# Development of the Fatigue Life Prediction Method for IB Type Single Spot-Welded Lap Joint by Maximum Stress Function

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In this study, the reliability of various fatigue strength evaluation methods suggested so far was verified by the theory of Weibull probability distribution function for practical application to fatigue design of the spot-welded structure. It was found that the maximum stress has high reliability than other fatigue strength evaluation method. The reliability of fatigue strength evaluation method by the maximum stress method was 92.91%. Therefore, the fatigue strength evaluation method by the maximum stress was applicable to fatigue life prediction of spot-welded joint. The maximum stress function was introduced based on the relationship between influence of geometrical factor and maximum stress at the nugget edge of IB type spot-welded joint. From the maximum stress function and $\Delta\sigma - N_f$ relation, fatigue life of spot-welded joint was predicted without any additional fatigue test. By the proposed fatigue life prediction method, fatigue life of spot-welded joint having various shape of dimension and loading condition could be economically predicted. The predicted fatigue life has good agreement with the tested fatigue life.

**Key Words** : Fatigue Life Prediction, IB Type Spot Welded Lap Joint, Maximum Stress Function, Fatigue Strength, Nugget Edge

**Nomenclature**

$a$ : Shape parameter
$b$ : Scale parameter
$c$ : Location parameter
$j$ : Jth sample number
$N$ : Total number of sample
$x$ : Number of cycle to failure
$t$ : Thickness of specimen
$t_0$ : Thickness of standard specimen
$W$ : Width of specimen
$W_0$ : Width of standard specimen
$L$ : Length of specimen
$L_0$ : Length of standard specimen
$\theta$ : Joint angle of specimen
$\theta_0$ : Joint angle of standard specimen
A, B, C, D, E, F, G, H : Constant
$P$ : Applied load
$A$ : Area

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## 1. Introduction

Spot-welding has been known as a very useful technology in the fabricating process of the thin sheet structure such as automobile and train. Fatigue strength of the spot-welded joint is however, considerably lower than that of base material due to the stress concentration occurring at the nugget edge of spot-welded joint. Therefore, many researchers have numerically and experimentally investigated the fatigue strength of the spot-welded joint, so far. (Bae, 1991, JSAE, 1987) Furthermore, the fatigue strength data on spot-weld lap joints having various dimensions have been also considerably accumulated. (Tada, 1985)

The most typical and traditional fatigue strength evaluation method is $\Delta P-N_f$ curve. However, it is difficult to predict fatigue strength systematically considering the various geometrical factors, since the data plotted in $\Delta P-N_f$ relation are too scattered. To improve this problem, the new approaches, such as the $\Delta\sigma-N_f$ relation by the maximum stress at the nugget edge, have been recently suggested for systematic fatigue strength evaluation of spot-welded lap joint. (Sohn, 1998a, 1999a, 1999b, 1999c)

By the way, in order to estimate fatigue strength of specific spot-welded lap joint, it is necessary to conduct many additional fatigue tests for obtaining its $\Delta P-N_f$ curve. This indicates that a lot of cost and time for additional testing has to be considered. However, if fatigue life of spot-welded lap joint can be predicted from the fatigue data already accumulated and the reliable fatigue strength evaluation method, then the fatigue design criterion can be established without any additional fatigue test according to welding condition and joint type.

In this study, the reliability of various fatigue strength evaluation methods suggested so far was verified by the theory of Weibull probability distribution function for practical application to fatigue design of the spot-welded structure. (Patrick, 1995, Crowder, 1991, Kao, 1995) It was found that the maximum stress has high reliability than other fatigue strength evaluation method. The reliability of fatigue strength evaluation method by the maximum stress was 92.91%. Therefore, the fatigue strength evaluation method by the maximum stress was applicable to fatigue life prediction of spot-welded joint.

The maximum stress function was proposed based on the relationship between influence of geometrical factor and maximum stress at the nugget edge of IB type spot-welded joint. From the maximum stress function and $\Delta\sigma-N_f$ relation, fatigue life of spot-welded joint was predicted without any additional fatigue test. Using the proposed fatigue life prediction method by the maximum stress function, fatigue life of spot-welded joint having various shape of dimension and loading condition could be economically predicted.

## 2. Fatigue Strength of IB Type Spot-welded Joint

### 2.1 Analysis model

When tensile shear load is applied to the IB type single spot-welded lap joint, in-plane shear force (P), in-plane bending force (Q), and out of plane bending moment (M) act on the spot welding point. Thus, there occurs very complicate deformation by the forces. By this deformation mechanism, fatigue cracks are generally initiated from the nugget edge on the inner surface of the plate by stress concentration and propagated to the outer surface of the plate. Therefore, it is very important to calculate the accurate stress distribution and strain conditions around the nugget edge for reasonable fatigue strength evaluation.

In this study, dimension of the standard model was plate thickness (t) = 30mm, plate width (w) =

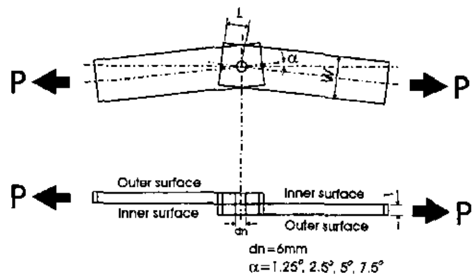



**Fig. 1** Dimension of standard model of IB type spot-welded lap joint

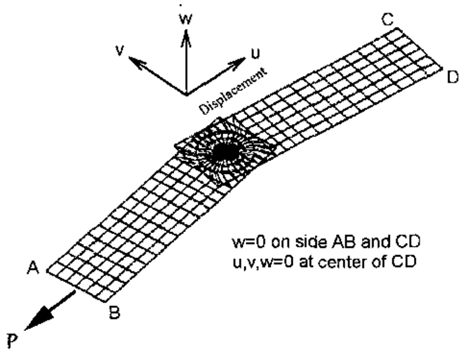



**Fig. 2** Finite element analysis model of IB type spot-welded lap joint

**Table 1** Chemical composition of specimen (Wt. %)

| | C | Si | Mn | P | S | Ni | Al | Fe |
|---|---|---|---|---|---|---|---|---|
| SPCC | 0.012 | 0.01 | 0.127 | 0.015 | 0.007 | 0.025 | 0.045 | Remain |

**Table 2** Mechanical properties of specimen

| | Tensile Strength (MPa) | Yield Strength (MPa) | Elongation (%) |
|---|---|---|---|
| SPCC | 307.0 | 168.4 | 47 |

30mm, lapped length (2L) = 30mm and joint angle ($\theta$) = 2.5 shown in Fig. 1. Three-dimensional finite element analysis model shown in Fig. 2 was used to calculate the stress distribution. This IB type single spot-welded lap joint model is to simulate the spot-welded bus window pillar joint sustaining in-plane force by warping of the body structure. The chemical compositions and mechanical properties are illustrated in Tables 1 and 2.

Eight nodes plain strain elements were entirely used. Mesh generation for the upper and lower plate was symmetrically performed. Particularly, the weld nugget was formed by getting together the node numbers of the elements contacting each other on the inner surfaces of the upper and lower plate. The elastic modulus value applied for the nugget was same with base metal due to that its influence upon the numerical results was not large. (Sohn, 1998b)

For convenience, tensile shear load of 9.81MPa was applied to the direction illustrated in Fig. 1. But, when systematically estimated fatigue strength with the maximum stress at the nugget edge of single spot-welded joint, they were calculated with actual loads applied to each fatigue specimen. The total number of elements and nodes were about 1164 and 1992, respectively. MSC. Nastran program was used for the stress analysis.

### 2.2 Results of stress analysis

Stress distribution around the spot-welded joint subjected to tensile shear load was shown in Fig. 3. When tensile shear load is applied to the specimen, three kinds of the components, which are in-plane force, in-plane bending, and out of plane bending moment, are partially act at the spot-welded joint. These components cause very complicate deformations, and affect to stress distribution around the nugget. By this mechanism, stress concentration is occurred at the nugget edge on the loading side of the plate, and also, its distribution range is influenced by joint angle of IB type spot-welded lap joint.

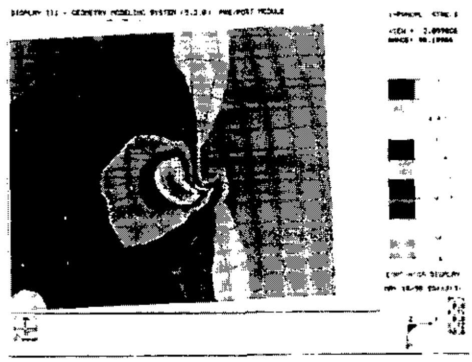


**Fig. 3** Stress distribution around the spot-welded lap joint at inner surface

The range of the maximum stress generated around the nugget edge by the combination of these three kinds components was occurred in $-20° \sim +40°$from the center line of the plate to the opposite direction against the rotating direction of the lower and upper plate by tensile shear force. It was also shown that the tensile stress was widely distributed in the region of the loading side and this region was removed to the rotating direction of the plate according to increase of the magnitude of applied load. When the joint angle was larger than 5 degree, tensile stress was simultaneously generated at the edge on the opposite side as well as on the loading side. This was due to out of plane bending and in-plane bending deformation increased with joint angle increase. (Sohn, 1998b)

In this study, in order to determine the systematic and reasonable fatigue design criterion for IB type spot-welded lap joints having various dimensions and subjected to tensile shear fatigue load, the $\Delta P - N_f$ relation was systematically rearranged with the maximum stress at the nugget

**Table 3** Welding condition of specimen

| | Electrode Force (F) | Welding Current (I) | Welding Time (cycle) |
|---|---|---|---|
| Welding Condition | 1962N | 8.3kA | 15cycles |

**Table 4** Fatigue test condition

| Test Condition | IB type specimen |
|---|---|
| Load Ratio ($R=P_{min}/P_{max}$) | 0 |
| Frequency(Hz) | 25 |
| Wave pattern | Sine wave |

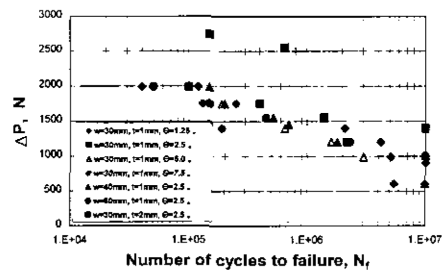



**Fig. 4** $\Delta P-N_f$ relation of IB type spot-welded lap joint

edge of the welding point.

## 2.3 Fatigue strength estimation of spot-welded joint

Material and configuration of fatigue test specimen are same with numerical analysis model. The spot welding condition to prepare fatigue test specimen is as shown in Table 3. (RWMA, 1985) Fatigue tests were conducted using a servo-hydraulic power system (MTS 10ton) under the condition of Table 4. Fatigue test results presented in the $\Delta P-N_f$ relation for various IB type spot-welded lap joints were shown in Fig. 4. Fatigue cracks were mostly generated in the maximum stress region. Fatigue limit of the specimen was decided by fatigue load that fatigue crack did not initiate until $10^7$ cycles.

The effects of geometrical factors on the fatigue strength of the specimen were estimated. In high load and short life range, their influences beside the plate thickness could be not clearly estimated due to the complicate deformation characteristics of the thin plate. However, in the low load and long life range, their influences were clearly revealed. Fatigue strength considerably increased with the plate thickness increase under the same loading condition due to increase of bending rigidity of the plate. Under the same thickness condition, fatigue limit was decreased with the joint angle of the specimen increase. This was due to that torque of the nugget by in-plane bending deformation linearly increased with the joint angle increase. But, the influence of the plate width on fatigue strength was not clear.

Therefore, the data of the $\Delta P-N_f$ relation shown in Fig. 4 was too scattered by the influence of geometrical factors of the specimen. Although the influence of geometrical factors on the fatigue strength of the specimen could be compared from these results, it was very difficult to establish a fatigue design criterion systematically considering the influence of geometrical and mechanical factors on the fatigue strength. Thus, it is necessary to determine a systematic fatigue design criterion considering the influence of geometrical and mechanical factors.

The stress categories using in fatigue analysis are generally nominal stress, structural hot spot stress, and notch stress considering stress concentration effect. The choice of stress category depends on the method used to express the fatigue strength data that will be used in the fatigue assessment. (Niemi, 1995)

Nominal stress of the plate $\left(\sigma_n=\frac{P}{A}=\frac{P}{W_t}\right)$ can be considered as the mechanical parameter for systematic fatigue strength estimation of spot-welded joints. But, because this shows big difference according to geometrical factors such as the plate thickness and the width of the plate, it is difficult to use as the parameter. (Sohn, 1998b)

The stress range ($\Delta\sigma$) is the main parameter to be determined for fatigue strength estimation. And, this can be applied for any of the three stress categories mentioned above. In the case of constant amplitude loading, the stress range is

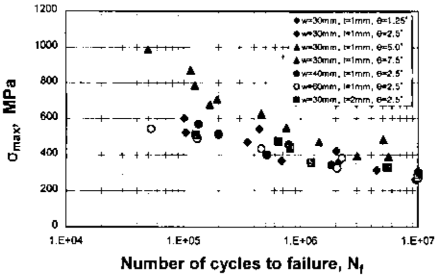



**Fig. 5** $\sigma_{max}-N_f$ relation of IB type spot-welded lap joint

defined as follows :

The fatigue load range : $\Delta P=P_{max}-P_{min}$ (1)

The stress range : $\Delta\sigma=\sigma_{max}-\sigma_{min}$ (2)

The load ratio : $R=\dfrac{\sigma_{min}}{\sigma_{max}}\left(=\dfrac{P_{max}}{P_{min}}\right)$ (3)

when R=0, $\sigma_{min}=0$ thus $\Delta\sigma=\sigma_{max}$ (4)

and, $R\neq 0$, $\sigma_{min}=R\cdot\sigma_{max}$,

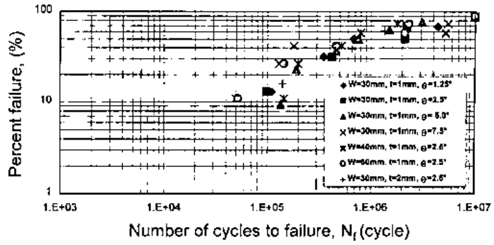
thus $\Delta\sigma=\sigma_{max}-R\cdot\sigma_{max}=(1-R)\cdot\sigma_{max}$ (5)

In this study, the load ratio(R) is zero as illustrated in Table 4. Therefore, from Eq. (4), the stress range ($\Delta\sigma$) for fatigue strength assessment of spot-welded joint is equal to the maximum stress ($\sigma_{max}$). The fatigue strength data was arranged using this maximum stress that was calculated from numerical analysis of prior section.

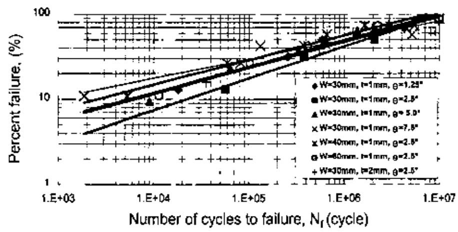
Figure 5 show the $\sigma_{max}-N_f$ relation rearranged the $\Delta P-N_f$ relation. It was shown that the $\Delta P-N_f$ data was more systematically rearranged by the maximum stress ($\sigma_{max}$) at the nugget edge of the spot-welded joint. This result means that, if the $\sigma_{max}-N_f$ relation for the spot-welded joint having specific dimensions and shapes like Fig. 5 is previously obtained, the other joints having optional dimensions can be predicted by it. From Fig. 5, the fatigue limit of various IB type spot-welded lap joints subjected to tensile shear load was predicted in about 36 MPa.

## 3. Reliability Verification of the Fatigue Strength Evaluation Methods

It is well known that the theory of Weibull probability distribution function is useful in statistical approach on the fatigue strength and




(a) 2-parameter Weibull probability distribution function




(b) 3-parameter Weibull probability distribution function

**Fig. 6** Comparison of Weibull probability distributioin function

fatigue life prediction of weldment by arc welding or gas welding. (Schijve, 1993, Zhao, 1998) However, it seldom happen that the theory of Weibull probability distribution function is applied to estimate of spot-welded lap joint.

Thus, in this study, the application of Weibull probability distribution function in fatigue strength evaluation and fatigue life prediction of spot-welded lap joint was investigated. Used model, as an example, was IB (in-plane bending) type single spot-welded lap joint of Fig. 1.

Comparison of 2-parameter and 3-parameters Weibull probability distribution function on the fatigue data of spot-welded lap joint having various dimension were shown in Fig. 6. It has known that Weibull probability distribution function presented in log-log scale is desirable to be presented in a straight line for obtaining more accurate result (Patrick, 1995). 3-parameters Weibull distribution of Fig. 6(b) is a straight line but 2-parameter Weibull distribution function of Fig. 6 (a) is not. Therefore, it was assumed that fatigue strength evaluation and fatigue life prediction of spot-welded lap joint by 3-parameter Weibull probability distribution function was more preferable than 2-parameter.

Therefore, reliability estimation of the fatigue strength evaluation methods by the maximum stress was verified using 3-parameters Weibull distribution function. The values of median rank, transformation function and Weibull reliability were calculated from the following equations:

$$\text{Median rank}: F=\frac{J-0.3}{N+0.4}\times 100(\%) \quad (6)$$

$$\text{Transformation function}: T=-\ln(1-F) \quad (7)$$

$$\text{Weibull reliability}: R(x)=\exp\left[-\left(\frac{x-c}{b}\right)^{a}\right] \quad (8)$$

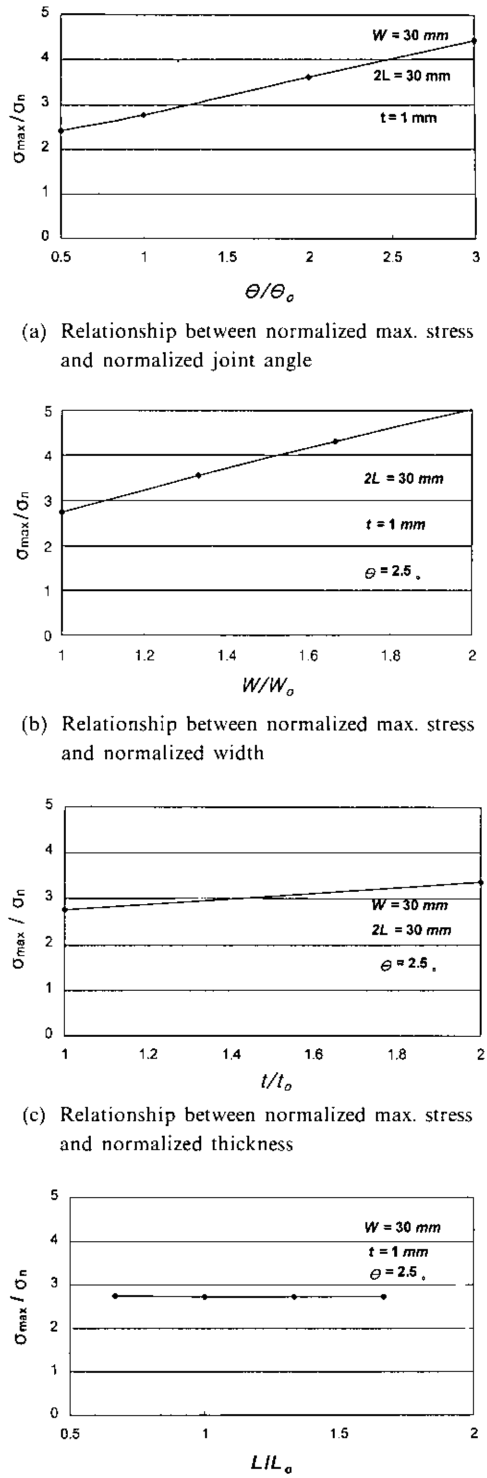
In this study, the reliability of various fatigue strength evaluation methods suggested so far was verified by the theory of Weibull probability distribution function for practical application to fatigue design of the spot-welded structure. (Sohn, 1998a, 1999a, 1999b, 1999c) It was found that the maximum stress has high reliability than other fatigue strength evaluation method. The reliability of fatigue strength evaluation method by the maximum stress was 92.91%.

## 4. Fatigue Life Prediction Method

### 4.1 Mechanical parameter for fatigue life prediction

In developing the fatigue life prediction method, it is necessary a mechanical parameter that can systematically consider the various geometrical factors of spot-welded lap joint. Fig. 7 shows relationship between the normalized maximum stress and the various geometrical factors of IB type spot-welded lap joint. (Sohn, 2000) From the Fig. 7, it was found that the maximum stress at the nugget edge was influenced by the geometrical factors of IB type spot-welded lap joint. This indicates that the maximum stress at the nugget edge of spot-welded lap joint can be defined as a function of applied load and the geometrical factors.

By using relationship between the maximum stress and the geometrical factors of Fig. 7, shape coefficient ($F_S$) can be defined as follows:




(a) Relationship between normalized max. stress and normalized joint angle

(b) Relationship between normalized max. stress and normalized width

(c) Relationship between normalized max. stress and normalized thickness

(d) Relationship between normalized max. stress and normalized lapped length

**Fig. 7** Relationship between normalized max. stress and geometrical factors

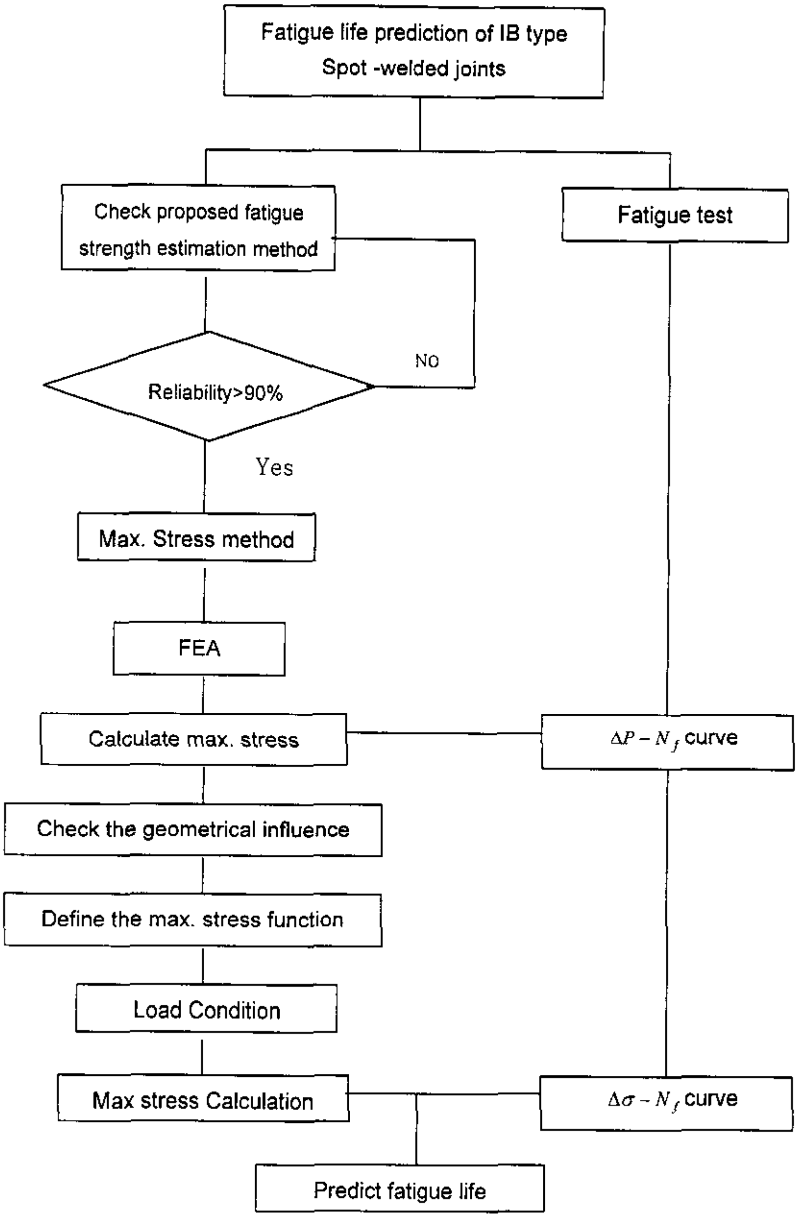



**Fig. 8** The procedure of fatigue life prediction method

$$F_s=\left\{A\left(\frac{L}{L_0}\right)+B\left(\frac{t}{t_0}\right)+C\left(\frac{W}{W_0}\right)+D\left(\frac{\theta}{\theta_0}\right)^2+G\left(\frac{\theta}{\theta_0}\right)+H\right\} \quad (9)$$

It is also necessary to make a transformation function presenting $\Delta\sigma-\Delta P$ relation based on and relation. Since the maximum stress at the nugget edge of the spot-welded lap joint is related to the fatigue load (P), which applied to the specimen as well as shape coefficient ($F_s$), the maximum stress function can be defined as follows:

$$\sigma_{max}=F_s \cdot \frac{P}{A} \quad (10)$$

Thus, the maximum stress function can be obtained as follows:

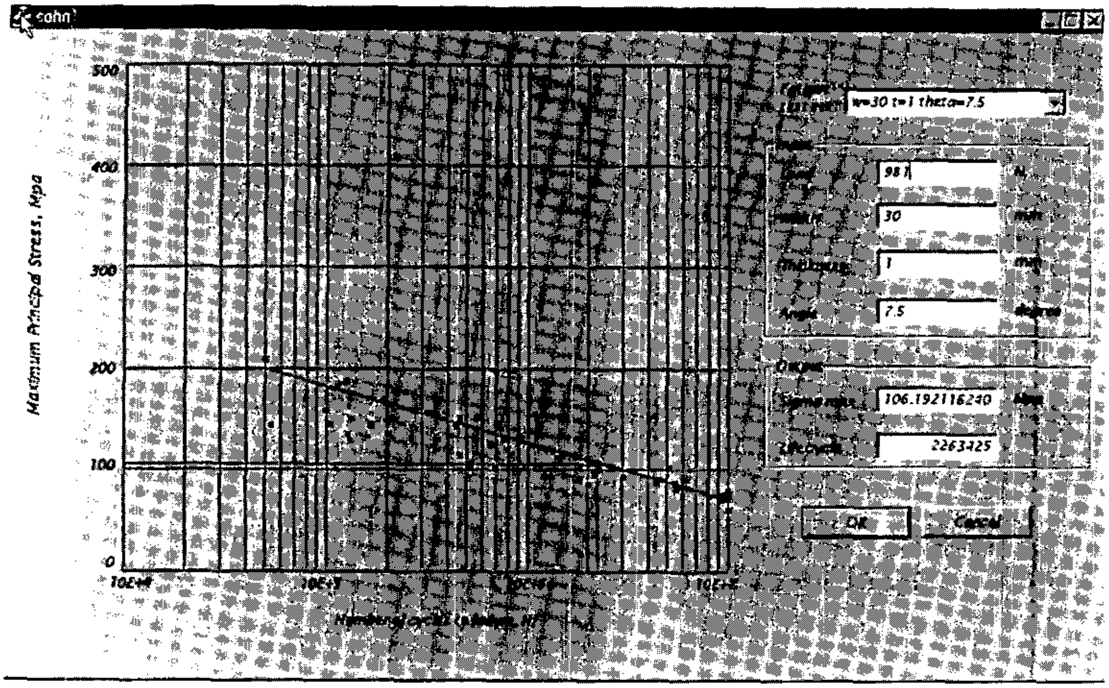



**Fig. 9** Case study of fatigue life prediction method with existed fatigue data

$$\sigma_{max}=\left\{2.31\left(\frac{t}{t_0}\right)+1.98\left(\frac{W}{W_0}\right)-0.20\left(\frac{\theta}{\theta_0}\right)^2+0.56\left(\frac{\theta}{\theta_0}\right)-2.21\right\}\cdot\frac{P}{W\cdot t} \quad (11)$$

### 4.2 Fatigue life prediction

In this study, the possibility of fatigue life prediction method for spot-welded lap joint having specific dimension and loading condition from the $\Delta P-N_f$ curve was verified. When the fatigue strength is estimated by $\Delta P-N_f$ curve in Fig. 4, the fatigue data was too scatter. Therefore, this means that fatigue strength of spot-welded lap joint can be overestimated or underestimated by the above method.

For improving this problem, the fatigue life prediction method was proposed by the maximum stress function. Figure 8 illustrates the procedure of fatigue life prediction method to predict the fatigue life of spot-welded lap joint having specific dimension and loading condition without additional fatigue test.

The first step that has to be done is to obtain the $\Delta P-N_f$ curve from fatigue test considering geometric factors and load condition. And next, the maximum stress at the nugget edge was calculated through finite element analysis. By using these results, $\Delta\sigma-N_f$ relation can be obtained which is the result of systematically re-arrangement of $\Delta P-N_f$ curve. Meanwhile, according to the relationship between maximum stress and geometric factors, the maximum stress function can be expressed by Eq. (11). Therefore, fatigue life of IB type spot-welded lap joint corresponding to the maximum stress range can be finally predicted from $\Delta\sigma-N_f$ curve.

The possibility and reliability of the proposed fatigue life prediction method was investigated with several cases. As an example, the case study of fatigue life prediction method with existed fatigue test data is shown as Fig. 9. When $\Delta P=785N$ was applied to IB type spot-welded lap joint of the specimen which dimension is $W=30$mm, $t=1$mm, $\theta=7.5°$, maximum stress from the FEA was 82. 4MPa but maximum stress from the maximum stress function was 85. 4MPa in Table 5. The stress by the maximum stress function has some difference with FEA result. Certifying this result, it was compared the predicted fatigue life with the tested fatigue life when $\Delta P=$

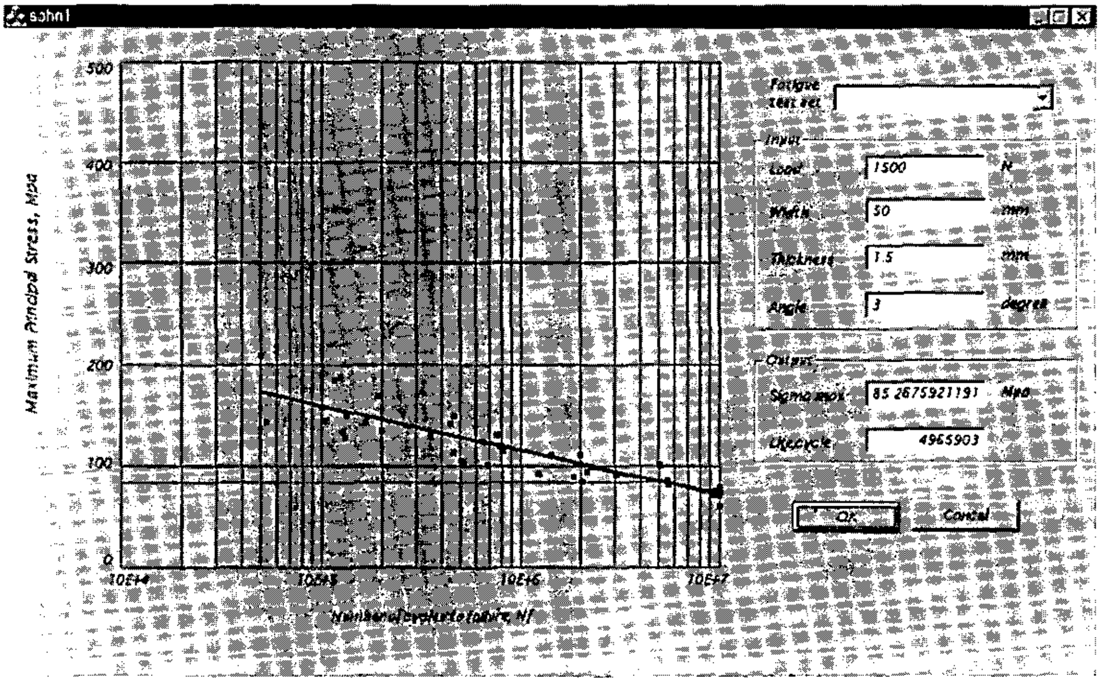



**Fig. 10** Case study of fatigue life prediction method without additional fatigue test

**Table 5** Comparison of max. stress from max. stress function and the FEA

| Specimen | Max. stress from max. stress function | Max. stress from FEA |
|---|---|---|
| Case study 1 | 85.4MPa | 82.4MPa |
| Case study 2 | 85.3MPa | N/A |

Remark>.Cast study 1 : $W=30$mm, $t=1$mm, $\theta=7.5°$, $\Delta P=785N$

.Cast study 2 : $W=50$mm, $t=1.5$mm, $\theta=3.0°$, $\Delta P=1500N$

$785N$ was applied to the specimen which dimension is $W=30$mm, $t=1$mm $\theta=7.5°$. From the $\Delta P-N_f$ curve, the tested fatigue life was $5.6\times10^6$ cycles and the predicted fatigue life was $5.3\times10^6$ cycles. At the same specimen and loading condition, it was almost the same fatigue life.

And next, in this time, it was predicted to the specimen of $W=50$mm, $t=1.5$mm, $\theta=3.0°$ and $\Delta P=1500N$ by the maximum stress function is shown as Fig. 10. The maximum stress at the nugget edge from Eq. (11) was 85.3MPa, and its fatigue life was predicted as $5.0\times10^6$ cycles from $\Delta\sigma-N_f$ curve.

To increase the reliability, some case of predicted fatigue life and tested fatigue life was compared. Table 6 shows the comparison of predicted fatigue life and tested fatigue life. It shows good agreement the predicted fatigue life with the tested fatigue life.

From the compare results, the maximum stress has some difference with FEA result, but the predicted fatigue life has good agreement with the tested fatigue life. It was known that the fatigue life of IB type spot-welded lap joint having specific dimension could be more reasonably predicted by the maximum stress function and $\Delta\sigma-N_f$ curve. Therefore, the maximum stress function can be economically predicted the fatigue life of spot-welded thin sheet structure without any additional fatigue test.

## 5. Conclusion

In order to develop the fatigue life prediction method and to establish the long life fatigue design criterion of IB type spot-welded lap joint subjected to tensile shear fatigue load, the reliabil-

**Table 6** Comparison of tested fatigue life and predicted fatigue life

| Specimen and load condition | Tested fatigue life (cycle) | Predicted fatigue life (cycle) |
|---|---|---|
| Case study 1 | $5.6\times10^6$ | $5.3\times10^6$ |
| $W$=30mm, $t$=1mm, $\theta$=2.5°, $\Delta P$=981$N$ | $1.0\times10^7$ | $1.0\times10^7$ |
| $W$=60mm, $t$=1mm, $\theta$=2.5°, $\Delta P$=1570$N$ | $4.6\times10^5$ | $4.8\times10^5$ |
| $W$=30mm, $t$=1mm, $\theta$=2.5°, $\Delta P$=1374$N$ | $9.6\times10^6$ | $1.0\times10^7$ |
| Case study 2 | N/A | $5.0\times10^6$ |

Remark>.Cast study 1 : $W$=30mm, $t$=1mm, $\theta$=7.5°, $\Delta P$=785$N$
.Cast study 2 : $W$=50mm, $t$=1.5mm, $\theta$=3.0°, $\Delta P$=1500$N$

ity estimation of fatigue strength evaluation by the maximum stress was verified using 3parameter Weibull probability distribution function.

It was found that the maximum stress has high reliability than other fatigue strength evaluation method. The reliability of fatigue strength evaluation method by the maximum stress method was 92.91%. Therefore, the fatigue strength evaluation method by maximum stress was applicable to fatigue life prediction of spot-welded joint.

From the above status, an economic and systematic fatigue life prediction method by the maximum stress function was proposed. Obtained conclusions are as follows:

(1) The maximum stress function considered the relation of the maximum stress, fatigue load and the effect of geometrical factors of the IB type spot-welded lap joint were suggested.

(2) The maximum stress function for IB type spot-welded lap joint was defined as follows ;

$$\sigma_{max}=\left\{2.31\left(\frac{t}{t_0}\right)+1.98\left(\frac{W}{W_0}\right)-0.20\left(\frac{\theta}{\theta_0}\right)^2+0.56\left(\frac{\theta}{\theta_0}\right)-2.21\right\}\cdot\frac{P}{W\cdot t}$$

(3) The predicted fatigue life by the maximum stress function has good agreement with the actual fatigue test results.

(4) The proposed fatigue life prediction method provided design flexibility and will be cost effective method.

## Reference


Bae, D. H., 1991, "Fracture Mechanical Fatigue Strength Evaluation of Spot-Welded Lap Joint Under Tension-Shear Load," *Journal of KSAE*, Vol. 13, No. 5, pp. 42~50.

Crowder, M. J., et al, 1991, *Statistical Analysis of Reliability Data*, Champman & Hill, pp. 69~103.

Kao, J. H. K., 1995, "A Graphical Estimation of Mixed Weibull Parameter in Life Testing of Electron Tube," *Technometrics*, Vol. 1, pp. 389~407.

Niemi, E., 1995, "Stress Determination for Fatigue Analysis of Welded Components," *The International Institute of Welding*, Abington Publishing. pp. 3~52.

Patrick, D. T., et al, 1995, "Practical Reliability Engineering," $2^{nd}$ edition, International Thomson Publishing Inc., pp. 81~103.

Schijve, J., 1993, "Normal Distribution or a Weibull Distribution for Fatigue Lives," *Fatigue & Fracture Engineering Materials & Structure*, Vol. 16, No. 8, pp. 851~859.

Sohn, I. S., et al, 1998a "Fracture Mechanical Fatigue Strength Evaluation of IB-Type Spot-Welded Lap Joint Under Tension-Shear Load," *journal of KSAE*, Vol. 6, No. 5, pp. 20~27.

Sohn, I. S., 1998b, "A Study on the Fatigue Design Methods and Expert System Development for Thin Steel Sheet Spot Welded Lap Joint," Ph. D. Thesis, Sungkyunkwan University. Seoul.

Sohn, I. S., et al, 1999a, "Fatigue Strength Evaluation on the IB Type Spot-Welded Lap joint of 304 Stainless Steel, Part 1 : Maximum Principal Stress," *journal of KWS*, Vol. 17, No. 6, pp. 380~385.

Sohn, I. S., et al, 1999b, "Fatigue Strength Evaluation of IB Type Single Spot Welded Lap Joint by Strain Energy Density Factor Under Tension-Shear Load," *Proceeding of autumn conference, KSME*, pp. 54~59.

Sohn, I. S. and Bae, D. H., 1999c, "Fatigue Strength Evaluation of In-Plane Bending Type Single Spot Welded Lap Joint by Strain Energy Density," *Journal of KSME-A*, Vol. 23, No. 9, pp. 1435~1441.

Sohn, I. S. and Bae, D. H., 2000, "A Study on the Fatigue Life Prediction Method of the Spot-Welded Lap Joint," *journal of KSAE*, Vol. 8, No. 3, pp. 110~118.

Tada, H., Paris, P. C. and Irwin, G. R., 1985, "The Stress Analysis of Cracks Handbook," $2^{nd}$ edition, Paris Production Inc, pp. 2. 1~2. 22.

The Committee of JSAE, 1987 "Fatigue Strength Data Book of Spot-Welded Lap Joint," JSAE.

The Committee of RWMA, 1985, "Resistance Welding Manual," Vol. 1, pp. 119~143, RWMA.

Zhao, Y. X., 1998, "A Statistical Investigation of the Fatigue Lives of Q235 Steel Welded Joint," *Fatigue & Fracture Engineering Materials & Structure*, Vol. 21, No. 8, pp. 781~790.