -K_4 & 0 \\ 0 & 0 & -K_4 & K_4+K_5 & -K_5 \\ 0 & 0 & 0 & -K_5 & K_5+K_6 \end{bmatrix} \tag{13}$$

The overall displacement vector $\{u\}$ is represented by

$$\{u\}=[x_1\ x_2\ x_3\ x_4\ x_5]^T \tag{14}$$

Here x_2 and x_4 are selected as the primary degrees of freedom. The displacement vector partitioned as the primary and the secondary degrees of freedom becomes

$$\{u\}=\begin{Bmatrix} \begin{Bmatrix} x_2 \\ x_4 \end{Bmatrix} \\ \begin{Bmatrix} x_1 \\ x_3 \\ x_5 \end{Bmatrix} \end{Bmatrix}=\begin{Bmatrix} \{u_p\} \\ \{u_s\} \end{Bmatrix} \tag{15}$$

The partitioned stiffness matrices become Eqs. (16) ~ (19).

$$[K_{pp}]=\begin{bmatrix} K_2+K_3 & 0 \\ 0 & K_4+K_5 \end{bmatrix} \tag{16}$$

$$[K_{ps}]=\begin{bmatrix} -K_2 & -K_3 & 0 \\ 0 & -K_4 & -K_5 \end{bmatrix} \tag{17}$$

$$[K_{sp}]=\begin{bmatrix} -K_2 & 0 \\ -K_3 & -K_4 \\ 0 & -K_5 \end{bmatrix} \tag{18}$$

$$[K_{ss}]=\begin{bmatrix} K_1+K_2 & 0 & 0 \\ 0 & K_3+K_4 & 0 \\ 0 & 0 & K_5+K_6 \end{bmatrix} \tag{19}$$

By using Eq. (7), the condensed stiffness matrix $[\bar{K}]$ is obtained as

$$[\bar{K}]=\begin{bmatrix} AK11 & AK12 \\ AK21 & AK22 \end{bmatrix} \tag{20}$$

where

$$AK11=\frac{K_1K_2(K_3+K_4)+K_3K_4(K_1+K_2)}{(K_1+K_2)(K_3+K_4)}$$

$$AK12=AK21=-\frac{K_3K_4}{K_3+K_4}$$

$$AK22=\frac{K_3K_4(K_5+K_6)+K_5K_6(K_3+K_4)}{(K_3+K_4)(K_5+K_6)}$$

The stiffness influence matrix condensed by using stiffness influence coefficients is obtained as the following procedure. As the primary degrees of freedom are x_2 and x_4, the mass-spring system in Fig. 1 can be represented again by the equivalent mass-spring system as shown in Fig. 2. A displacement vector for the stiffness influence vector and the corresponding load vector are

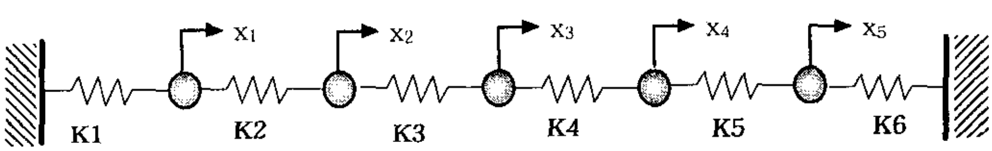


Fig. 1 5 DOF mass-spring system

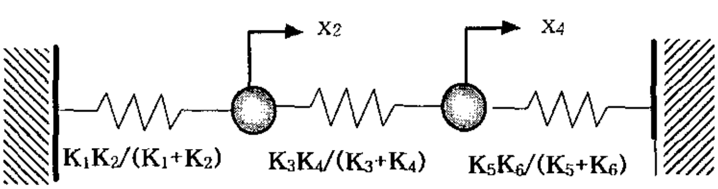


Fig. 2 Equivalent condensed mass-spring system

respectively,

$$\begin{Bmatrix} x_2 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \tag{21a}$$

$$\begin{Bmatrix} F_2 \\ F_4 \end{Bmatrix} = \begin{Bmatrix} \dfrac{K_1K_2(K_3+K_4)+K_3K_4(K_1+K_2)}{(K_1+K_2)(K_3+K_4)} \\ -\dfrac{K_3K_4}{K_3+K_4} \end{Bmatrix} \tag{21b}$$

Similarly the other pair are respectively

$$\begin{Bmatrix} x_2 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \tag{22a}$$

$$\begin{Bmatrix} F_2 \\ F_4 \end{Bmatrix} = \begin{Bmatrix} -\dfrac{K_3K_4}{K_3+K_4} \\ \dfrac{K_3K_4(K_5+K_6)+K_5K_6(K_3+K_4)}{(K_3+K_4)(K_5+K_6)} \end{Bmatrix} \tag{22b}$$

Therefore the stiffness influence matrix $[S]$ becomes

$$[S] = \begin{bmatrix} AS11 & AS12 \\ AS21 & AS22 \end{bmatrix} \tag{23}$$

where

$$AS11 = \frac{K_1K_2(K_3+K_4)+K_3K_4(K_1+K_2)}{(K_1+K_2)(K_3+K_4)}$$

$$AS12 = AS21 = -\frac{K_3K_4}{K_3+K_4}$$

$$AS22 = \frac{K_3K_4(K_5+K_6)+K_5K_6(K_3+K_4)}{(K_3+K_4)(K_5+K_6)}$$

The matrix $[K]$ obtained from a common static condensation and the stiffness influence matrix $[S]$ by the influence method are the same.

2.3 Characteristic matrix of joints

The joint region of a structure shown in Fig. 3 is modeled finely as a finite element model considering clamping force, friction force, etc. If the stiffness influence matrix is obtained for the selected primary degrees of freedom by applying the method mentioned above, the condensed stiffness matrix is obtained for the reduced degrees of freedom,

$$[\bar{K}]_{TOTAL} = [S] \tag{24}$$

In the next step, the joint region is considered to be divided into joint member A and joint member B, then the condensed stiffness matrix $[\bar{K}]_A$ and $[\bar{K}]_B$ are easily obtained for each joint member A and B respectively by applying a common static condensation. As the difference of the condensed stiffness matrix in joined state obtained by Eq. (24) and the condensed stiffness matrices for each joint member only is obtained, the condensed stiffness matrix of the joint itself can be finally obtained by the expression

$$[\bar{K}]_{JOINT} = [\bar{K}]_{TOTAL} - \begin{bmatrix} [\bar{K}]_A & 0 \\ 0 & [\bar{K}]_B \end{bmatrix} \tag{25}$$

So the characteristic matrix condensed to the primary degrees of freedom including the effects of clamping forces or friction forces due to contact elements is obtained by Eq. (25). And if the selected primary degrees of freedom are the same, then we have the merit that the same stiffness matrix is always obtained even though the joint region is defined in a different manner.

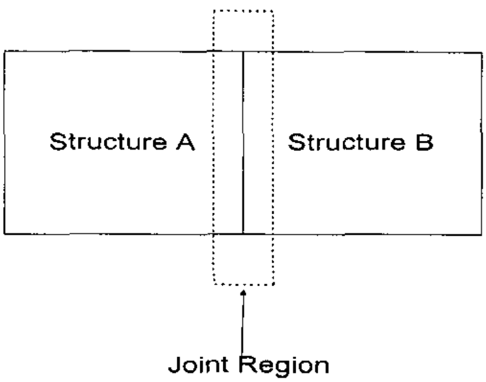


Fig. 3 Definition of the joint region

3. Analysis of a Beam-Type Structure with Bolted Joint

3.1 Application model

In order to apply the proposed joint modeling method, a beam-type structure is selected as a benchmark example (see Fig. 4). Two beams which have the same rectangular cross section (25 mm width ×5mm thickness) and 200 mm length are joined with single M 6 bolt by lapping 40 mm long. The 40 mm lapped region is defined as joint region, which is modeled in detail by using a general purpose finite element program. This model is divided into 5 parts as the joint member A (the upper plate), joint member B (the lower plate), bolt, nut, and contact elements. The previous 4 parts are modeled by 3 dimensional struc-

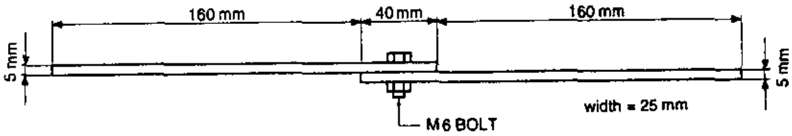
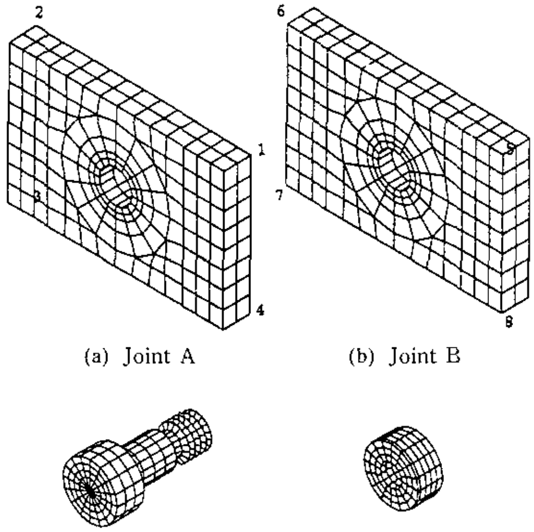


Fig. 4 Configurations of beams with a bolted joint



(a) Joint A (b) Joint B

(c) Bolt (d) Nut

Fig. 5 Modeling of structural components

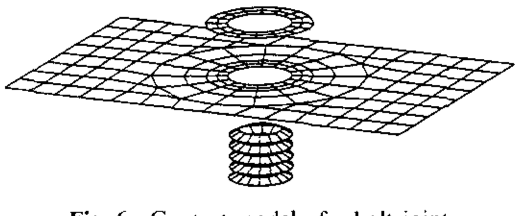


Fig. 6 Contact model of a bolt joint

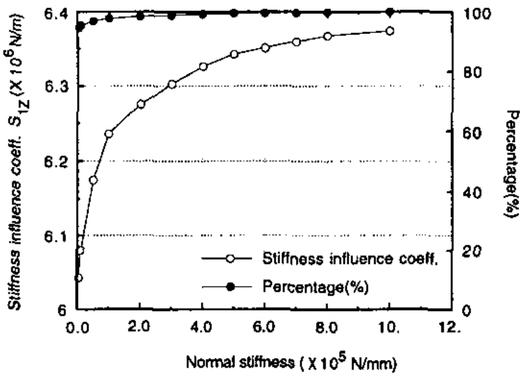


Fig. 7 Effects of stiffness of the contact elements

tural solid elements as shown in Fig. 5. And the joint (between joint members A and B, between bolt head and joint member A, between nut and joint member B, and between bolt and nut) is modeled by using contact elements as shown in Fig. 6. The numbers of elements are 304 for joint member A, 304 for joint member B, 496 for bolt, 240 for nut, and 565 for contact elements and total number of degrees of freedom is 6006.

3.2 Calculation of the stiffness influence matrix

The stiffness of the contact elements is required to be modeled for the contact elements in Fig. 6 and its effect on the stiffness influence matrix is considered. As this stiffness is varied from $1.0\times 10^4 N/mm$ to $1.0\times 10^6 N/mm$, the stiffness influence coefficient $S_{1Z,1Z}$ is calculated. The friction coefficient and the displacement are set to 0.3 and $1.0\times 10^{-3} mm$, respectively. Figure 7 shows the variation of the stiffness influence coefficients according to the stiffness of contact elements. Their deviations are less that 5% in the analysis range, and they are affected relatively small by the stiffness of the contact elements.

Modeling the contact elements also requires the friction coefficients of the contact region, and the stiffness influence coefficients may be dependent on this friction coefficients. The stiffness influence coefficient $S_{1Z,1Z}$ is obtained while the friction coefficients are varied from 0.1 to 0.5. The stiffness of the contact element and the given displacement are set to $5.0\times 10^5 N/mm$ and $1.0\times 10^{-3} mm$, respectively. Figure 8 shows that the deviation is less than 2% in the analysis range and the effect of friction coefficients is very small.

In the calculation of the stiffness influence coefficients, the reaction forces are obtained by assuming an optional magnitude of displacement on each degree of freedom. The reactions are then converted into those corresponding to unit displacement. So the stiffness influence coefficient $S_{1Z,1Z}$ is evaluated while the displacements are varied from $1.0\times10^{-10}mm$ to $1.0\times10^{-3}mm$. At this time the stiffness of the contact element and the friction coefficient are set to $5.0\times10^{5}N/mm$ and 0.3, respectively. Figure 9 shows the stiffness influence coefficients plotted against magnitude of displacement. The variation is about 10% in the

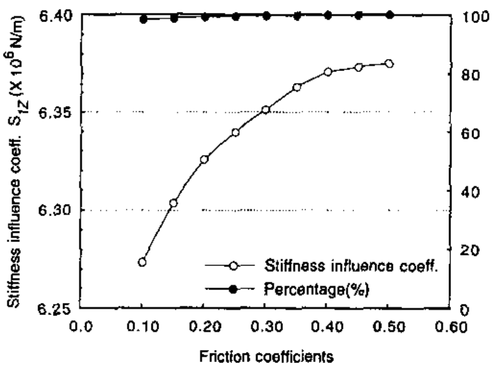


Fig. 8 Effects of the friction coefficients

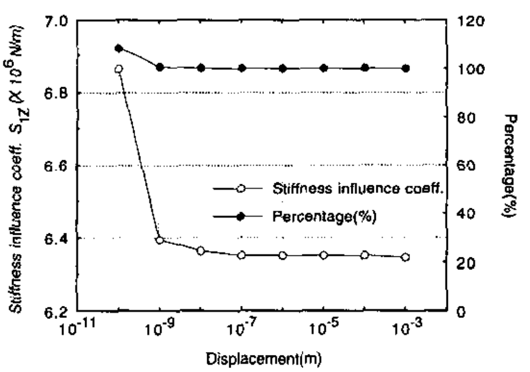


Fig. 9 Effects of the initial displacements

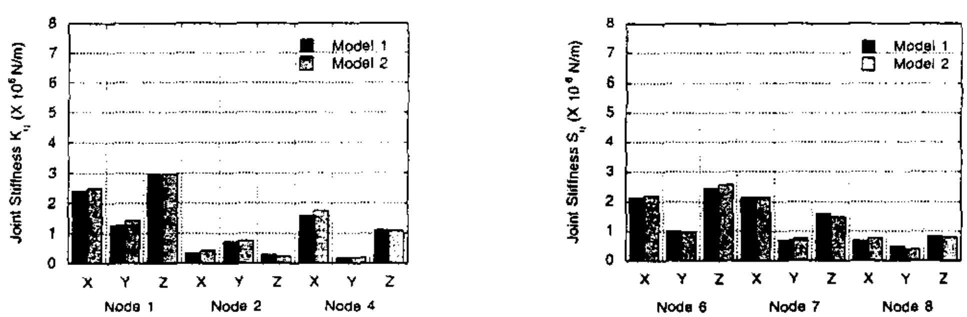
analysis range and it may be considered almost same except for very small displacement.

In order to identify the change of the joint characteristic matrix according to the definition of the joint region, the case of modeling[model 1] as shown in Fig. 4 and the case of modeling [model 2] where joint region was defined 10 mm longer on each plate than in model 1 are compared. Figure 10 shows that joint characteristic values for nodes 1, 2, 4 and 6, 7, 8 of model 1 and model 2. They are in good agreements and the differences are less than $0.1\times10^{6}N/m$. This means that the same joint characteristic matrix is obtained even though the joint region is defined in a different manner.

3.3 Analysis results

Eigenvalue problem is solved by the proposed modeling method and by the rigidly joined modeling. In the rigidly joined modeling, finite element analysis is performed under the assumption that two beams are united as solid.

The natural frequencies of the results are compared with those of modal experiments in Table 1. The proposed method shows more accurate results than rigidly joined modeling. Also the order of the fourth and the fifth mode are reversed in the results of rigidly joined method. So the rigidly joined modeling may not be good in this case. Figures 11 and 12 show the lower 4 mode shapes of the structure obtained by the proposed modeling method and those by the modal experiments, respectively. As shown in two figures, the mode shapes are in good agreement.



(a) Nodes 1, 2 and 4

(b) Nodes 6, 7 and 8

Fig. 10 Joint characteristics of the model 1 and 2

Table 1 Comparison of the natural frequencies (Hz)

Mode	Experimental	Proposed	Rigidly joined
1 (1st bending)	204.3	210.0(+2.8%)	221.5(+8.4%)
2 (2nd bending)	562.6	573.3(+1.8%)	567.3(+0.8%)
3 (3rd bending)	1080.4	1122.8(+3.9%)	1183.8(+9.5%)
4 (1st torsion)	1749.0	1761.1(+0.7%)	1860.2(+6.4%)
5 (4th bending)	1786.7	1768.6(−1.0%)	1840.8(+3.0%)
		RMS error 2.4%	RMS error 6.5%

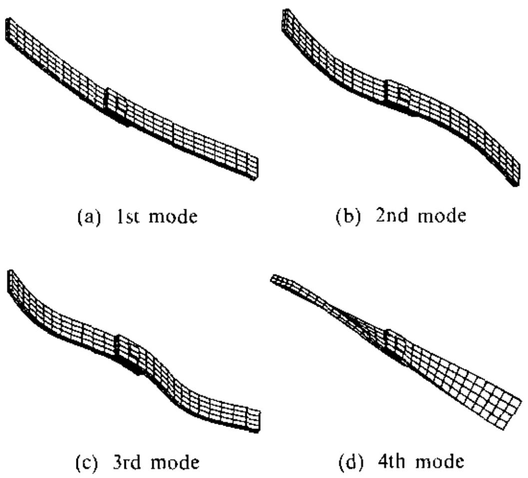


Fig. 11 Mode shapes of the beam with a bolted joint by proposed analysis

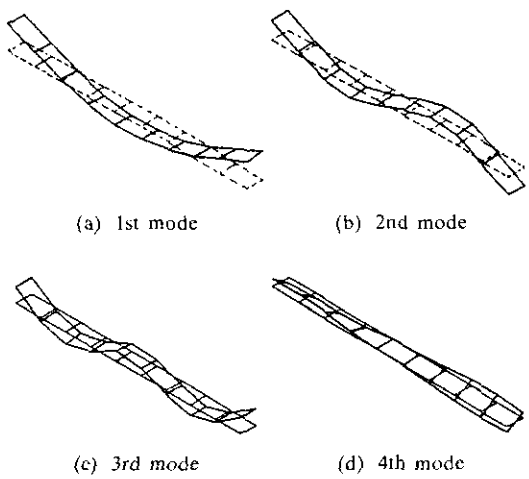


Fig. 12 Mode shapes of the beam by modal testing

4. Conclusion

A modeling method is proposed in order to reduce errors in eigenvalue analysis due to structural joints which make the modeling work complex and increase the number of degrees of freedom in many cases. A beam type structure with bolted joint is selected as a benchmark example to verify the proposed method by numerical analyses and experiment. Summaries of the research are as follows.

(1) If the primary degrees of freedom in joint are the same, the same joint stiffness matrices are obtained even though joint region is defined in a different manner.

(2) The stiffness of the contact elements and the friction coefficient exert a small effect on the overall stiffness matrix.

(3) In the natural frequency result by the proposed method, the root mean square error and the relative errors of many modes except for the second are less than those of rigidly joined modeling.

(4) Natural frequencies of a structure with joint are lower than those without joint and the effect of joint is different for each mode.

It is anticipated that this method can be effectively applied to substructure synthesis method where the joint modeling is important.

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