◈ **Application Papers**

Optimal Strategies for Robust Design of Products or Processes

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ABSTRACT

There is more than a single quality characteristic and they are often of varying or mixed target types. The purpose of this paper is to develop general strategies for solving the multiple response robust design problem. The desirability function provides an important tool to solve problems that have different types of target since the desirability values all the range between zero and one. Several combinations of arithmetic averages, geometric averages, and standard deviations are used in the various strategies to find the best design point.

1. Introduction

This paper considers robust design strategies for off-line quality control, with the use of experimental design and response surface methodology, in situations where all products have multiple quality characteristics. Previously, researchers have assumed that there was only a single quality characteristic that the customer would consider to be important for manufactured products or that the type of target for each of the quality characteristics were all of the "nominal-is-better" variety. They used conventional and sometimes controversial methods and strategies involving ANOVA, perMIA, S/N ratios, etc. to identify important factors in the location or in the dispersion of product quality, and to make their products and processes robust under various noise environments. However, for always most manufactured products or process, there is more than a single quality characteristic and they are often of varying or mixed target types.

Although the robust design problem with a single quality characteristic has moved into the limelight since 1980, the multiple response robust design problem has not attracted the attention of statisticians and quality engineers because of its complexity. The multiple response robust design problem involves not only with the performance of the average and the variability of the product and process quality characteristics, but also with the various types of the targets.

The purpose of this paper is to develop general strategies for solving the multiple response robust design problem. The desirability function provides an important tool to solve problems that have different types of target since the desirability values all the range between zero and one.

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To develop general strategies, the methodology will be presented in chapter 3, how quality characteristics having different scales can be transformed into a common scale, how the performance averages and variabilities for several quality characteristics can be calculated and combined. In chapter 4, several combinations of arithmetic averages, geometric averages, and standard deviations are used in the various strategies to find the best design point. To give evidence to the claim that the new strategies are workable and are potentially useful tools for practitioners in the final chapter, the example from an application area will be constructed as test beds for the strategies.

2. The Method of Desirability

In the method of desirability, the desirable statements for values (to attain a goal) are formalized. This is done by constructing a scale converting the desirable statements into a [0, 1] numerical rank. It is a map from a desirability value d_k to [0,1], preserving the designer's desirable order over d_k.

Such a map can be constructed, if the designer can elicit the desirable ordering over d_k, (an assumption which must be met to be able to apply the method of desirability). A rigorous method for doing so is the lottery method (Krantz et al, 1971), by asking the designer to first identify over all d_k the minimal acceptable and most preferred points d_k^-, d_k^+.

This method provides a rigorous basis for constructing performance ranks of a design configuration based on ability and performance. For example, assume that a point d_k^- gives the minimal acceptable bound on a response d_k. If $d_k \geq d_k^-$, d_k is acceptable. Otherwise, d_k is unacceptable and desirability value d_k is 0. A point d_k^+ gives the best acceptable bound on a response d_k. If $d_k \geq d_k^+$, desirability value d_k is 1.

Table 1: Method of General Desirability Axioms

$P(0, \dots, 0)=0, P(1, \dots, 1)=1$	boundary conditions
$P(d^1, \dots, d^k, \dots, d^N) \leq P(d^1, \dots, d^{k'}, \dots, d^N)$ iff $d^k \leq d^{k'}$	monotonicity
$P(d^1, \dots, d^k, \dots, d^N)= \lim_{d^{k'} \to d^k} P((d^1, \dots, d^k, \dots, d^N)$	continuity
$P((d^1, \dots, 0, \dots, d^N)=0$	annihilation
$P(d, \dots, d)=d$	idempotentcy
$P(d^1, \dots, a,\dots,b,\dots d^N)=P(d^1, \dots, b,\dots,a \dots, d^N)$	commutativity
$P(1,\dots,1,d^k,1,\dots,1)=d^k$	identity
$P(P(d^1,\dots, d^{N-1}), d^k)=P(d^k,P(d^1,\dots, d^{N-1}))$	associativity

The desirability of each response is combined into the overall desirability, D, associated with a combination of variables. Usually, the geometric mean of individual desirabilities is used to give an overall assessment of the desirability of the combined responses. The range of D is in the interval [0,1]. Table 1 lists the restrictions the general axioms satisfied.

In robust design, there are three types of target, larger-is-better, smaller-is-better, and

nominal-is-better. For larger-is-better and smaller-is-better cases, one-sided transformation (Derringer and Suich, 1980)can be used. For any response y_i, one specific one-sided
transformation can be given by:

$$d_i = \begin{bmatrix} 0 & \text{if} & y_i \le A_i \\ [\frac{B_i - y_i}{B_i - A_i}]^r & \text{if} & A_i \le Y_i \le B_i \\ 1 & \text{if} & y_i \ge B_i \end{bmatrix} \tag{2.1}$$

where y_i is the response variable, the value A_i gives the minimal acceptable bound on the response y_i and the value B_i is the minimum most desirable value on the response y_i. The selection of a suitable value of r offers the user flexibility in the definition of desirability functions (Mayer and Montgomery, 1995).
For nominal-is-better case, two-sided desirability function can be used. A two-sided desirability function can be developed for situations where there is a specific target either a point or an interval for y_i and where there are upper and lower desirability limits on y_i, such as $A_i < T_i < B_i$, where A_i is the maximum least desired value for a minimum constraint and B_i is that for a maximum constraint.
One specific two-sided transformation can be given by:

$$d_i = \begin{bmatrix} [\frac{y_i - A_i}{T_i - A_i}]^s & \text{if} & A_i \le y_i \le T_i \\ [\frac{y_i - B_i}{T_i - B_i}]^r & \text{if} & T_i \le Y_i \le B_i \\ 0 & & \textit{otherwise} \end{bmatrix} \tag{2.2}$$

By selecting the power s and t, one can attribute various levels of desirability to Y.

3. Methodology

Suppose that y_{ijk} is the response associated with the jth noise variable for the kth quality characteristic at the ith product design, and $d_{ijk}(0 \le d_{ijk} \le 1)$ is a desirability value for y_{ijk} obtained from the kth desirability function.
the basic procedure for comparing products or processes using desirability functions is as follows: For each response, y_{ijk}, transform y_{ijk} into d_{ijk} using the kth desirability function and assess the desirability of the product or process based upon some function of the d_{ijk}.

3.1 The desirability function for location

Assume that there are k quality characteristics associated with some product. Each quality characteristic has m observations.

Let d_{ik}^* be a geometric average of d_{i1l}, ..., d_{imk}: that is,

$$d_{ik}^* = (d_{i1k} \times \cdots \times d_{imk})^{1/m} \tag{3.1}$$

where i=1,2,...,k and l=1,2,...,p

The quantity d_{ik}^* is the geometric average, over all noise variable, of the desirability values of the kth quality characteristic at the ith product design, x_i.

Let $\bar{y}_{il}$ be the average of the individual response, y_{i1l}, ..., y_{imk}, that is,

$$\bar{y}_{ik} = \frac{1}{m}(y_{i1k} + \cdots + y_{imk}) \tag{3.2}$$

where i=1,2,...,o and l=1,2,...,p. The average response value, $\bar{y}_{ik}$, is the arithmetic average of the response values of the kth quality characteristic at the ith product design over m noise variables.

Let $\bar{d}_{ik}$ be the desirability value of average response value, $\bar{y}_{il}$, for the lth quality characteristic at the ith combination of the product design variables.

3.2 The Overall Quality

Let $\bar{d}_{il}$ be the desirability value of the average response value, $\bar{y}_{ik}$, for the lth quality characteristic at the ith combination of the product design variables. Let G($G(\bar{d}_{i.})$ be the geometric average of the desirabilities, $\bar{d}_{ik}$, that is,

$$G(\bar{d}_{i.}) = \sqrt[p]{\bar{d}_{i1} \times \cdots \times \bar{d}_{ip}} \tag{3.3}$$

where i=1,2,...,k

Let d^*_{ik} be the geometric average of d_{i1l},...,d_{imk}. Let $G(d^*_{i.})$ be the geometric average of the geometric averages, d^*_{ik}, for the desirability values in the lth response at the ith product design over the noise space. The geometric average, $G(d^*_{i.})$, can be expresses as

$$G(d^*_{i}) = \sqrt[p]{d^*_{i1} \times \cdots \times d^*_{ip}} \tag{3.4}$$

where i=1,2,...,o, j=1,2,...,m, and l=1,...,p

3.3 The desirability function for dispersion

Desirability functions for will be considered here. We let σ^2_k represent variance of the original responses over noise variables for the lth quality characteristic. We let e^2_k represent the desirability values for the variance, σ^2_k, of the lth quality characteristic.

The combination of controllable variables with the maximum desirability value is considered as the combination of controllable variables that is the least sensitive to dispersion effects. The smaller the variation by noise variables, the larger the desirability value. This implies that if the desirability value for dispersion is larger, the product or process is more robust.

Let s_{ik} be the estimated standard deviation of the kth response at the ith experiment point. That is,

$$s_{ik}=\sqrt{\frac{\sum_{j=1}^{m}(y_{ijk}-\bar{y}_{ik})}{m-1}} \quad (3.5)$$

where y_{ijk} is the response of the kth quality characteristic at the ith product design under the jth noise variable and $\bar{y}_{ik}$ is the arithmetic average of the kth quality characteristic at the ith design over the m noise variable.

Let e_{ik} be the desirability value of s_{ik}. Let $G(e_{i.})$ be the geometric average for desirability values of the standard deviation of each quality characteristic at the ith product design.

$$G(e_{i.})=(e_{i1}\times\cdots\times e_{ip})^{1/p} \quad (3.6)$$

4. Response Model Strategy

A response model strategy first models the observed responses, and then determines the best combination of controllable variables setting from the fitted models. After fitting predictive models, methods that achieve the best design point from the fitted models should be considered. As a tool for solving these problems that to find the best design point from the fitted models, optimization methods or Response Surface Methodology(RSM) can be used.

The weighting method of multi-objective optimization methods is to assign weights to the various objective functions and to combine these functions into a single objective function. Mathematically, the weighting method can be described as follows:

$$\text{Max } Z(x)=w_1z_1(x)+w_2z_2(x)+\cdots+w_pz_k(x) \quad (4.1)$$
$$\text{subject to} \quad x \in X$$

where $z_i(x)$ is the ith objective function and the coefficient w_i operating on the ith objective function, $z_i(x)$, is called a weight. The coefficient w_i can be interpreted as "the relative weight or worth" of that objective when compared to the other objectives.

In other words, this multi-objective problem has been transformed into a single optimization problem for which solution methods exist. We assume that all weights are equal to one.

Let $G(\bar{d})$, $G(d^*)$ and $G(e)$ denote the geometric average of the desirabilities for the arithmetic average, the geometric average for the desirability values of each qulaity characteristic, and the geometric average of the desirabilities for the standard deviation of each quality characteristic, respectively.

4.1 Strategy(1)

This response model strategy involves three regression equations. The basic idea is that the $G(\bar{d})$, $G(d^{*})$ and $G(e)$ can have an effect on the multiple response robust design. This strategy is to find the best combination of controllable variables that maximize three predicted models for $G(\bar{d}_{i.})$, $G(d_{i.}^{*})$ and $G(e_{i.})$ subject to a set of constraints for controllable variables. That is, fit the three predicted response models for $G(\bar{d}_{i.})$, $G(d_{i.}^{*})$ and $G(e_{i.})$, then optimize these equations using RSM or multi-objective linear programming. Since the objective is to find the best combination of controllable variables that maximizes the values of these fitted models, this type of problem can be considered as the maximization problem of the multiobjective programming. Mathematically, this strategy can be stated as follows.

Let $\bar{\bar{d}} = g_0 + \sum_{i=1}^{k} g_i x_i$, $\bar{\bar{e}} = h_0 + \sum_{i=1}^{k} h_i x_i$, and $\bar{d}^{*} = w_0 + \sum_{i=1}^{k} w_i x_i$, be linear response model fitted using the regression technique on the data, $\bar{d}_{i.}$, $e_{i.}$ and $d_{i.}^{*}$. Then the objective function is

$$\text{Max} \quad \bar{\bar{d}}(x) + \bar{e}(x) + \bar{d}^{*}(x) \qquad (4.2)$$

$$\text{Subject to} \quad l_i \le x_i \le u_i$$

where l_i and u_i are the lower and upper bounds respectively on x_i and i=1, 2, ..., k.

As another method, response surface methods can also be considered if the various objective functions can be combined into a single objective function. Since the three predicted response models for $G(\bar{d}_{i.})$, $G(d_{i.}^{*})$ and $G(e_{i.})$ are used to find the optimal setting that maximizes the values of fitted models, three functions could be combined into a single function, $\bar{\bar{d}}(x) + \bar{e}(x) + \bar{d}^{*}(x)$. Using the steepest ascent method, the best combination of controllable variables this single function could be found.

4.1.1 Procedure

The first step of this strategy is to design an experiment over an initial region of the design variables. The desirability function for location(for the individual observations), another desirability function for location (for the arithmetic average) and the desirability function for dispersion are also constructed. It could be done in the opposite order.

The second step is to collect data sets at product design points. The data are transformed into measures of desirability functions for location and for dispersion.

The third step is to calculate the average, $\bar{y}_{ij}$, and standard deviation, s_{il}, for each

quality characteristic at design point i (i=1, 2, ..., o). The calculated averages and standard deviations are transformed into desirability values, $\bar{d}_{ij}$, for the arithmetic average, and desirability values, e_{il}, for the standard deviation, sil, using the desirability function for location and dispersion. The geometric average, d^*_{ik}, for desirability values, d_{ijk}, (j=1, 2, ..., m), of each quality characteristic are calculated. After obtaining $\bar{d}_{ij}$, d^*_{ik}, and eil, these variables are transformed into a single measure, $G(\bar{d}_{i.})$, $G(d_{i.}^*)$ and $G(e_{i.})$, respectively.

The fourth step is fit the three predicted response model for $G(\bar{d}_{i.})$, $G(d_{i.}^*)$ and $G(e_{i.})$ and combined three fitted models into a single function, $\bar{\bar{d}}(x) + \bar{e}(x) + \bar{d}^*(x)$.

The fifth step is to determine the path of the steepest ascent and run additional experiments.

The direction of steepest ascent could be followed until one is sufficiently far from the initial experimentation region or until the predictive response model no longer adequately predicts the response along the ascent direction gradient. In such situations, a new local region for experimentation would be then defined.

On a new local region, a new set of experiments is performed. This procedure continues until little or no further improvement in response can be achieved from the method. Finally, in the region where the most desirable response values are suspected to be found, additional experiments are performed to verify that this is so.

4.2 Strategy (2)

The idea of this second response model strategy is that the best design point may be found by two multiple response predictive models. Two multiple response predictive models are constructed by $G(\bar{d}_{i.})$, $G(d_{i.}^*)$ and $G(e_{i.})$, at experimental point i. Since for $G(\bar{d}_{i.})$, and $G(d_{i.}^*)$ have different properties, the best design point may be found by multiple response predictive models that are obtained from the geometric average for for $G(\bar{d}_{i.})$ and $G(e_{i.})$ and for $G(d_{i.}^*)$ and $G(e_{i.})$.

Let $G(\bar{d}_{i.}, e_{i.})$ be the geometric mean of $G(\bar{d}_{i.})$ and $G(e_{i.})$, $(G(\bar{d}_{i.}) \text{X} G(e_{i.}))^{1/2}$.

Let $G(d^*_{i.}, e_{i.})$ be the geometric mean of $G(d_{i.}^*)$ and $G(e_{i.})$. To find the best combination of controllable variables, fit regression models to $G(\bar{d}_{i.}, e_{i.})$ and $G(d^*_{i.}, e_{i.})$, and optimize these models using RSM techniques.

5. Direct Strategy

The response model strategy that uses an appropriate statistical model for the optimal setting can be employed. As an alternative strategy, a direct strategy is appealing because it provides directly the best design point using the product of desirability values for averages and variance of quality characteristics.

To solve multiple response design problems, Herrington(1965) and Derringer and Suich (1980) advocated the geometric average for the desirability values of responses as a criterion for selecting the optimal design point. To find the optimal design point, they considered on;y the geometric average for the desirability values of quality characteristics. Since our objective is to find the optimal robust design point, our strategies will consider not only the location but also the dispersion of quality characteristics.

This direct strategy to obtain robust design is based on the idea that the best combination of controllable variables may be found by considering simultaneously the relationship between the geometric average, G(d^*_i), of the location desirabilities for the individual responses and the geometric average of dispersion desirability value, G($e_{i.}$). That is, this strategy is based on the assumption that only G($e_{i.}$) and G($d^*_{i.}$) are needed for determining the best combination of the controllable variables for multiple response robust design.

The objective of this strategy is to find a design point that has the minimum dispersion among responses and the maximum location desirability value simultaneously. To implement this strategy, two type of desirability functions are required: (1) the desirability function for dispersion to evaluate the dispersion effect for quality characteristics, and (2) the desirability function for the location of each individual response.

6. Example

The proposed strategies for the traffic Light system model are tested to show that the strategies developed can be used on applications in a variability of different problem areas. In addition to illustrating the method in a realistic scenario, the example presents a platform to measure the performance of the method.

The example that is presented here is an adaptation of the single-lane traffic analysis model in Pritsker(1995). To make it suitable for a multiple response robust design problem, the goal of the analysis is reformulated.

We will suppose that the objective of this problem is to reduce the variability of the waiting time, the queue length of the cars in both directions and to minimize the average waiting time and the average queue length for cars moving in both directions.

For this model, two response variables are considered. Engineer who designs the system want that the average waiting time for cars in direction 1 and in direction 2 and the average queue length for cars in both directions are as small as possible to reduce the number of waiting cars on road and to protect neighbors from air pollution caused by waiting cars.

In this example, there are four controllable variables the green time in direction 1 (x_1: 45 ~75 seconds), the green time in direction 2 (x_2: 45~75 seconds), the length of time that both signals are red following a green signal in direction 1 (x_3: 45~75 seconds), and the length of time that both signals are red following a green signal in direction 2 (x_4: 45~75 seconds).

For noise variables, the mean interarrival time of cars in direction 1, and the mean interarrival time of cars in direction 2 are selected. The mean interarrival time of cars in direction 1 (N_1) is exponentially distributed with a mean of 8 to 12 seconds. The mean interarrival time of cars in direction 2 (N_2) is exponentially distributed with a mean of 5 to 9 seconds.

In order to estimate the regression equations for $G(\bar{d}), G(d^*)$ and G(e*), initial starting points in the experimental region need to be specified. To measure the effects of the changes of the xi, a Resolution IV, 2^{4-1} fractional factorial design is chosen, A 22 fractional design is the choice for the Noise Matrix. To measure S, both N1 and N2 are perturbed according to a 2^2 factorial design pattern for each response.

Designer's desire wants to design a traffic system that the average waiting time of all cars passing this road is only in 30 seconds. However, if the average waiting time of cars is over 150 seconds, the system should be redesigned. For the average queue length of cars at the traffic light, designer wants to be as small as possible. The limitation of queue length that can be accepted is the range from 3 to 13. If the waiting line of cars is over 13 cars, this system is also rejected. we assume that all desirability functions are linear.

To compare with the results of the various strategies, improvement rate was used. Improvement rate is a criterion that represents how much the final result of RSM is improved more than the desirability values at the initial design point.

	Starting point	Result of RSM	Improvement
Response model strategy(1) (Desirability Value)	60, 60, 60, 60 0.6184	48.3, 30.1, 39.6, 31.4 0.7978	29.01%
Response model strategy(2) (Desirability Value)	60, 60, 60, 60 0.6540	59.6, 30.4, 41.6, 32.7 0.7654	17.03%
Direct Strategy (Desirability Value)	60, 60, 60, 60 0.5964	42.9, 35.8, 38.7, 33.7 0.8396	40.33%

Direct strategy made the maximum improvement rate (40.33%) among the strategies. The response model strategy that consisted of the combination of three first-order regression equations had the improvement rate of 29.01%. the second response strategy had the least improvement rate, 17.03%.

7. Conclusion

This paper considered robust design strategies for off-line quality control, with the use of experimental design and response surface methodology, in situations where all products have multiple quality characteristics. Previously, researchers have assumed that there was only a single quality characteristic that the customer would consider to be important for manufactured products or that the type of target for each of the quality characteristics were all of the "nominal-is-better" variety.

However, this paper developed general optimal strategies for solving the multiple response robust design problem. To develop general strategies, we explained how quality characteristics having different scales can be tranformed into a common scale, how the performance averages and variabilities for several quality characteristics can be calculated and combined, and why desirability functions for the average and dispersion can be utilized.

The contributions of this paper are (1) new approach for multiple response robust design problem (Up to now, there has been no multiple response robust design strategy that dealt with both performance for average and dispersion and with the various types of target. Hence, the new approach developed in this research produces a new paradigm for solving multiple response robust design problem.) (2) new tool for robust response product design (This research presents not only a new method for product designers in the quality control domain but also a new strategy for system designers in the manufacturing domain.).

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