Development of Global Function Approximations for Desgin Optimization Using Evolutionary Fuzzy Modeling

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This paper introduces the application of evolutionary fuzzy modeling (EFM) in constructing global function approximations to subsequent use in non-gradient based optimization strategies. The fuzzy logic is employed for express the relationship between input and output training patterns in form of linguistic fuzzy rules. EFM is used to determine the optimal values of membership function parameters by adapting fuzzy rules available. In the study, genetic algorithms (GA's) treat a set of membership function parameters as design variables and evolve them until the mean square error between defuzzified outputs and actual target values are minimized. We also discuss the enhanced accuracy of function approximations, comparing with traditional response surface methods by using polynomial interpolation and backpropagation neural networks in its ability to handle the typical benchmark problems.

Key Words : Global Function Approximations, Fuzzy Modeling, Genetic Algorithms

1. Introduction

Emerging computation strategies such as genetic algorithms (GA's) and simulated annealing (SA) have found increased use in the problems of engineering design optimization. In many practical design problems, the design space may contain continuous, discrete and integer design variables. Furthermore, it may be multimodal, or even disjointed, making it very difficult to identify the global optimum. So the GA based optimization strategy has been developed to be especially effective (Hajela and Lee, 1995). This method does not require gradient information, and due to a global nature of the search in which design data from widely dispersed points are used to make design modifications, has the increased probability of locating the global optimum. However, one of the major drawbacks in genetic algorithms is the significant increase in computational resource requirements when the underlying analysis is inherently nonlinear, and the design problem is characterized by a large number of design variables and constraints. In such cases, it is required that the design optimization algorithm be used in conjunction with an approximate analysis.

Recently there have been considerable advances in the use of approximate analysis methods in engineering design optimization. Function approximations are useful in modeling the behavior of engineering systems that cannot be readily defined by analytical formulations. Recently, with a growing interest of manufacturing considerations in the early stage of integrated design, a technique for modeling manufacturing processes typically characterized by inadequate or vague information, has become necessitated. Unlike gradient based methods, the search in GA progresses from a multiple set of designs to another set, incorporating information from all points to establish the direction of move. While such an approach offers the increased probability of locating the global optimum, the computational cost is high due to multiple design point evaluations. A

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Taylor series type approximation scheme is unacceptable in GA's as multiple approximations must be constructed. Furthermore, the integrity of these approximations is questionable, as the method does not impose explicit limits on design changes during iterative process. Response surface methods (RSM) based on polynomial functions and/or neural networks have been studied in the context of global function approximations (Carpenter and Barthelemy, 1993; Hajela and Berke, 1992). RSM is extremely potent in those problems where the combination of numerical and experimental data is used to construct an approximation model. A problem associated with the use of RSM is that when using polynomial interpolation, it becomes difficult to determine the best order of the polynomial, and the amount and distribution of its terms that must be used in developing a suitable response surface within an acceptable range of approximation accuracy. A similar situation exists in the use of neural networks; it is also difficult to establish the architecture of the network as indicated by the number of hidden layers of neurons, the number of neurons in each layer.

The present paper discusses the efficient method of developing evolutionary fuzzy modeling (EFM) based global function approximation tools for subsequent use in design optimization, where global search strategies such as genetic algorithms are used. EFM is an optimization process for determining the types of membership functions and their parameters of interest by adapting fuzzy rules, where the optimization process is conducted by evolutionary computing methods such as genetic algorithms (Satyadas and KrishnaKumar, 1994; Cordon and Herrera, 1995) as well. In practice, it is also found that the small (or insufficient) amount of deterministic data is provided, and most of analyses or experimental data are given in form of heuristic or linguistic information. When the training data is the type of fuzzy rules, we obtain a function approximation model by determining the optimal parameters for input and output membership functions describing the conditions and actions in fuzzy rules, respectively. In EFM, GA's will treat a set of membership function parameters as design variables and evolve them until the error between predicted outputs and actual target values are minimized. In the present study, a global function approximation with EFM approach is implemented and compared with the conventional RSM by both artificial neural networks and polynomials to see its generalization capability. Fuzzy logic and neural networks are generally categorized as soft computing techniques where global function approximation tools using machine learning paradigms were explored (Hajela and Lee, 1997). In the paper we employ simple response functions with single-input and single-output to verify the proposed approach in modeling nonlinear and multimodal design problem. In the subsequent sections we will briefly discuss the theoretical aspects in fuzzy system and the evolutionary strategy for developing global function approximations.

2. Fuzzy System

The fuzzy inference system (FIS) is a computing framework based on the traditional concepts of fuzzy set theory, fuzzy if-then rules and fuzzy reasoning (Jang, Sun and Mizutani, 1997). The fuzzy logic and fuzzy inference system have been widely applied in control systems design (Hines, 1997), and have received recent attention in multiobjective optimization of structural and mechanical systems (Rao, 1987; Dhingra and Rao, 1992). The basic structure of FIS has three components; fuzzy rules are expressed by linguistic rule base information, fuzzy membership functions are introduced to represent a set of fuzzy rules, and a reasoning mechanism performs the inference procedure upon the rules and given facts to derive a reasonable output or conclusion.

Using fuzzy sets, the linguistically expressed rules can be defined for a given set of input and output variables. The fuzzy rules use the conditional statements of if-then rules. For example, a standard fuzzy if-then rule assumes the following form:

$$\text{If } x \text{ is } A, \text{ then } y \text{ is } B. \tag{1}$$

In the above, A and B are the linguistic values of the input variable x and the output variable y, respectively. The if-part of the rule "x is A" is called the antecedent or condition, and the then-part of the rule "y is B" is called the consequent or action. In the case of boolean logic, if the antecedent part of the if-then rule is true, then the consequent part of the if-then rule is also true. However, the fuzzy if-then rules do not operate in the same manner since they use the fuzzy statement. Instead, in fuzzy if-then rules, if the antecedent is partially true to some degree, then the consequent is also partially true to the same degree. If-then rules can also have more than one part in both the antecedent and consequent. In this case, all antecedent parts are calculated simultaneously and generate a single value by using the logical operators. This results from the antecedent part and affects all consequents equally by an implication function.

The definition of a fuzzy set is the simple extension of the classical set definition where the characteristic function has any values between 0 and 1. From the definition of a fuzzy set, for example, the linguistic values of input variable in the antecedent in Eq. (1) could be expressed as a set of ordered pairs:

$$A=\{(x, \mu_A(x)) \mid x \in X\} \quad (2)$$

where, $\mu_A(x)$ is called the membership function for the fuzzy set A. The membership function can be selected as any arbitrary curve according to one's subjective perception based on the behavior of a function. We can introduce such membership function to the consequent as well. Notice that the basic FIS can take either fuzzy or crisp inputs, while the outputs it produces are mostly fuzzy sets. It is necessary to have a crisp output, especially in a case where FIS is used as a decision-making device. Therefore, a method of generating an aggregated decision value, referred to as defuzzification is needed to extract a crisp value best representing a fuzzy set. The procedure for fuzzy rule aggregation and subsequent defuzzification is summarized for completeness.

Consider a case where a number of fuzzy rules are made of more than one part in the antecedent and a single part in the consequent. For each of fuzzy rules, all antecedent parts and are calculated simultaneously to generate a single value by using the logical operators; this process results from the antecedent parts and the then affects the consequent equally by an implication function. The defuzzification is then performed based on the multiply aggregated values of output fuzzy sets from each of fuzzy rules. When the maximum method as an implementation of aggregation process is considered for example, the order in which the rules are aggregated does not matter for more than two output fuzzy sets due to its commutative characteristic in aggregation. The aggregation of two output fuzzy sets returned by the implication process generates another fuzzy set. In order to extract useful information from newly aggregated fuzzy set, it must be defuzzified to obtain a single value as well. Although the conversion of a fuzzy set into a single crisp value is available in several ways, the present study adopted the centroid method to calculate the center of a region generated by all aggregated output fuzzy sets. FIS with a crisp output is shown in Fig. 1, where a basic FIS transforms an aggregated output fuzzy set into a single crisp value.

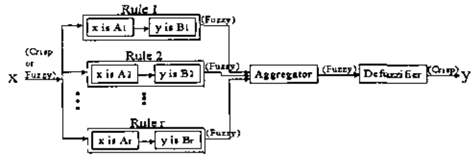


Fig. 1 Fuzzy inference system

3. Evolutionary Fuzzy Modeling

Evolutionary fuzzy modeling (EFM) employs evolutionary algorithms to evolve the fuzzy model of a nonlinear and/or multimodal system. The general approach for using a parameter optimization technique for fuzzy modeling has been used to tune the parameters of predefined rules. In the study, genetic algorithms are used to obtain near-optimum fuzzy membership parameters and fuzzy rule structure through an iterative procedure using appropriate performance index and available system information. Building an optimal and robust fuzzy model is critical to the performance of the fuzzy logic. In order to tune a fuzzy model, the EFM approach is introduced in the present study. Even though the type of membership functions and the number of rules can vary during the GA evolution, the only tuning parameters in this work are membership parameters used to define the shape of each membership function. Notice that the additional consideration of selecting the type of membership functions produces more intelligent EFM framework and, resulting in the increase in the computational costs during the GA based optimization process. GA's treat a set of membership function parameters as design variables and evolve them until the error between defuzzified outputs and actual target values are minimized. The GA based optimization statement for optimal tuning of a model with m number of inputs and n number of outputs can be written as follows:

$$\text{minimize} \quad F=\frac{1}{n}\sum_{j=1}^{n}(y_j-t_j)^2 \tag{3}$$
$$x_i^L \le x_i \le x_i^U,\ i=1,\cdots,m$$

The objective function was considered as the mean square error between the response predicted output, y_j and the actual output, t_j. It should be noted that actual output is obtained from exact analyses, while the predicted output is generated by fuzzy membership functions and their parameters. The design variables in this approach are membership function parameters, limited by proper lower and upper bounds; each design variable in the EFM approach represents a parameter for use in defining the membership function. The solution for this optimization problem is the set of membership parameters generating the most accurate approximation. The next section introduces the benchmark functions to be approximated by EFM; they are typical in terms of function modality. The general procedure for EFM based optimal membership parameter extraction is shown in Fig. 2.

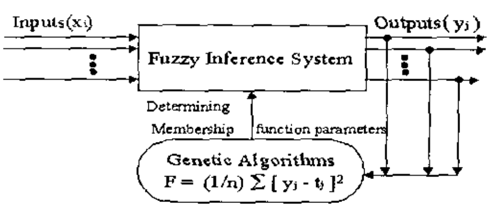


Fig. 2 GA based EFM for function approximations

4. Numerical Examples

The present study explores two types of functions to verify the proposed strategy in response approximations. The first case is a half-sine wave, a simple harmonic function that has the unimodality in function response. The second is a multimodal function, typically obtained through a vibration analysis of mechanical systems; this may have a higher degree of complexity in function approximations.

4.1 Unimodal function

Consider the following half-sine wave equation that behaves in a unimodal and continuous manner over the entire design space:

$$y=\sin(\theta),\ 0\le\theta\le\pi \tag{4}$$

Fuzzy model requires some basic knowledge about the function to be approximated before initializing the model. For the fuzzy logic model working properly, a set of fuzzy rules, and the type and number of membership functions for input and output variables should be defined based on the original function. Assuming that the minimal knowledge about the half-sine wave function is available, the following three rules can be generated to construct the proper membership functions:

Rule 1 If θ is small, then y is small.
Rule 2 If θ is medium, then y is large.
Rule 3 If θ is large, then y is small.

In this function approximation, three Gaussian (small, medium, large) and two triangular (small, large) membership functions can be used to describe variations of input and output values, respectively;

$$\mu_{Gaussian}(x,c,\sigma)=e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \tag{5}$$

$$\mu_{triangular}(x,\alpha,\beta,\gamma)=\begin{cases}0, & x\le\alpha\\ (x-\alpha)/(\beta-\alpha), & \alpha\le x\le\beta\\ (\gamma-x)/(\gamma-\beta), & \beta\le x\le\gamma\\ 0, & \gamma\le x\end{cases} \tag{6}$$

where, c and σ determine the center and width of Gaussian membership function, respectively, and the parameters $\{\alpha, \beta, \gamma\}$ define the x coordinates of three corners of the triangular membership function. It should be reminded that these parameters are treated as design variables in EFM approach.

4.2 Multimodal function

The second function to be approximated is more complex in modality than a simple harmonic function. The present study adopted the following typical equation:

$$y=5e^{-0.68t}\sin(2\pi t),\ 0.0\leq t\leq 4.0 \qquad (7)$$

Multimodality of Eq. (7) requires at least ten rules to express the function behavior over the design space as follows:

Rule 1 If t is Zero, then y is Zero.
Rule 2 If t is VeryVerySmall, then y is PositivelyGreat.
Rule 3 If t is VerySmall, then y is NegativelyGreat.
Rule 4 If t is Small, then y is PositivelyBig.
Rule 5 If t is Medium, then y is NegativelyBig.
Rule 6 If t is Big, then y is PositivelyMedium.
Rule 7 If t is Great, then y is NegativelyMedium.
Rule 8 If t is VeryGreat, then y is PositivelySmall.
Rule 9 If t is VeryVeryGreat, then y is NegativelySmall.
Rule 10 If t is VeryVeryVeryGreat, then y is Zero.

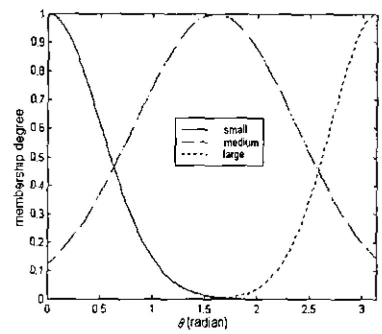
This multimodal function can also be approximated using Gaussian and triangular membership functions in Eq. (5) and (6) to describe antecedents and consequents, respectively.

5. Results and Discussion

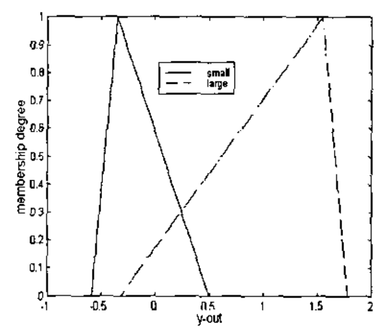
EFM based approximate analysis was simulated for a half-sine wave function, where there are two cases in numerical experiments. In the first case, only a small number of data set is provided for training to effectively approximate the output for the entire range of input. Three training patterns are selected at three distinct positions of interest, $\theta=0$, $\pi/2$, and π, which covers the entire design space; these training patterns exactly match

Table 1 Lower and upper bounds on membership function parameters

	Parameter	Unimodal Response			Multimodal Response		
		L bound	U bound	Precision	L bound	U bound	Precision
Input Membership Function	c	0.0	3.14	0.01	0.0	4.0	0.01
	σ	0.0	1.0	0.01	0.0	1.0	0.01
Output Membership Function	α	−0.8	1.8	0.01	−6.0	8.0	0.01
	β	−0.8	1.8	0.05	−6.0	8.0	0.01
	γ	−0.8	1.8	0.01	−6.0	8.0	0.01



(a) Input membership function



(b) Output membership function

Fig. 3 *Optimized membership function for* unimodal response (3 training data)

for each of fuzzy rule described in Eq. (4). Figures 3(a) and 3(b) show the optimized membership functions for input (condition) and output (action) fuzzy rules, respectively. Lower and upper bounds on membership parameters for use in GA evolution are given in Table 1. Note that Gaussian input membership function values are as close as 1.0 around $\theta=0$, $\pi/2$, and π, while output triangular membership function values

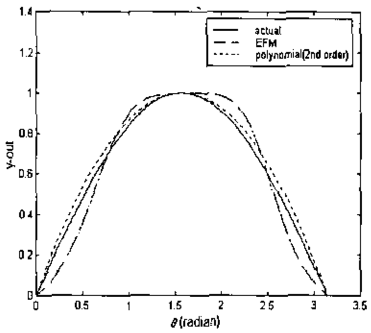
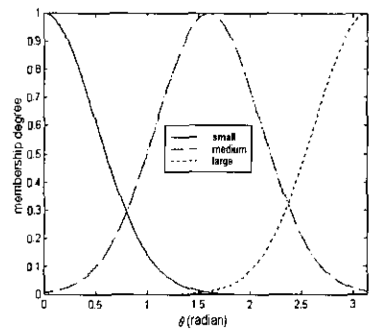
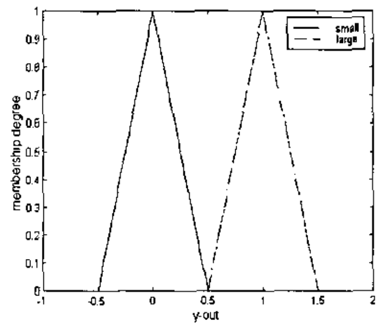


Fig. 4 Unimodal response (3 training data)



(a) Input membership function



(b) Output membership function

Fig. 5 Knowledge based membership functions for unimodal response

have their crisp values within the prescribed bounds on membership parameters. Function approximations are generalized using these optimal values of membership function parameters. Figure 4 shows approximated results obtained from EFM and polynomial interpolation; the 2nd order polynomial was selected according to the nature of response characteristics in actual output. This result shows that the polynomial is more effective in global approximation than that in EFM. One can deduce the way of increasing the approximation accuracy by looking at the membership function behavior; the optimized input membership function in Fig. 3(a) has the crisp values around $\theta=0$, $\pi/2$, and $_2$, as described before. However, the crisp values for the output membership function in Fig. 3(b) do not exactly match with 0.0 and 1.0, which correspond to actual minimum and maximum values of the half sine wave, respectively. Since the fuzzy logic can effectively take advantage of the user's knowledge, a more uniformly distributed set of membership functions are generated as shown in Fig. 5. This does not require any optimization process for the fuzzy model since all membership parameters are specified by the user's subjective understanding of the function behavior over the design space. The fuzzy logic approximation based on the user's knowledge is shown in Fig. 6, where the fuzzy logic model generated by the non-optimized user's knowledge produced worse in approximation than even the tuned model with only three training patterns. Nevertheless, the model may still provide a useful approximation tool in a situation where no analytical formulation is available.

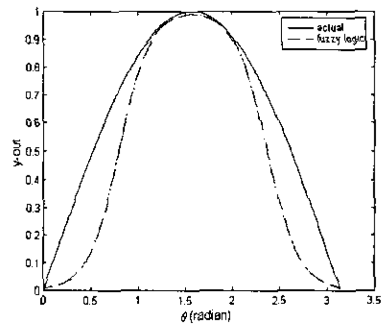
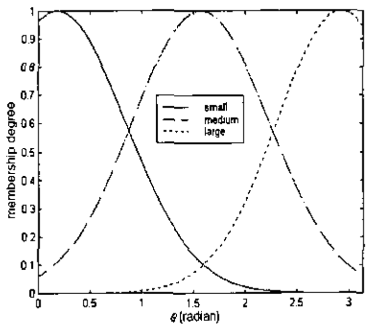


Fig. 6 Unimodal response (fuzzy logic only)

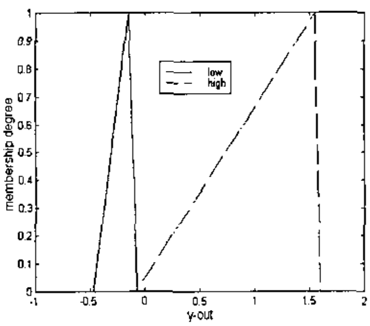
Now, we need consider the second case of increasing the number of training patterns in function approximation of a half sine wave. A set of uniformly distributed 20 patterns were employed for both the EFM and polynomial approachs. The optimized membership functions and approximated results using 20 training data are displayed in Fig. 7 and 8, respectively. The use of more training data in EFM approach generates the approximate result as close as the

Table 2 Comparision of approximation performance for unimodal response

Traing Data / Modle type	Testing Error(%)	
	3	20
EFM	5.95080	1.3596
Polynomial	2.89046	1.6402
Fuzzy logic	9.70524	N/A



(a) Input membership function



(b) Output membership function

Fig. 7 Optimized membership functions for unimodal response (20 training data)

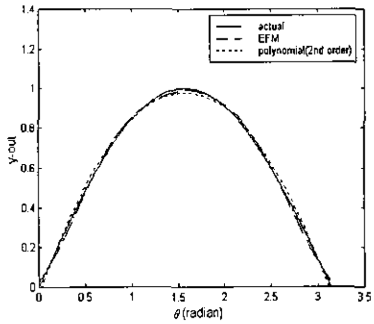


Fig. 8 Unimodal response (20 training data)

exact function value. The overall comparison over this unimodal function is also shown in Table 2 in terms of approximation strategy and the number of training data. The evolutionary fuzzy modeling approach has shown the marginally comparable approximation results compared to the polynomial interpolation in a case where the function to be approximated is of the type unimodal that is much simpler in modeling.

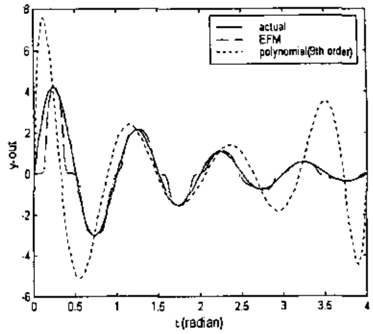


Fig. 9 Multimodal response(10 training data)

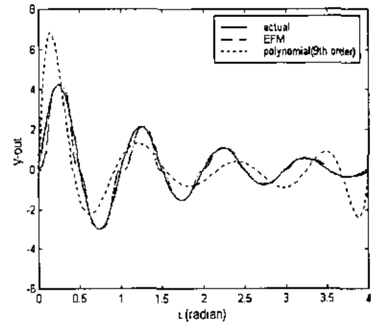


Fig. 10 Multimodal response (20 training data)

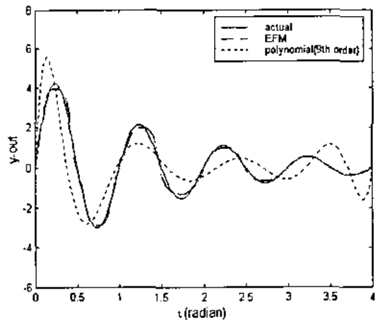


Fig. 11 Multimodal response (40 training data)

Consider a different type of the function with multimodality as shown in Equation (7); this function requires 10 fuzzy rules to express the membership functions, resulting in at least 10

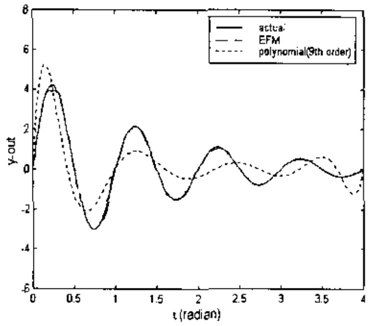


Fig. 12 Mutimodal response (80 training data)

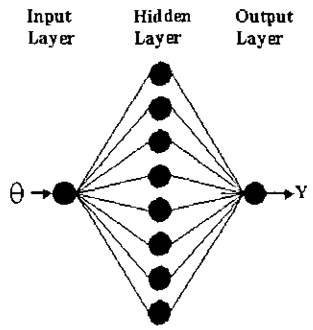
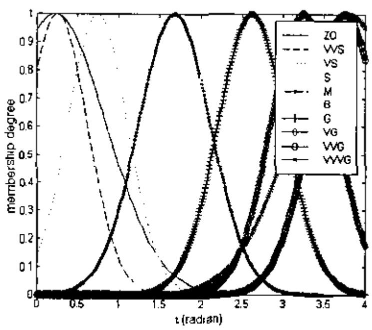
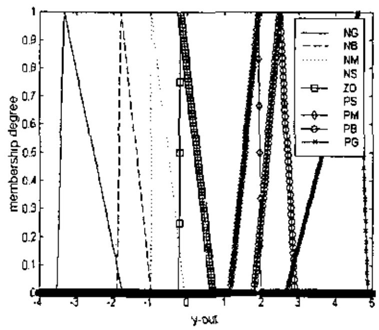


Fig. 14 Backpropagation neural network



(a) Input membership function



(b) Output membership function

Fig. 13 Optimized membership functions for multimodal response (80 training data)

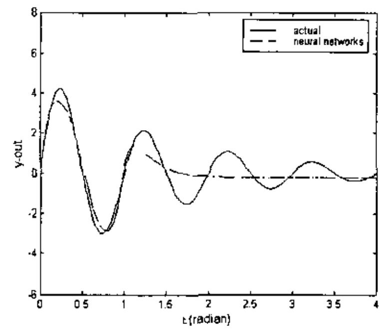


Fig. 15 Use of neural networks (300 training data)

training patterns to find optimal membership parameters. Based on the fact that the increase in the number of training data will contribute the increase in the accuracy of EFM based approximation, the multimodal function problem employs four different cases in term of the number of training data. Lower and upper bounds on membership parameters for multimodal response are also shown in Table 1. Approximation results using the different number of training data are depicted in Figs. 9 to 12; the 9th order of polynomial was used to compare with EFM approach. The optimized membership functions for the multimodal response are also shown in Fig. 13. The optimal values of membership parameters are placed near the crisp values of interest. These numerical experiments indicate that EFM approach is especially efficient in a case where the response to be approximated is multimodal, and the traditional response surface method (RSM) based on the polynomial function has difficulty in global approximation over the entire design space. EFM with 40 training patterns agree well with the exact function, while the 9th order of polynomial interpolation do not follow any of modality regardless of the number of training data.

The benefit of EFM approach is more evident when compared to artificial neural networks based RSM (Lee and Hajela, 1996). A number of backpropagation neural network (BPN) architectures (Hajela and Lee, 1996) were generalized to approximate the multimodal function with one

Table 3 Comparision of Approximation Performance for Multiimodal Response

Traing Data / Order of Polynomial	Testing Error(%)			
	10	20	40	80
6th	135.70	110.63	105.43	104.11
7th	95.60	92.88	99.33	91.85
8th	90.12	87.77	83.54	82.79
9th	88.06	81.83	80.11	67.88
EFM	23.19	13.51	8.620	5.741

neuron in the input and output layers, and a multiple number of neurons in one hidden layer, referred to as 1-H-1 architecture as shown in Fig. 14. Approximated result by BPN with 300 training data and 1-8-1 architecture is shown in Fig. 15, wherein the approximation performance is comparable to the exact function and EFM approach around first two modalities only. Approximation results obtained from different approaches are shown in Table 3. EFM approach with a smaller number of training data provides the better performance in global function approximation over the entire design space.

6. Closing Remarks

The proposed paper describes the application of evolutionary fuzzy modeling in global function approximations for use in design optimization. In this approach, relationship between input variables and output responses is expressed by linguistically expressed fuzzy rules. The genetic algorithm based optimization has been conducted to determine the optimal membership parameters, minimizing the variance of actual outputs and defuzzified outputs obtained from fuzzy inference system. Especially, in a case where no data or methematical formulation is available, fuzzy logic provides a convenient way to incorporate designer's knowledge into modeling the problem. Function approximations by evolutionary fuzzy modeling have shown their remarkable performance when a response function to be approximated over the entire design space is nonconvex, and multimodal. The optimization procedure in fuzzy system increase the approximation accuracy with the smaller number of training data over the interpolation based approximation tools such as polynomials and neural networks. Continuing studies are being extended into large dimensionality design problems. Research pertinent to such problems modifies the fuzzy rules by increasing the portion of condition parts and/or action parts according to the number of design variables and response functions of interest. Generalization of FEM based approach in terms of the number of fuzzy rules and membership functions are discussed in greater details in a separate publication.

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