▩ 연구논문

제품생산을 위한 공구이송장치의 이용횟수 최소화

Minimizing the number of tool transporter's movements for processing parts

송 창 용
Song, Chang Yong[1]

요 지

본 연구에서는 공구이송장치(Tool Transporter)의 이용횟수를 최소화하는 공구교환문제(Tool Switching Problem)를 다룬다. 한 기계에서 생산해야 하는 제품들이 요구하는 공구의 총수가 기계에 장착할 수 있는 공구의 수보다 많은 경우 각 제품을 생산할 때마다 공구이송장치를 이용한 공구교환이 필요하다. 공구저장소(Tool Crib)와 기계간에 한 번에 이송할 수 있는 공구의 수가 제한되어 있기 때문에 공구이송장치의 이용률에 따라서 기계의 생산성이 크게 달라진다. 그러나 기존의 연구에서는 공구이송장치의 능력을 배제하고 공구교환횟수만을 고려하고 있다. 따라서 본 연구에서는 공구이송장치의 이용횟수의 최소화를 목적함수로 하는 공구교환문제를 수리적으로 모형화하고 이 문제에 대해 기존의 최적 공구교환방법을 적용하여 얻어지는 해는 최적해를 보장할 수 없다는 점과 공구가 필요한 시점보다 먼저 장착하는 경우(Early Insertion)를 고려할 때 기존 방법에 의한 해보다 좋은 해를 얻을 수 있다는 특성을 간단한 예를 통하여 보여준다. 또한, 이러한 특성을 고려한 새로운 공구교환방법을 제시하고 다양한 유형의 예제를 통하여 기존의 방법과 비교한다.

1. Introduction

Tool management affects the productivity of many flexible manufacturing systems (FMSs). Tool switching is a key problem of tool management at the single machine level[4, 10]. It is common in current flexible manufacturing systems that some operations cannot be performed even on a very highly versatile machine because the required tools are not available on the tool magazine. As the total number of tools required to process a set of parts is generally larger than available tool magazine capacity, it is sometimes necessary to switch tools on the tool magazine between two successive parts in a production sequence. Tools that required but not loaded on the tool magazine are delivered from a tool crib by a tool transporter. The tool switching is a time consuming operation and delays production.

For a given set of parts and the set of tools required to process them on a single flexible machine,

1) Dept. of Business Administration, Tamna University

the tool switching problem is defined as follows: (1) how the parts should be sequenced (part sequencing) and (2) which tools to switch on the machine prior to processing each part (tooling). This paper focuses mainly on the latter problem.

The tool switching problem has received considerable attention from many researchers[1, 2, 4-9]. Most of the studies carried out until now considered the minimization of the number of tool switches as the objective of the problem. However, some FMSs are provided with a tool transporter that can carry several tools together between the tool magazine and the tool crib[3, 10]. Since the transportation time are not negligible, the movements of the tool transporter for tool switching take the idle time of the machine and cause the processing of parts to be delayed. And when several machines use the tool transporter, there is a distinct possibility that this transporter becomes overloaded[2]. Then, minimizing the number of tool transporter's movements can reduce the amount of the time when the machine is not producing and improve the productivity of FMSs. Therefore, this paper considers the minimization of the number of tool transporter's movements as the objective of the tool switching problem.

2. Problem formulation

Consider a single flexible machine that can perform any processing operation on any part if appropriate tools are available in a tool magazine. N parts to be processed are given at the beginning of a production period. Each part requires a subset of M tools, which has to be placed in the tool magazine before the part can be processed. The tool magazine can accommodate at most C tools, so it is necessary to change tools between two parts in a sequence. If the tools required by each part are not available in the magazine, they must be brought from a tool crib by a single tool transporter and switched with other tools on the magazine. We assume that the tool transporter can carry D tools simultaneously.

The further assumptions are made as follows:

(1) Each tool fits in one slot of the magazine.

(2) No part requires more than C tools.

(3) The tool switching does not occur during the processing of the parts.

The following notations for parameters and decision variables are introduced for the mathematical formulation.

< Notations >

N number of parts to be processed

M number of tools needed to produce all parts

C capacity of the tool magazine

D capacity of the tool transporter

n the instant just after processing the nth part, but before any tools are switched

a_{it} = 1 if part i requires tool t

= 0 otherwise

x_{in} = 1 if part i is scheduled at the nth position

= 0 otherwise

y_{tn} = 1 if tool t is on the magazine at instant n
= 0 otherwise

p_{tn} = 1 if tool t is inserted at instant n
= 0 otherwise

With the above notations, the problem can be formulated as follows:

$$\text{Min} \quad \sum_{n=0}^{N-1} \left\lceil \frac{\sum_{t=1}^{M} p_{tn}}{D} \right\rceil \tag{1}$$

$$\text{s.t.} \quad \sum_{t=1}^{M} y_{tn} \le C \qquad n=1, \ldots, N \tag{2}$$

$$a_{it} \cdot x_{in} \le y_{tn} \qquad i,\ n=1, \ldots, N \quad t=1, \ldots, M \tag{3}$$

$$\sum_{i=1}^{N} x_{in} = 1 \qquad i=1, \ldots, N \tag{4}$$

$$\sum_{n=1}^{N} x_{in} = 1 \qquad n=1, \ldots, N \tag{5}$$

$$y_{t,n+1} - y_{tn} \le p_{tn} \qquad t=1, \ldots, M \quad n=1, \ldots, N-1 \tag{6}$$

$$y_{t1} \le p_{t0} \qquad t=1, \ldots, M \tag{7}$$

$$x_{in},\ y_{tn} = 0 \text{ or } 1 \qquad i,\ n=1, \ldots, N \quad t=1, \ldots, M \tag{8}$$

$$p_{tn} = 0 \text{ or } 1 \qquad n=0, \ldots, N \quad t=1, \ldots, M \tag{9}$$

The objective function is to minimize the number of tool transporter's movements among the machine and the tool crib. Since the number of tool switches is given by $\sum_{t=1}^{M} p_{tn}$ at each instant and the tool transporter can carry D number of tools, the number of movements of the transporter can be expressed as equation (1). (where, $\lceil A \rceil$ means the minimum integer no less than A.)

Constraint (2) ensures that no more than C tools can be placed on the machine at any instant. Constraint (3) indicates that if part i is the nth part to be processed, then all the tools required by part i must be on the machine at instant n. Constraints (4) and (5) assign exactly one part to exactly one instant. Constraints (6) and (7) are to count the tool switches occur at instant n. Finally, constraints (8) and (9) denote binary integer variables for each x_{in}, y_{tn}, and p_{tn}.

Unfortunately, this model has tremendous computational complexities. In real manufacturing environments, it is likely that an optimal solution can be obtained in a reasonable amount of time even for a small-sized problem. For this reason, we would like to discuss the sub-problem of tooling for a given sequence.

3. The tooling problem

The tooling problem is the special case of the tool switching problem in which the part sequence is given, and the tooling decisions (y_{tn}) are the only decisions to be considered. The objective of this problem is to determine the set of tools to be placed on the machine at each instant so that the total number of tool transporter's movements is minimized.

In Bard[1] and Tang and Denardo[9], the KTNS (Keep Tool Needed Soonest) policy was proven to yield an optimal tooling policy for a given part sequence. However, this is not true under the objective function of this problem, as will be seen.

Observation 1. When the objective of the tool switching problem is to minimize the number of tool transporter's movements, the KTNS policy applied to complete parts does not guarantee optimality.

Proof. To prove the observation, it is sufficient to show that there is a solution to the problem that is better than that obtained by the KTNS policy applied to complete parts. The KTNS policy, in the proof that it is an optimal policy, is shown to have no early insertions of tools. However, because an early tool change has the potential for minimizing the number of the tool transporter's movements in this new problem, the KTNS policy is no longer an optimal policy.

This idea is now shown in the following example. A set of twelve tools (M = 12) is required to process a sequence of eleven parts (N = 11) on a flexible machine with a tool magazine capacity of six tools (C = 6). The tool transporter can carry three tools simultaneously (D = 3) between the machine and the tool crib. The tool requirements for each part in a given part sequence are given in Table 1.

Table 1. Tool requirements and a part sequence

sequence(n) part / tool(t)	1 (A)	2 (B)	3 (C)	4 (D)	5 (E)	6 (F)	7 (G)	8 (H)	9 (I)	10 (J)	11 (K)
1	1	1	1			1	1	1		1	
2			1		1		1				1
3	1			1		1			1	1	
4		1	1	1							1
5	1					1	1		1	1	
6		1		1	1			1			1
7	1						1	1			1
8				1	1	1					
9						1				1	
10	1		1	1			1		1		1
11	1						1		1		
12				1	1						

Applying the KTNS policy to the given sequence of twelve parts yields the solution given in Table 2. This solution yields a total of 24 switches and 12 movements of the tool

Table 2. Solution by the KTNS policy

	y_{tn}										
t \ n	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1			1	1	1	1	1	1
2			1		1		1				1
3	1	1	1	1	1	1			1	1	
4		1	1	1							1
5	1	1				1	1	1	1	1	
6		1	1	1	1			1			1
7	1						1	1	1	1	1
8				1	1	1					
9						1				1	
10	1	1	1	1	1	1	1	1	1	1	1
11	1						1	1	1		
12				1	1						
tool switches	6	2	1	2	1	3	3	1	1	1	3
tool transporter's movements	2	1	1	1	1	1	1	1	1	1	1

transporter. If early insertions of tools at some instants are considered, the solution in Table 2

Table 3. Solution with early insertions of tools

	y_{tn}										
t \ n	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1		**1***	1	1	1	1	1	1
2		**1***	1		1	1	1				1
3	1	1	1	1	1	1			1	1	
4		1	1	1							1
5	1					1	1	1	1	1	
6		1	1	1	1			1			1
7	1						1	1	1	1	1
8				1	1	1					
9						1				1	
10	1	1	1	1			1	1	1	1	1
11	1						1	1	1		
12				1	1						
tool switches	6	3	0	2	2	2	3	1	1	1	3
tool transporter's movements	2	1	0	1	1	1	1	1	1	1	1

*early inserted tool

can be improved as shown in Table 3. This solution yields only 11 movements, a clear improvement over 12. Therefore, an early insertion can reduce the number of movements of the tool transporter if the early insertion is made concurrently with another insertion.

Observation 2. There may be early insertions at the instant n satisfying that $\min\left(C - |\{t \mid L(t, n) = n,\ t = 1, \ldots, M\}|,\ D\lceil \frac{|N(n)|}{D} \rceil - |N(n)|\right) > 0$, where $L(t, n)$ = the first instant at or after instant n at which tool t is needed, $Y(n)$ = the set of tools on the machine at instant n, and $N(n) = \{t \mid L(t, n) = n,\ t = 1, \ldots, M\} \setminus Y(n-1)$

proof. For n = 2 in Table 1 and Table 3, $\{t \mid L(t, 2) = 2,\ t = 1, \ldots, 12\} = \{1, 4, \text{and } 6\}$, $N(2) = \{t \mid L(t, 2) = 2,\ t = 1, \ldots, 12\} \setminus Y(1) = \{4, 6\}$. Hence,

$C - |\{t \mid L(t, 2) = 2,\ t = 1, \ldots, M\}| = 6 - |\{1, 4, 6\}| = 3$, $D\lceil \frac{|N(2)|}{D} \rceil - |N(2)| = 3\lceil \frac{2}{3} \rceil - 2 = 1$, and

$$\min\left(C - |\{t \mid L(t, n) = n,\ t = 1, \ldots, M\}|,\ D\lceil \frac{|N(n)|}{D} \rceil - |N(n)|\right) = 1 > 0$$

Therefore, there is early inserted tool at instant 2.

Observation 2 indicates that an optimal solution contains an early insertion if and only if there is at least one tool not on the magazine that is needed before at least one tool on the magazine, and there is time for a beneficial early insertion.

Based on two observations, we propose a new tooling policy for the tool switching problem with the modified objective as follows.

Procedure

Step 0. Set $n = 1$, $r = 0$ and $c = 0$

Step 1. For all t, if $L(t, n) = n$, set $J_t = 1$, $r = r + 1$, and $c = c + 1$. Otherwise, set $J_t = 0$.

Step 2. If each t having $L(t, n) = n$ also has $J_t = 1$, go to Step 5.

Step 3. Pick t having $L(t, n) = n$ and $J_t = 0$. Set $J_t = 1$, $r = r + 1$ and $c = c + 1$

Step 4. If $c > C$, set $J_k = 0$ for k having a maximum value of $L(p, n)$ over $\{p \mid J_p = 1,\ p = 1, \ldots, M\}$

and set $c = c - 1$

Go to Step 2.

Step 5. If $r = D\lceil \frac{r}{D} \rceil$, go to Step 9.

Step 6. Pick q having a minimum values of $L(p, n)$ over $\{p \mid J_p = 0,\ p = 1, \ldots, M\}$

Set $m = L(q, n)$

Step 7. If $|\{t \mid L(t, n) \le m \text{ and } J_t = 1,\ t = 1, \ldots, M\}| = m$, go to Step 9.

Step 8. If $c > C$, set $J_k = 0$ for k having a maximum value of $L(p, n)$ over $\{p \mid J_p = 1,\ p = 1, \ldots, M\}$

and set $c = c - 1$

Go to Step 5.

Step 9. Set $p_{tn} = J_t$ for all t, $r = 0$ and $n = n+1$. If $n = N+1$, stop. Otherwise, go to Step 2.

4. Computational results

The objective of the numerical experiments is to compare the performances of two policy for the tooling problem. The solutions to the tooling problem are only useful in conjunction with a part sequence. For the part sequencing, a 'multiple-start greedy' heuristic presented in Crama *et al.*[2] was used in this tests.

For the tests, we generated 32 types of problem instances under each combination of parameters, characterized by (N, M, R, C, D), where

N = number of parts,
M = number of tools,
R = number of tools required by each part,
C = tool magazine capacity, and
D = tool transporter's capacity.

The various instance types are displayed in Table 4. For each type, 20 instances were randomly generated, resulting in a total 640 instances. Two levels for N (10 and 30)and M (20 and 40) were included. Two levels for R were used: sparse case ([0.1M, 0.3M]) and dense case ([0.4M, 0.6M]) depending on the value of M. For example, in case of M = 20, the value R was drawn from the discrete uniform distribution over [2, 6] and [8, 12]. Two levels for tightness of tool magazine capacity were used depending on the maximum value of R: tight (T) and not tight (NT). Also, two levels for D (2 and 4) were used.

Table 4. Instance types

N	M	R	C	D
10	20	[0.1M, 0.3M]	T(Tight)	2
30	40	[0.4M, 0.6M]	NT(Not Tight)	4

The computational results are summarized in Table 5. For each type, we report averages over 20 instances in terms of the number of tool switches and tool transporter's movements. As expected, the new policy had fewer number of tool transporter's movements, but had more tool switches. In particular, our policy yields better than the existing method the case of sparse instances for tools and large capacity of tool transporter.

5. Conclusions

Table 5. Comparison between NEW and KTNS policy

N	*M*	*R*	*C*	*D*	Average number of tool transporter's movements		Average number of tool switches	
					NEW	KTNS	NEW	KTNS
10	20	[2, 6]	T(6)*	2	11.60	12.45	21.40	20.95
				4	8.00	9.90	22.50	20.95
			NT(9)	2	9.55	11.65	18.50	17.95
				4	5.75	9.30	20.00	18.25
		[8, 12]	T(12)	2	17.70	18.15	32.70	32.00
				4	11.15	11.70	33.40	32.10
			NT(16)	2	12.15	13.25	23.15	22.15
				4	7.30	9.20	23.65	21.75
	40	[4, 12]	T(12)	2	23.30	23.60	43.40	42.50
				4	12.45	13.60	45.15	42.90
			NT(18)	2	18.50	19.70	36.35	35.75
				4	10.00	12.15	37.90	35.85
		[16, 24]	T(24)	2	35.25	35.65	67.45	66.75
				4	19.65	20.30	69.70	67.90
			NT(30)	2	25.10	26.10	49.35	48.00
				4	14.05	16.00	53.30	49.20
30	20	[2, 6]	T(6)	2	27.25	31.75	48.70	46.15
				4	19.60	27.45	50.35	44.75
			NT(9)	2	16.70	23.60	32.55	29.55
				4	11.65	21.55	36.60	29.95
		[8, 12]	T(12)	2	38.70	40.35	68.75	66.45
				4	28.35	31.65	71.50	67.25
			NT(16)	2	18.25	20.90	33.30	29.65
				4	13.60	17.35	34.30	29.85
	40	[4, 12]	T(12)	2	56.75	59.80	109.15	106.25
				4	32.30	37.15	113.40	104.60
			NT(18)	2	35.90	40.55	71.30	67.85
				4	20.15	31.45	77.60	68.00
		[16, 24]	T(24)	2	81.75	83.35	154.45	152.00
				4	45.85	46.90	155.85	150.30
			NT(30)	2	43.25	46.25	85.20	79.95
				4	25.60	34.90	92.75	78.95

* The figures in parentheses are *C* values actually used in the tests.

This paper has presented a new tooling policy for a modified tool switching problem. When the objective is to minimize the number of movements of a tool transporter, the proposed policy is shown to be superior to the KTNS policy on many sets of randomly generated instances.

But the new policy has not been proven to provide an optimal solution in this paper. Therefore, we need to study the optimal policy for a modified tool switching problem on a flexible machine.

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