◆ **Research Paper**

A Probabilistic Model for the Prediction of Burr Formation in Face Milling

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Abstract

A probabilistic model of burr formation in face milling of gray cast iron is proposed. During a face milling operation, an irregular pattern of the edge profile consisting of burrs and edge breakouts is observed at the end of cut. Based on the metal cutting theory, we derive a probabilistic model. The operational bayesian modeling approach is adopted to include the relevant theory in the model.

1. Introduction

Depending on the material properties and cutting conditions, fracture may occur when a machining tool exits a workpiece. This phenomenon results in edge breakout. A burr, which is defined as plastically deformed material left attached on the workpiece after machining, may also be created. The existence of burrs on a workpiece causes several problems such as decreasing the fit and ease of assembly of parts, damaging the dimensional accuracy and surface finish, increasing the cost and time of production due to removing burrs (or deburring). Depending on the application, edge breakout may be more desirable than a burr because the deburring time could be significantly reduced. Consequently, it is important to be able to construct a model for predicting burr occurrences since we can select cutting conditions which reduce the formation of undesirable burrs by using the model.

Burr technology is not simple, contrary to common beliefs that burr problems are not serious. Most attention has been paid to burr removal rather than prediction and minimization. The research field of burrs encompasses areas such as finite element analysis [1, 2], metal cutting mechanics [3, 4, 5], computer simulation [6, 7, 8], statistical experimental design [9], etc. There have been some efforts to analyze the behavior of

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material at the end of the cut [10, 11]. However, most studies have dealt with ductile materials which exhibit almost homogeneous material properties and necking under uniaxial tension and for the most part, produce burrs. On the other hand, gray cast iron is brittle, and necking does not occur under uniaxial tension testing at ambient temperature. Generally gray cast iron is a heterogeneous material, and the proportion in which the various constituents are present materially affects the manner in which it machines. Hence, at the end of a cut in face milling of gray cast iron, an irregular edge profile consisting of burrs and edge breakouts is usually observed. This edge profile of burrs and breakouts follows discernable patterns. Due to the complex material behavior of gray cast iron, in order to explain the irregular edge profile at the exit stage of the cut, it is appropriate to propose a probability modeling based approach.

Over several decades, mathematical models useful in solving engineering reliability problems related to system design have been developed. The models are probabilistic. That is, from information concerning the underlying probability distributions, deductions are made concerning optimal engineering system design. In order for the resulting mathematical deductions to be safely made, any probability modeling should be based on plausible physical considerations [12]. These considerations have been expressed in terms of the "physics" related to engineering problems. Therefore, in contrast to a curve-fitting statistical modeling approach, a probability model for an engineering problem should be constructed based on the relevant physics.

However, lacking a probabilistic modeling methodology, some engineering system designers are reluctant to use physics in building their own probability models. Instead, they tend to observe data and fit them to one of the pre-existing probability distributions whose parameters do not necessarily have any physical meaning. Here, we use the bayesian probabilistic modeling approach. It is appropriate for updating the probability of interest using observed and available data. In addition, it allows a probability model to include the relevant physics by way of using an uncertain but essential parameter governing a system's behavior. Under the bayesian modeling procedure, the so-called "prior" distribution expresses our initial opinion about the parameter. And we keep updating its form according to the data available from the experiments or real observations. In this way the prior becomes a "posterior" distribution of the parameter. In this way, the bayesian probabilistic modeling procedure includes a learning process [13].

In Section 2, the characteristics of gray cast iron in orthogonal cutting during the formation of chips and burrs is discussed. In particular, the role of shear plane angle resulting in the burr or breakout formations is explained. In Section 3, orthogonal metal cutting theory is applied to a face milling operation. The probability of burr formation is derived and an updating procedure for the probability is built in Section 4. In Section 5, the derived probability model in this study is discussed while explaining how to evolve the model.

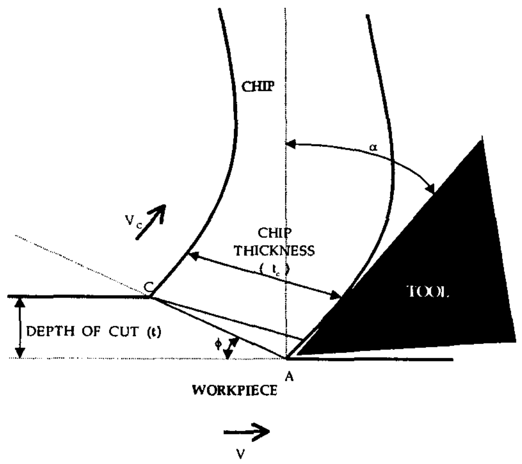

Figure 1: Shear plane (AC), the shear plane angle (ϕ) and the rake angle (α): V and V_c represent cutting velocity and chip velocity respectively.

2. Burr Formation of Gray Cast Iron in Orthogonal Cutting

All machining processes involve formations of chips by deforming the work material ahead of a cutting tool. In general, in most machining processes, two types of chips are formed without considering the influence of the build-up-edge (BUE); continuous and discontinuous chips. During the cutting of ductile materials such as carbon steel, copper, brass and aluminum alloys, etc., continuous chips are produced. The pressure of the tool makes the material along the cutting edge deform plastically forming a primary shear zone ahead of the tool. The deformed material then slides over the tool rake face for some distance and leaves the tool as a chip. Friction and continuous pressure between the chip and the tool may produce a secondary shear zone in the chip. On the contrary, discontinuous chips are produced during the cutting of the brittle materials such as gray cast irons. In this case, fracture occurs in the primary shear zone where the chip becomes segmented. Moreover, with an increase in temperature due to cutting conditions, especially, cutting speed, the yield strength of brittle materials decreases and may give a larger segment of chip or a continuous but inhomogeneous chip.

During orthogonal cutting, the workpiece material ahead of the tool tip plastically deforms and slides up the rake face of the tool. The zone of plastic deformation lies between the chip and the undeformed or only elastically deformed workpiece. At high cutting speeds, the size of the plastically deformed zone is small and approximates a thin shear plane. Therefore, it is assumed that the workpiece material shears across a plane and forms a chip. The plane is called the shear plane and the angle that it makes with the cutting velocity vector (V) is called shear plane angle, or shear angle (ϕ)(See Figure 1).

The shear angle has many important features not only for metal cutting but also for burr formation. Information with regard to material behavior of the work material as well as cutting conditions is reflected in the shear angle. In steady-state cutting, there is negligible change in the volume of the work material in plastic deformation. Thus, based on the geometry of orthogonal cutting, as shown in Figure 1, the shear angle ϕ can be determined by

$$\phi = \tan^{-1}\left(\frac{r_t \cos\alpha}{1 - r_t \sin\alpha}\right) \tag{1}$$

where r_t is the chip thickness ratio and α represents the rake angle of the tool. r_t is determined by t/t_c or V_c/V with depth of cut t, chip thickness t_c, cutting velocity V and chip velocity V_c.

The brittle behavior of gray cast iron in machining makes the analysis of both chip and burr/breakout formation more difficult than for ductile material in general. In case of orthogonal machining of ductile materials, an empirical equation has been introduced to predict the overall burr size based on the shear angle variation along with the deflection of the edge of the workpiece [10, 11, 14], and finite element analysis confirms the results [2]. According to the finite element study, the fundamental burr formation mechanism is divided into four stages: initiation, initial development, pivoting point, and final development stages.

During the burr formation process, yielding occurs at the edge of the workpiece as the tool moves toward the edge. This is known as the initiation stage. After the initiation stage, the variation in shear stress near the tool edge related to shear angle would be the most drastic change in the initial development stage. During the initial development stage, the transition in the cutting process occurs around the tool. In ductile materials, as the tool approaches the edge of the workpiece, the shear angle tends to decrease, and the shear plane gradually disappears due to the development of a high negative shear stress in front of the tool edge, similar to the plowing process. At the end of the initial development stage, depending on the cutting conditions, edge breakout may occur when high shear stress developing in front of the tool edge triggers the fracture. This phenomenon takes place when the shear angle decreases and goes below a critical shear angle (ϕ^*). If this occurs, then the stress field around the tool edge appears to exhibit the contact mechanism characteristics which could trigger fracture.

In the machining of brittle materials such as gray cast iron, shear angle variation would be expected, but the trend would not necessarily follow that of ductile materials. Due to the nature of discontinuous chip formation, the process of shear angle formation would not be so clear as in ductile materials in the initial development stage, and the edge breakout would take place instantaneously depending on how the shear angle develops in this stage. Therefore, the formation of a burr or edge breakout can be determined as:

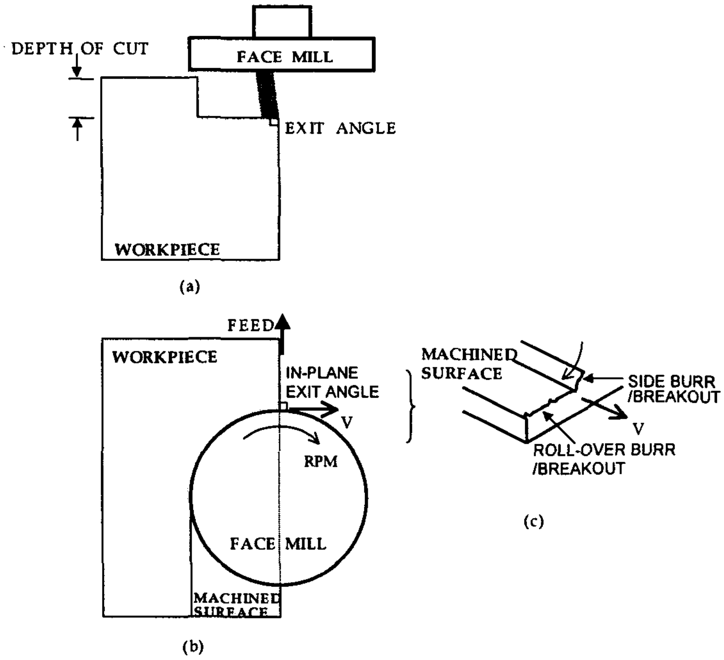

Figure 2: Face milling operation: Exit and in-plane exit angles are 90° respectively.

$$\begin{cases} \text{if } \phi > \phi^{*}, \textit{ then a burr forms.} \\ \textit{otherwise, edge breakout occurs.} \end{cases}$$

Although the fundamental mechanism of burr/breakout formation formed in gray cast iron would be similar to that in a ductile material, it involves some uncertainties due to the peculiarities of machining a brittle material. The irregular edge profile consisting of a combination of burrs and breakouts in gray cast iron is an example of this. These uncertainties largely arise from the peculiar material behavior of gray cast iron during the chip and burr formation processes. The uncertainties arising from the material properties would drive the uncertainties in the shear angle variation. Also, the uncertainties related to the critical shear angle would play an important role with regard to the observed random occurrences of each burr/breakout formation.

3. Application of Orthogonal Cutting Theory to Face Milling

In face milling, a cutter is equipped with multiple inserts rather than a single cutting edge in order to increase the cutting efficiency. The cutting speed is generated by cutter rotation. Moving the workpiece in a plane normal to the axis of spindle rotation

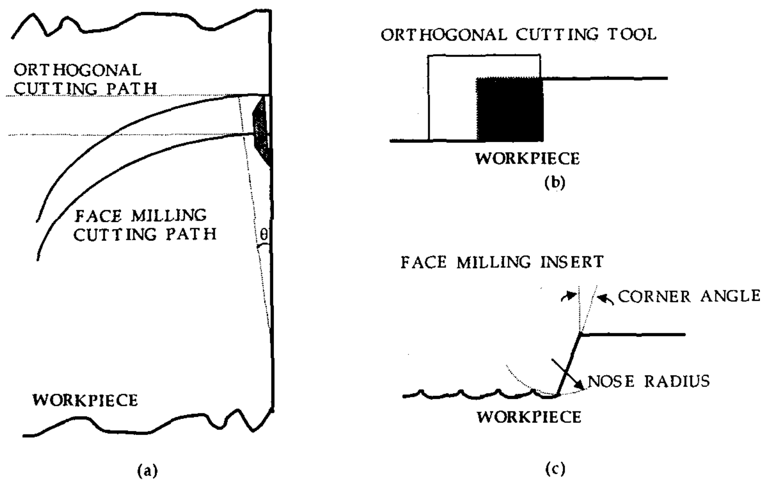

Figure 3: Similarity between face milling operation and orthogonal cutting at the exit stage of the cut.

creates the feed motion. Face milling is more complicated than turning and orthogonal cutting because the geometry of the cutting edges changes as the cut progresses.

In order to examine the behavior of gray cast iron with regard to burr/breakout formation, a single tooth face milling operation is used in this study because a wide range of cutting parameters can be used without the influence of insert variation and runout in multi-tooth machining. From the viewpoint of burr formation, the use of a single-toothed cutter rather than a multiple toothed cutter is not significant. Furthermore, the cutting process is constrained to create face milling as close to orthogonal cutting as possible at the end of cut. The exit angle, defined as the angle between the cutting velocity vector V and the edge of the workpiece, is 90°, and the in-plane exit angle, defined as the angle between the cutting velocity vector V and the edge of the workpiece to be machined in the machining plane, is 90°. Hence under these machining conditions, only one force component is considered at the tool exit stage (See Figure 2). Two independent parameters are considered in this study, feed rate and cutter rotation. The cutter rotation determines the cutting speed, and the feed rate provides the unit cutting path(UCP) on the edge of the workpiece for each feed motion of the table.

As previously mentioned, face milling is more complicated than turning and orthogonal cutting because the characteristics of the process, the chip thickness and forces on an edge, are constantly changing. However, the angle (θ) from burr initiation to tool exit is small (See Figure 3), and the diameter of the cutter is large enough that the cutting path in face milling is assumed to be straight after the burr initiation. Therefore, only one force component in the direction of the cutting velocity is considered, and the chip thickness during burr formation remains essentially constant. Under these circumstances, the characteristics in face milling would be similar to those in the orthogonal cutting process during the burr/breakout formation process in this study (See Figure 3). The length of a unit cutting path (UCP), l, at the tool exit stage is determined by

$$l\ (mm/tooth) = \frac{nf}{R}, \tag{2}$$

where n is 1 $rev/tooth$ in this study, f is the feed rate (mm/min) and R is the rotation of face mill (revolutions per minute, RPM). Therefore, N, the number of UCPs in the sampling length L, can be determined by

$$N = \frac{L}{l}. \tag{3}$$

In this study, the length of a UCP, l, is equivalent to the maximum undeformed chip thickness. Also, only roll-over type burrs and edge breakout along the edge of the workpiece are considered. However, at the tool exit stage, curl-type or side burrs could be created (See Figure 2(c)). Because of the ratio of the chip thickness to width used in this study and the brittle behavior of gray cast iron during machining, side burr formation and the influence of the corner angle and other tool geometries (See Figure 3(c)) are assumed to have little effect on the formation of roll-over type burrs and breakout. It should be noted that the cutting conditions used in this study are employed mainly to allow the application of available metal cutting theory and would not necessarily be used in industry.

4. A Probabilistic Model for the Prediction of Burr Formation

Due to the variability in the edge profiles involving burrs and breakouts, a probabilistic model is proposed for the prediction of their occurrences. That is, the probability model proposed in this study is for predicting the occurrence of burrs or breakouts when machining gray cast iron based on the metal cutting theory including burr/breakout formation.

The unit cutting path (UCP) is a fundamental parameter for prediction of variability of edge profile in the probabilistic model. That is, unit cutting paths are the objects to be measured for depicting the random occurrences of burrs or breakout resulting from a face milling operation. Deriving the probability that a burr forms in a UCP is the fundamental step for the prediction, wherein the length of a UCP is determined by Equation (2).

For the purpose of selecting cutting conditions which reduce the formation of undesirable burrs, data should be collected for several combinations of cutting conditions such as the tool and specimen geometry and velocity. In any case, the probability of burr formation should be evaluated for a specific cutting condition. we define and derive the probability of burr formation based on metal cutting theory including burr/breakout formation by considering the uncertainty in shear angle as the primary cause to the edge-profile variability. An updating procedure of the probability is also constructed.

When a finite number of samples or observed data are available, we make use of them in updating the probability of burr formation. Since such available data are always finite, a probability model should not be a statistically asymptotic model, at least in the beginning of the modeling stage [14, 15]. A bayesian parametric modeling approach is adopted in this study. Parametric modeling is a representation of the probabilistic behavior of a system and has flexibility in the prediction of the uncertain behavior simply by changing the values (or distributions) of the relevant parameter. In this framework, the bayesian modeling approach is a powerful inference method for such prediction and is well-equipped with an updating procedure using the observed data.

4.1 Probability Distribution of Shear Plane Angle

In building the probability model for the prediction of burr formation, we start with the engineering knowledge relative to burr formation in face milling. As explained in Section 3, we apply orthogonal cutting theory to burr formation in the face milling operation with limitations due to the assumptions about the cutting conditions. Also, from the assertion in Section 2, we believe that the shear angle ϕ plays an essential role in determining the likelihood of burr formation and regard it as the main source of uncertainty in resulting burr/breakout occurrences.

During burr-formation, there exists uncertainty in the value of the shear angle (ϕ in radians) caused by the peculiar behavior of gray cast iron. Hence a probability distribution of shear angle is necessary to represent its uncertainty. We assume that the variation in shear plane during the burr formation process is continuous and the mechanical properties along the shear plane are identical. According to the assertion in Section 2, we define the probability, p, that a burr forms in a unit cutting path (UCP) as follows:

$$p = Pr\{burr\ formation\} = Pr\{\phi > \phi^*\}, \tag{4}$$

or equivalently,

$$1 - p = Pr\{breakout\} = Pr\{\phi < \phi^*\}. \tag{5}$$

Notice that the above criterion of burr/breakout occurrence is established by the available cutting theory, meaning the application of orthogonal cutting theory to face milling. In order to evaluate the probability p, we need the probability density of shear angle, $\rho(\phi)$. That is, we calculate p by way of the following equation

$$p = Pr\{\phi > \phi^*\} = \int_{\phi^*}^{\infty} \rho(\phi)\, d\phi. \tag{6}$$

Under the supposition that there is no prevailing shear angle over the prespecified range (ϕ_0, ϕ_1), one way to express the uncertainty about the shear angle is the application of uniform probability density over (ϕ_0, ϕ_1). That is, the probability density of shear angle, $\rho(\phi)$, is of the form

$$\rho(\phi) = \frac{1}{\phi_1 - \phi_0} \qquad \phi \in (\phi_0, \phi_1). \tag{7}$$

Then, from Equation (6), the burr formation probability, p, becomes

$$p = \frac{\phi_1 - \phi^*}{\phi_1 - \phi_0}. \tag{8}$$

Even though we have a lower bound for the shear angle resulting in burr formation, we still do not have its exact value. Hence a probabilistic expression for ϕ^* should be given in this case. Since there is no information about its value until experiments are performed, we again apply the same uniform probability density to the lower bound as to the shear angle itself. It follows that the burr-formation probability, p, is regarded as a parameter which has a probability density. According to the foregoing statements, ϕ^* is uniformly distributed and its density $\pi(\phi^*)$ becomes

$$\pi(\phi^*) = \frac{1}{\phi_1 - \phi_0}. \tag{9}$$

To recapitulate, because we do not have any engineering knowledge or direct observations about the value of ϕ which results in burr formation, we can not have any probability density for ϕ reflecting its physical variability determining burr/breakout occurrences here. It should be noted that the probability density regarding the shear angle ϕ resulting in burr formation could be obtained from the burr shape, such as burr height and thickness. On the other hand, we do have data on the occurrence of either burr formation or breakout in the observed UCPs. By assigning 1 to burr formation in a UCP and 0 to breakout, we have an indicator random variable. By considering the 0 - 1 type random variables and the definition of burr-formation probability, the data can be used for updating ϕ^*. This updating procedure can be done by applying the uniform density to ϕ^*. Thus, we have Equation (8) as burr-formation probability given ϕ^* and Equation (9) as the probability density of ϕ^*.

Notice that the probability density (9) is a special case of the Beta probability density. Because the burr-formation probability, p, takes its value over (0, 1), the Beta probability density of p with parameters a and b has the form

$$\pi(p \mid a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} p^{a-1}(1-p)^{b-1}, \tag{10}$$

where $a, b > 0$, and $\Gamma(\cdot)$ is the gamma function. When the parameters $a = b = 1$, the Beta probability density becomes a uniform density. By rescaling and relocating p through the transformation $\phi^* = \phi_1 - (\phi_1 - \phi_0)p$, ϕ^* has a uniform density over (ϕ_0, ϕ_1).

4.2 Updating Procedure of the Burr-Formation Probability

In this section, we construct an updating procedure for the burr-formation probability when a finite number of observations are available. Due to the lack of "physics" explaining the influence of the burr-formation (or edge-breakout) in one UCP on the burr-formation (or edge-breakout) in another UCP, we need to construct a structure working among the random variables of interest. This can be accomplished by judgment. Because all we can observe are the occurrences of burrs and breakouts in UCPs after the face milling operation, the judgment about the structure should be based on the observations and in turn the judgment should be used to derive the probability of burr/breakout occurrences.

In this study, we make an exchangeability judgment for the set of random variables representing burr or breakout formation, which means that the joint probability function of the set of random variables is invariant under permutation of the coordinates [15]. It should be noted that when no "physics" are available but some phenomena can be observed, the observed facts can be reflected in a probability model by defining an appropriate random variable and applying exchangeability to the set of random variables.

4.2.1 Updating Procedure

In order to reflect the 0 - 1 type observations in the updating procedure, we first determine experimentally the smallest number of unit cutting paths showing the same results such as burrs or breakouts and define that they form a "block". These blocks are the objects to which the exchangeability judgement can be applied.

Since we judge that the block consists of one UCP and its corresponding random variables X_i are exchangeable conditional on the burr-formation probability, using Equation (8), we have the following probability distribution of the number of UCPs including burrs (name them burr-UCPs), s, given n UCPs

$$\rho\left(s \mid n, \frac{\phi_1-\phi^*}{\phi_1-\phi_0}\right) \propto \left(\frac{\phi_1-\phi^*}{\phi_1-\phi_0}\right)^s \left(\frac{\phi^*-\phi_0}{\phi_1-\phi_0}\right)^{n-s}, \tag{11}$$

where $\propto$ denotes "proportional to".

By using Bayes' formula with Equations (9) and (11), we can update the burr-formation probability as follows:

$$\rho\left(\frac{\phi_1-\phi^*}{\phi_1-\phi_0} \mid s, n\right) \propto \left(\frac{\phi_1-\phi^*}{\phi_1-\phi_0}\right)^s \left(\frac{\phi^*-\phi_0}{\phi_1-\phi_0}\right)^{n-s} \left(\frac{1}{\phi_1-\phi_0}\right)$$

$$\propto \left(\frac{\phi_1-\phi^*}{\phi_1-\phi_0}\right)^s \left(\frac{\phi^*-\phi_0}{\phi_1-\phi_0}\right)^{n-s}. \tag{12}$$

Equation (12) represents the updated probability density of the burr-formation probability when s burr-UCPs are observed out of n UCPs. Additionally, after observing t more burr-UCPs from the additional m sampled UCPs, we can keep updating by following the above procedure, wherein Equation (12) plays the same role as Equation (8) in the previous updating. The resulting updated probability distribution is

$$\rho\left(\frac{\phi_1-\phi^*}{\phi_1-\phi_0} \mid s, n, t, m\right) \propto \left(\frac{\phi_1-\phi^*}{\phi_1-\phi_0}\right)^{s+t} \left(\frac{\phi^*-\phi_0}{\phi_1-\phi_0}\right)^{n+m-s-t} \tag{13}$$

4.2.2 Prediction of the Number of Burrs

As in most statistical inferences, the prediction of the future behavior from data is of interest . That is, we are interested in the probability that there will be k burr-UCPs out of m unobserved future UCPs, or $\rho(k \mid m)$. Let Y_j be the random variable indicating that a burr forms in the jth UCP and M be the number of UCPs within a specimen. From the exchangeability judgement among the set of random variables $(Y_1, Y_2, \cdots, Y_m)$ satisfying $\sum_{j=1}^{M} Y_j = K$, where K is the number of burr-UCPs within a specimen, we have the following discrete uniform distribution

$$\rho(y_1, y_2, \cdots, y_M \mid K) = \frac{1}{\binom{M}{K}}. \tag{14}$$

Consider k such that $\sum_{j=1}^{m} y_j = k$,

$$\rho(y_1, y_2, \cdots, y_m \mid K) = \frac{\binom{M-m}{K-k}}{\binom{M}{K}} \tag{15}$$

and

$$\rho(k \mid M, K, m) = \frac{\binom{K}{k}\binom{M-K}{m-k}}{\binom{M}{m}}. \tag{16}$$

Notice that Equation (16) is equal to $\binom{m}{k}$ X Equation (15) and is a hypergeometric distribution.

In most face milling operations, the number of UCPs within a workpiece, M, is big enough to be regarded as unbounded. With the exchangeability judgement for the infinite number of UCPs, de Finetti [16] has shown that

$$\rho(k \mid m) = \int_0^1 p^k (1-p)^{m-k} \pi(p)\ dp, \tag{17}$$

where the burr-formation probability p as defined in Equation (8) and $\pi(p)$ in Equation (17) represents the probability density of p which has been updated by using data available up to now.

5. Discussion

In this study, the bayesian probability modeling approach is used for explaining the uncertain physics of a phenomenon when some empirical knowledge is available. The 0 - 1 type random variable which indicates burr or breakout occurrence is used in the modeling. However, we believe that by including more observable and physical parameters in the modeling process, we can have a more physically defined probability model. For example, if burr/breakout occurrence in a unit cutting path can be represented by an engineering formula in terms of physical random quantities such as burr height and thickness, we would have a more physics-based probability model. It follows that we will be more able to explain the relevant process.

The models in this study were derived based only on such controllable parameters as the feed rate and cutting speeds. In the previous study of burr/breakout formation, it has been observed that the depth of cut plays an important role in determining whether a burr or breakout occurs. However this study demonstrates that the cutting speed is also one of the important parameters to determine the burr/breakout formation in gray cast iron. In general, as the cutting speed increases, temperature increases due to increased high strain-rate. Hence it can be deduced that a temperature increase due to an increase of the cutting speed could cause the shift of the probability densities of the shear angle. In order to further investigate the burr/breakout formation, it would be desirable to have temperature as an observable parameter along with the feed rate and cutter rotation. With information on temperature during burr formation, the temperature-dependent material behavior could be obtained for more precise prediction of the burr/breakout formation. The more observable parameters we have, the more physical knowledge can be obtained. Therefore, for this purpose, an engineering formula should be proposed with a probability model in the development stage.

References

[1] K. Iwata, K. Ueda, and K. Okada. Study of Mechanism of Burrs Formation in Cutting Based on Direct SEM Observation. J. Soc. Prec. Eng., 48(4):510-515, 1983.

[2] I.W. Park, S.H. Lee, and D.A. Dornfeld. Modeling of Burr Formation in Orthogonal Cutting by FEM. ESRC Technical Report 94-34, Univ. of California, Berkeley, Engineering Systems Research Center, Dec. 1994.

[3] K. Adachi et al. A study on Burr in Low Frequency Vibratory Drilling. Japan Soc. of Prec. Eng., 21(4):258-264, May 1987.

[4] M. Hashimura, P.Y. Chang, and D.A. Dornfeld. Analysis of Burr Formation Mechanism in Orthogonal Cutting. ESRC Technical Report 95-04, Univ. of California, Berkeley, Engineering Systems Research Center, 1995.

[5] K. Nakayama and M. Arai. Burr Formation in Metal Cutting. Annals of the CIRP, 36(1):33-36, 1984.

[6] D.A. Dornfeld et al. Intelligent Deburring of Precision Components. Technical Report, SME Technical Paper, No. MR 93-319-7, Univ. of California, Engineering Systems Research Center, Oct. 1993.

[7] R. Narayanaswami and D.A. Dornfeld. Burr Minimization on Face Milling of Flat Polygon Parts. ESRC Technical Report 94-23, Univ. of California, Berkeley, Engineering Systems Research Center, Oct. 1994.

[8] R. Narayanaswami and D.A. Dornfeld. Design and Process Planning Strategies for Burr Minimization and Deburring. NAMRI Transactions, SME, 22, May 1994.

[9] J. Stein and D.A. Dornfeld. An Analysis of the Burrs in Drilling Precision

Miniature Holes Using a Fractional Factorial Design. ESRC Rechnical Report 95-02, Univ. of California, Berkley, Engineering Systems Research Center, Jan. 1995.

[10]G.L. Chern and D.A. Dornfeld. Analysis of Burr Formation and Breakout Using a Scanning Electron Microscope in Micro-Machining Stage.Trans. ASME, J. Eng. Mats. Tech., submitted for publication, 1994.

[11]S.L. Ko and D.A. Dornfeld. A study on Burr Formation Mechanism. Trans. ASME, J. Eng. Ind., 98(1):66-74, 1991.

[12]R.E. Barlow and F. Proschan. Mathematical Theory of Reliability. John Wiley & Sons, Inc., 1965.

[13]B. de Finetti. Probability and Exchangeability from a Subjective Point of View. International Statistical Review, 47:129-135, 1979.

[14]G.L. Chern and D.A. Dornfeld. Burr/Breakout Development and Experimental Verification. Trans. ASME, J. Eng. Mats. Tech., submitted for publication, 1995.

[15]R.E. Barlow and M.B. Mendel. De Finetti-type Representations for Life Distributions. JASA, 87(420), 1992.

[16]Bruno de Finette. Theory of Probability, v. 2. John Wiley & Sons, Inc., New York, 1990.