# Experimental Investigation of Partial Rotor Rub

**Yeon-Sun Choi***
*School of Mechanical Engineering, Sungkyunkwan University*

Rubbing occurs when a rotor contacts with a stator during whirling motion of the rotor. Compared to full annular rub, partial rub against a nonrotating part is more common in practice. In this study, several partial rubbing phenomena of superharmonic and subharmonic vibrations and jump phenomenon are demonstrated experimentally for the cases of light and heavy rub for a flexible rotor. The orbit patterns of forward or backward whirling are also calculated using directional spectrum analysis. The occurrence of subharmonic vibration during heavy rub is demonstrated as one impact per two rotations both experimentally and numerically.

**Key Words** : Partial Rub, Subharmonic Vibration, Jump Phenomenon, Light and Heavy Rub, Directional Spectrum Analysis

## 1. Introduction

The performance of rotating machinery such as a motor or a gas turbine can be maximized by reducing the clearance between rotating and nonrotating elements of the machine. During the whirling motion of a rotor, the rotor can contact the stator of nonrotating elements. Contact between the rotor and stator during whirling motion is called rubbing. The rubbing generated by some perturbation of normal operating conditions of the machine can persist, become more severe, and lead to higher vibration levels. Rubbing can be one of the most damaging malfunctions in rotating machinery.

Once rubbing occurs during the operation of rotating machinery, runouts of the rotor and stator become large and the operation can potentally become destructive. Because of this disastrous phenomenon, many researchers have been interested in the safe and reliable design of rotating machinery. Black (1968) can be mentioned as a notable contributor to this problem. He tried to explain the physics of rubbing using the polar receptance of the rotor and stator on a radial symmetric rotor. Ehrich (1966) made a simple model and calculated the responses of the rotor and stator numerically. Choi and Noah (1987) treated this problem as a piecewise linear vibration problem, and proposed an algorithm to calculate the steady-state solution using FFT techniques. The results showed that superharmonic and subharmonic responses could be found due to rubbing. Crandall and Lingener (1990) showed a very specific pattern of whirling frequency responses during rubbing by an experiment, and developed a theory to explain backward rolling and backward slipping. Choi (1994) demonstrated rubbing phenomenon which has different ways of increasing and decreasing the operating speed, and explained the onset of backward whirling and the disappearance of backward slipping using the receptance of the rotor and stator.

Studies of full annular rub have been done more than studies on partial rub since the circumferential homogeneity of the full annular rub makes the problem simpler. However, partial rub is more common in practice. Intermittent impacts and friction during partial rub make the phenomenon complex. In this respect, Beatty (1985)

* **E-mail** : yschoi@yurim.skku.ac.kr
**TEL** : +82-31-290-7440 ; **FAX** : +82-31-290-5849
School of Mechanical Engineering, Sungkyunkwan University, 300 Chunchun-dong, Jangan-gu, Suwon-city, Kyunggi-do 440-746, Korea. (Manuscript **Received** June 5, 2000; **Revised** July 31, 2000)

provided a reliable rubbing detection methodology by relative harmonic frequency strength for the case of light rub. Muszynska (1984) demonstrated complex orbit patterns experimentally for a heavy rub and analyzed them theoretically. However, the difference between light and heavy rub is still not clear, especially from the viewpoint of experimental demonstrations.

In this study, several rubbing phenomena of super- and subharmonic vibrations and jump phenomenon are demonstrated for partial rub using an experimental apparatus. The experimental results are discussed by distinguishing between light and heavy rub. The orbit patterns are calculated using the method of directional spectrum analysis to demonstrate the forward and backward components of the whirling motions. Also, the phenomenon of one impact per two rotations during heavy rub, which demonstrates the occurrence of subharmonic vibration, is discussed experimentally and numerically.

## 2. Rubbing Phenomenon and Its Analytical Model

Destructive instability of rotors in high-speed and high efficiency rotating machinery with decreased clearance increases the possibility of rubbing due to thermal mismatch, rotor unbalance, aerodynamic and seal force, and misalignment at the minimum rotor/stator clearance locations, i. e., blade tips and seals. The occurrence of rubbing results in 1X, 1/2X, 1/3X or 2X, 3X components of vibration, high power consumption, high noise levels, and ultimately catastrophic failure of the machine itself.

Rubbing occurs in various ways depending on the design parameters and operation conditions of rotating machinery. Generally, rubbing has two types, full annular rub and partial rub. Partial rub involves intermittent contacts, whereas a full annular rub is characterized by continuous contact during the entire whirling motion. The whirling direction may be the same as the rotor spin direction, i.e., forward whirling, or opposite, i.e., backward whirling. In the case of forward whirling, the whirling frequency is usually the same as the rotor speed, called forward synchronous whirling. Forward whirl always accompanies slipping. On the other hand, backward whirl occurs with slipping or no slipping depending on the rotor speed. If there is no slipping at the contact surface, the rotor just rolls on the inner surface of the stator, called backward rolling. If slipping exists, it is called backward slipping (Crandall and Lingener, 1990, Choi, 1994).

The super- and subharmonic vibrations due to rubbing have been reported by numerical analysis (Choi, 1987) and experimental demonstrations (Beatty, 1985, Muszynska, 1984). The demonstrations were seen to be distinct in orbit pattern and Fourier spectrum. Subharmonic resonance is believed to be especially dangerous. The occurrence of superharmonic vibrations having higher harmonics can be deduced by the distortion of whirling orbits during rubbing. However, no other clear explanation on the occurrence of subharmonic response have been offered.

Forward and backward whirl, and super- and subharmonic vibrations have been reported to occur in both full annular and partial rub. For full annular rub only the friction force affects the whirling motion. However, for partial rub intermittent impact is added, which makes the analysis more complicated.

For the analysis of partial rub, Fig. 1 of an experimental apparatus was setup in order to investigate the effect of partial rub. The contact was intended to occur at the middle of the shaft. The whirling motion was measured at both ends of the shaft using gap sensors in the x and y directions. When runout of the rotor due to imbalance in or bending of the shaft reaches to the limit of the given clearance, the shaft comes into contact with a protrusion on the stator. The response of the rotor due to contact at a discrete

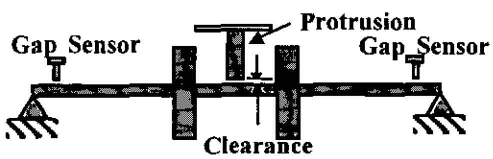



**Fig. 1** Schematic diagram of experimental apparatus for partial rubbing

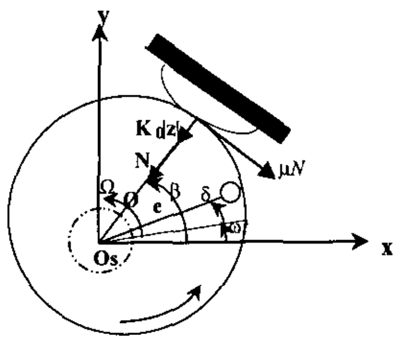



**Fig. 2** Analytical model of partial rub

protrusion would not be expected to be the same as contact at one point on a continuous circular stator. The contact between the rotor and stator causes the variation of stiffness of the rotor and the friction at the contact surface makes for complex whirling orbits. From the consideration of the effects of partial rub, an analytical model of Fig. 2 is assumed. From the analytical model, the following equation of motion is derived.

$$M\ddot{z}+C\dot{z}+Kz+F(\omega t)[K_c|z|+N(1+j\mu)]e^{j\beta} \\ =Me\omega^2e^{j(\omega t+\delta)} \\ \text{where } z=x+jy \qquad (1)$$

Where $M$, $C$, and $K$ are the rotor mass, damping and stiffness coefficients, and $F(\omega t)$ is the function which indicates the contact, i.e., if the rotor contacts, $F(\omega t)$ is unity, otherwise zero. $K_c$, $N$, $\mu$, and $\beta$ are the contact stiffness, normal force, frcition coefficient, and the angle of contact position, respectively. $e$ and $\omega$ are the eccentricity and exciting frequency, i.e., the rotor speed. Equation (1) can not be solved simply since the occurrence of contact cannot be predicted, and the contact force and friction force are impossible to predict or calculate without further definition of the contact conditions. In this respect, experimental investigations on rubbing become meaningful and important.

## 3. Experiment

In this research, the RK-4 rotor kit, manufactured by the Bently Nevada Co., was used to demonstrate various partial rubbing phenomena as shown in Fig. 3. The contact between the stator

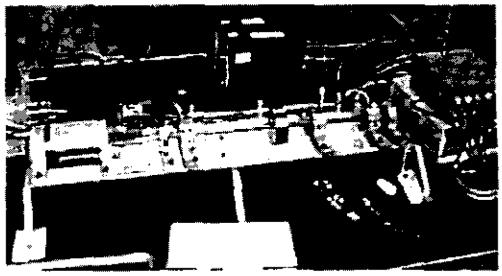


**Fig. 3** Experimental apparatus, RK-4 rotor kit

and rotor was accomplished by making a protrusion of a brass screw bolt at the top of the stator. If runout of the rotor exceeds the given clearance between the protrusion and the shaft during whirling motion of the rotor, the shaft comes into contact with the protrusion. The whirling orbit was measured using four gap sensors near the position of both bearing supports in the x and y directions, respectively, as shown in Fig. 1. The running speed of the rotor was measured by a photo sensor with a spur gear having 60 teeth. A key phasor was set up to indicate the initial point of the whirling orbit. The signals measured by several sensors were stored in a computer through analog-to-digital conversion, and analyzed using MATLAB (1996). The position of partial rub was set to occur at the middle of the shaft. However, the measuring points for the whirling motion were near the bearing support. The difference can be corrected by considering the mode shape of the rotor. Fortunately the range of operation was much lower than the second natural frequency. Thus, only the first mode was considered.

The natural frequency and damping coefficient of the rotor system were measured from an impact test and a free run test. An impact was exerted on the center of the shaft, where the contact happens, and the response was measured near the bearing position using gap sensors. The free run test implies a running test for the rotor system by increasing and decreasing the speed of the rotor. Since the tests were done after carefal adjustment of the rotor system, which makes the orbit circular, the vertical and horizontal results show the same value. The stiffness of the rotor was found by the static deflection test which measures the

**Table 1** System parameters of the rotor

| Mass, M | Viscous damping coefficient, C | Stiffness, K | Eccentricity, e |
|---|---|---|---|
| 0.50 (kg) | 2.60 (Ns/m) | 14,100 (N/m) | 0.125 mm |

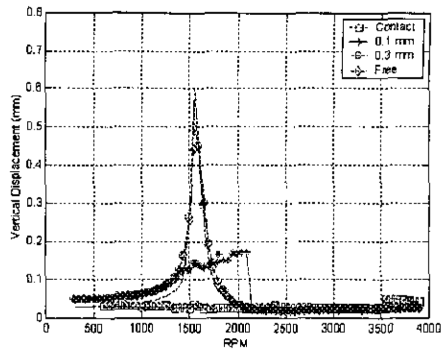
static deflection of the shaft when a force is exerted on the center of the shaft. The mass of the rotor was calculated from the natural frequency and stiffness of the shaft. The system parameters of the rotor kit are shown in Table 1.

## 4. Frequency Response

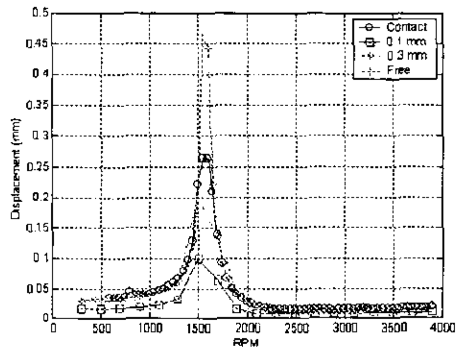
The runouts of the rotor were measured with varying clearances and rotor speeds. Fig. 4 shows the vertical and horizontal displacements of the rotor. "Free" in the Fig. 4 denotes free response without any contact with the stator, and "0.1 mm" and "0.3 mm" denotes the clearance size between the rotor and stator at the middle of the shaft, where the protrusion exists. "Contact" denotes zero clearance size, i. e., the shaft and protrusion are in contact with each other at the beginning of the rotor run.

The response of the free run shows a typical response of a single degree of freedom rotor system. The response for the 0.3 mm gap shows a similar shape as the free run case. However, the response for the 0.1 mm gap shows a very distorted response compared to the free run case. In particuler, the vertical response for the 0.1 mm gap shows a jump phenomenon. The responses to increasing speeds are different from those for decreasing speeds near the range of 2000 rpm. This suggests that the contact between the rotor and stator introduces a nonlinearity due to a hardening of the stiffness of the rotor system.

The slight contact of the 0.3 mm gap seems to be a light rub, and the deep contact of the 0.1 mm gap seems to represent a heavy rub. As the speed increases, the vertical and horizontal responses with variations in the gap size are almost similar. The horizontal response has little effect due to the protrusion for the case of the 0.3 and 0.1 mm gap size. However, the horizontal responses for zero clearance are greater than for the 0.3 mm clearance. This is because the orbit becomes flat as the




(a) Vertical



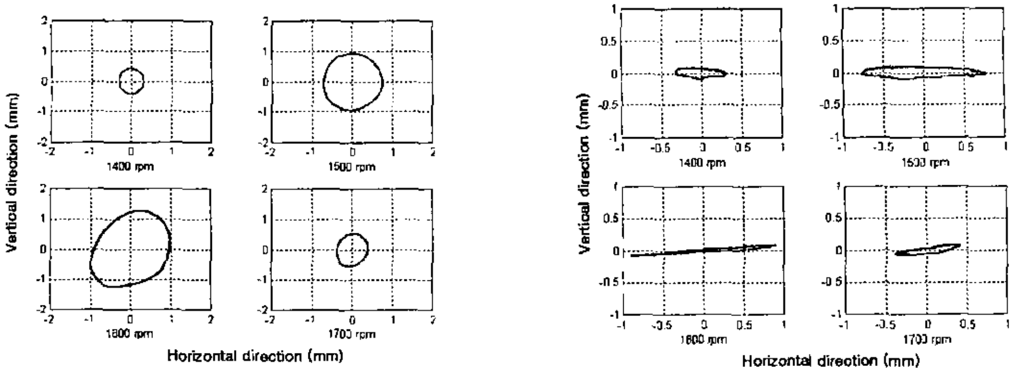

(b) Horizontal

**Fig. 4** Vibration levels for different clearances

duration of contact is longer in the case of zero gap. In the range greater than 3000 rpm, the vertical responses of the zero gap become larger as the irregularity of the orbit becomes larger.

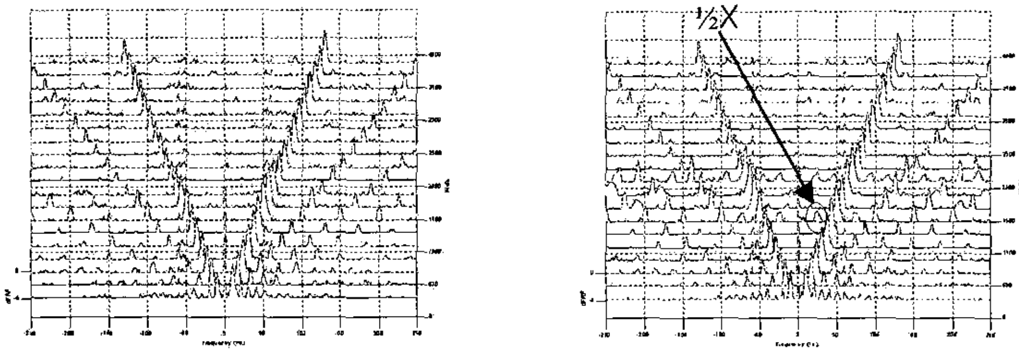
## 5. Whirling Orbits and Directional Spectrum

Figure 5 shows the whirling orbits during the partial rub for the 0.1 mm and 0.3 mm gaps near the range of the first critical speed. The orbit of the 0.3 mm gap is almost a circle, like the response of the free case; however, the orbit of the 0.1 mm gap is completely different from that of the free case. Therefore, it can be said that the response of the 0.3 mm gap represents a light rub and that of the 0.1 mm gap represents a heavy rub. From these experimental results, a partial rub due to a protrusion attempts to maintain its orbit during the whirling motion of the rotor for small interference; however, if the interference is not so mall, the orbit becomes a completely different




(a) For light rubbing (0.3 mm) (b) For heavy rubbing (0.1 mm)

**Fig. 5** Orbits during rubbing




(a) For light rubbing (0.3 mm) (b) For heavy rubbing (0.1 mm)

***Fig. 6*** *Directional waterfall diagram*

shape. The distortion of the orbit has the shape of a horizontally long irregular ellipse. The orbit feature of a figure-eight or star shape, which was demonstrated by Muszynska (1984) was not found in this experiment. It is believed that the shape and material of the protrusion, the system parameters, and the operation speed have effects on the orbit shape.

The distorted orbits are generally the combination of a forward and backward whirling motion with several higher modes. In this respect, the classification of a forward and backward whirling motion is done using the directional spectrum analysis, which was developed by Lee (1994).

$$w(t)=x(t)+jy(t) \tag{2}$$

$$w(t)=r_f e^{jwt}+r_b e^{-jwt} \tag{3}$$

Where $r_f$ and $r_b$ are the complex amplitude of the forward and backward whirling amplitudes, respectively.

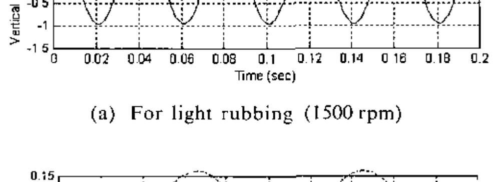
The directional spectrum regards the x and y directional responses as a complex response as in Eq. (3). The complex response is analyzed using FFT algorithm in the directional spectrum analysis. The results are shown in Fig. 6 as a waterfall diagram for the case of the 0.3 mm and 0.1 mm gaps, respectively.

As shown in Fig. 6 for the light rubbing of the 0.3 mm gap, the backward components are dominant after the contact, and the second order superharmonic vibrations 2X are shown in the backward components. For the heavy rubbing of the 0.1 mm gap, the backward components are more dominant. Also, several higher harmonics are gathered near the first critical speed. It is believed that the strong nonlinear effects of impact and friction due to the contact between the rotor and stator create the complex orbits. Consequently the directional spectrum analysis on the response of partial rubbing shows the existence of backward whirling components and higher harmonic terms. However, the existence of the subharmonic term, which is believed a typical phenomenon of rotor rubbing, was not so clear in this directional spectrum analysis, even though the 1/2X term for 1500 rpm was indicated at Fig. 6(b).

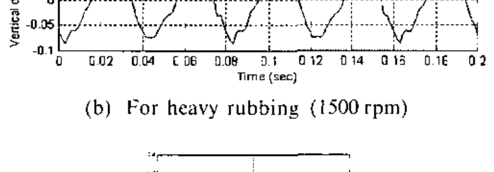
## 6. Subharmonics

Figure 7 shows the time response of vertical displacements of the whirling rotor during partial rubbing. As shown in Fig. 7(a) for the case of light rub, the time response is almost a sinusoidal function without any large distortion due to the contact. However, for the case of heavy rub in Fig. 7(b), the time response shows a distorted shape from a sinusoidal function, which results from the inclusion of higher harmonics as shown in the directional spectral analysis of Fig. 6. Also, the highest peak appears once per two revolutions of the whirling motion. This is shown more clearly in Fig. 7(c) of the whirling orbit, where the orbit is drawn for more than 10 revolutions. Nevertheless, the orbit is not compeletely overlayed. This is the experimental second order subharmonic response, which is indicated in Fig. 6 (b).

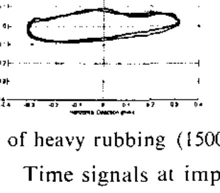
In order to verify the second order subharmonic vibration, i.e., one contact per two revolutions, an iterative numerical analysis for the equation of motion (1) was done using the system parameters of Table 1. Since the equation of motion (1) is a nonlinear equation due to the contact, the steady-state response can be different depending on initial conditions. In this respect, to find all the steady-state responses is not easy. The normal force and friction coefficient in the Eq. (1) can not be predicted. However, it is known that the displacement at the contact is the gap size, and only the velocities before and after contact are unknown. Here, the concept of the coefficient of restitution can be utilized. The initial displacement at the contact and an assumed initial velocity after the contact are input data for the free run condition. The displacement after one or two revolutions is then checked for agreement with the original estimate. If not in agreement, the initially assumed velocity needs to be modified. Although the calculation can be done analytically, a fourth-order Runge-Kutta method and shooting method for modifying the initial velocity were used. The numerical results for light rubbing show only the 1X component; however, for the case of heavy rubbing as shown in Fig. 8(a), there is one contact for only every two revolu-




(a) For light rubbing (1500 rpm)

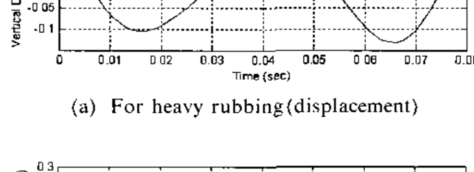



(b) For heavy rubbing (1500 rpm)

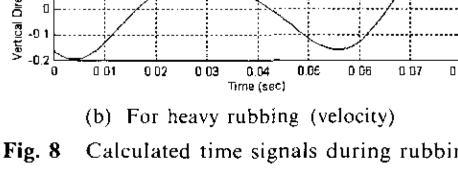


(c) Orbit of heavy rubbing (1500 rpm)

**Fig. 7** Time signals at impact




(a) For heavy rubbing (displacement)




(b) For heavy rubbing (velocity)

**Fig. 8** Calculated time signals during rubbing tions. Figure 8(b) shows the velocities for the steady-state response. In this case, the coefficient of restitution was 0.76, which is the measured value from the experiment by checking the velocities before and after the contact. Therefore, the analysis using this coefficient of restitution can be said to be valid. Consequently, the 1/2X component of the subharmonic vibration during partial rubbing was found experimentally and was verified by numerical analysis.

## 7. Conclusion

In this paper, partial rub of a rotor due to contact with a nonrotating protrusion is investigated by an experiment for the clarification of nonlinear characteristics, such as jump phenomena, forward and backward whirling motion, and super- and subharmonic responses. The rubbing phenomena are demonstrated on the RK-4 rotor kit of the Bently Nevada Co. The experiment shows the following results:

(1) Partial rub is characterized by nonlinear phenomenon in the form of superharmonics, subharmoincs, and jump phenomenon, which were demonstrated experimentally.

(2) As long as the interference of the whirling shaft with a nonrotating part is not so large, the rotor shows a light rub with a similar response to the free run case. However, for the case in which the interference is large, the rotor demonstrates a heavy rub, giving a flat ellipse shaped orbit with distortions.

(3) During the rub, impacts and friction generate a backward whirling motion, which was demonstrated by directional spectrum analysis.

(4) The second order subharmonic response near the critical speed is due to the contact occurring once every two revolutions.

(5) The coefficient of restitution can be utilized successfully to analyze the impact phenomenon of partial rub.

## Acknowledgement

This work was supported by a grant from the Critical Technology 21 Project of the Ministry of Science and Technology, Korea.

## References

Beatty, R. F., 1985, "Differentiating Rotor Response Due to Radial Rubbing," *Journal of Vibration, Acoustics, Stress, and Reliability in Design*, Vol. 107, pp. 151～160.

Black, H. F., 1968, "Interaction of a Whirling Rotor with a Vibrating Stator across a Clearance Annulus," *J. of Mechanical Engineering Science*, 10, pp. 1～12.

Choi, Y. -S. and Noah, S. T., 1987, "Nonlinear Steady-State Response of a Rotor-Support System," *Journal of Vibration, Acoustics, Stress, and Reliability in Design*, Vol. 109, No. 4, pp. 225～261.

Choi, Y. -S., 1994, "Dynamics of Rotor Rub in Annular Clearance with Experimental Evaluation," *KSME Journal*, Vol. 8, No. 4, pp. 404～413.

Crandall, S. H. and Lingener, A., 1990, "Experimental Investigation of Reverse Whirl of a Flexible Rotor," *ITFoMM Third International Conference on Rotordynamics*, Lyon, France, pp. 13-18.

Erich, F. F. and O'Connor, J. J., 1966, "Stator Whirl with Rotor in Bearing Clearance," ASME Paper 66, WA/MD-8.

Lee, C. W., 1994, "Development of the Use of Directional Frequency Response Functions for the Diagnosis of Anisotropy and Asymmetry in Rotating Machinery: Theory," *Mechanical Systems and Signal Processing*, Vol. 8, No. 6, pp. 665～678.

The MathWorks, 1996, "MATLAB User's Guide".

Muszynska, A., 1984, "Partial Lateral Rotor to Stator Rubs," *ImechE* C281/84, pp. 327～335.