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The Comparison Analysis of an Estimators of Nonlinear Regression Model using Monte Carlo Simulation

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Abstract

In regression model, we estimate the unknown parameters by using various methods. There are the least squares method which is the most general, the least absolute deviation method, the regression quantile method and the asymmetric least squares method. In this paper, we will compare each others with two cases: firstly the theoretical comparison in the asymptotic sense and then the practical comparison using Monte Carlo simulation for a small sample size.

Key Words: regression estimators, consistency, asymptotic normality, Monte Carlo simulation

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1. Introduction

Generally the nonlinear regression model is

$$y_t = f(x_t, \theta_o) + \varepsilon_t, \qquad (1.1)$$

where $t=1,2,\cdots,T$ and $f(x_t, \theta_o)$ is a real valued nonlinear function defined on $R^{p_1+p_2}$, x_t is a $(1\times p_2)$ observed vector, the error terms ε_t are independent and identically distributed(i.i.d.) with finite variance. The parameter vector θ_o which is interior point in a compact parameter space $\Theta \subset R^{p_1}$ is unknown and to be estimated.

Given an observation y_t, any vector $\widehat{\theta_T}$ in Θ minimizing the following objective function

$$S_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} (y_t - f(x_t, \theta))^2 \qquad (1.2)$$

will be called a least squares estimators (LSE) of θ_o based on $\{y_t\}_{t=1}^{T}$.

Jennrich(1969) first rigorously proved the existence of the LSE and showed the strong consistency and asymptotic normality of the LSE with the several assumptions including the following assumption : $F_T(\theta_1, \theta_2)$ converges uniformly to a continuous function $F(\theta_1, \theta_2)$ and $F_T(\theta_1, \theta_2)=0$ if and only if $\theta_1 = \theta_2$, where

$$F_T(\theta_1, \theta_2) = \frac{1}{T}\sum_{t=1}^{T}(f_t(\theta_1) - f_2(\theta_2))^2.$$

Wu(1981) gave sufficient conditions under which the LSE converges to θ_0 strongly, when the condition of the above requirement of F_T is replaced by the following assumption : $f_t(\theta)$are Lipschitz function on Θ and

$$\sup_{\theta_1 \neq \theta_2} \frac{|f_t(\theta_1) - f_t(\theta_2)|}{|\theta_1 - \theta_2|} \le M \sup_{|\theta - \theta_o| \ge \delta} |f_t(\theta) - f_t(\theta_o)|,$$

for some $\delta > 0$ and for all t, where M is independent of t and $|\theta_1 - \theta_2|$ is the Euclidean distance between θ_1 and θ_2.

On the other hand, in spite of the theoretical and practical merits, a certain criticisms of procedures based on the least squares method in the past have been pointed to the robustness even with a single outlier or a slight departure from the normality assumption on the errors. When the error distribution is heavy-tailed such as Laplace or Cauchy distributed errors and the least squares method is judged inadequate, automatically the least absolute deviation estimator (LAD) which is defined as follows have attracted considerable attention in the part of the robust regression analysis.

Given an observation y_t, any vector $\widetilde{\theta_T}$ in Θ which minimize the residual sum of absolute difference

$$S_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} |y_t - f(x_t, \theta)| \qquad (1.3)$$

will be called a least absolute deviation estimators (LAD) of θ_o based on $\{y_t\}_{t=1}^{T}$. With this plausible properties, during the past few years there has been an increase interest in robust estimation procedure applied to the regression model. The general discussion of robustness are given in Huber(1981) and Hampel(1986).

Historically, the LAD estimation for regression coefficient started from Boscovich and Laplace, but for a long time it never attracted much attention. One reason is the difficulty of computing the LAD estimation and another is the lack of adequate sampling theory of such estimators. In this linear model, the problem of computing is successfully solved and the asymptotic sampling theory of the LAD estimation is well developed, although for applications we need to develop the small sample theory. Asymptotic results of the LAD for linear model are given various authors; Koenker and Basset (1978), Amemiya(1982), Nyquist(1983), Dupacova and Wets(1988) and Chen and Wu(1988), among others. In the nonlinear regression model, various authors have provided conditions which ensure the existence, consistency and asymptotic normality of the LAD. Oberhofer(1982) gave weak consistency results of the LAD. Wang(1995) studied the asymptotic properties of the LAD estimators.

The concept of the periodicity in time series is of fundamental interest, since it provides a means for formalizing the notions of dependence or correlation between adjacent points. In this paper we think about a sum of sinusoidal components:

$$y_t = \sum_{r=1}^{q} \{A_{ro}\cos(\omega_{ro}t) + B_{ro}\sin(\omega_{ro}t)\} + \varepsilon_t. \quad (1.4)$$

where $\theta_o = (A_{1o}, B_{1o}, \omega_{1o}, \cdots, A_{qo}, B_{qo}, \omega_{qo})$, and for $q \geq 1$, A_{ro}, B_{ro}'s are some fixed unknown constants, ω_{ro} is unknown frequency lying between 0 to π $(1 \leq r \leq q)$ and in this case the observed value x_t means t.

But the above formula does not satisfy Jennrich (1969)'s assumptions nor Wu's Lipschitz type condition, the methods which are proposed by Jennrich(1969) and Wu(1981) are not available. For this reason Walker(1971) obtained the asymptotic properties of an approximate LSE. Hannan(1973) generalized the results of the Walker(1971). Hannan(1973) considered the case when ε_r is generated by a strictly stationary random variable process. Rice and Rosenblatt(1988) discussed some computational issues of a similar kind of model. Kundu(1993) and Kundu and Mitra (1996) gave the direct proof of the strong consistency and asymptotic normality and observed that the approximate LSE and the LSE are asymptotically equal. And Oberhofer (1982) studied the weak consistency about the LAD estimators with the assumptions from B1 to B6 in his paper, but the assumption B5 in his paper is equivalent to assumption of Jennrich(1969). And then using the different methods mentioned previous methods, the asymptotic properties of LAD of this model is proved by Kim T.S., Kim H.K. and Choi S.H.(2000).

But the LSE and LAD are inadequate for asymmetric model. In this case the asymmetric model means that the error's distribution $G(\varepsilon_t)$ satisfies $G(0) \neq \frac{1}{2}$. Accordingly we need a substitute approach. Koenker, R. and Bassett, G.(1978) introduced Regression Quantile Estimators and Whitney K. Newey and James L. Powell(1987) studied the Asymmetric Least Squares Estimators which are defined as follows. For the nonlinear regression time series model (1.4), consider the next objective function, when $\beta \neq \frac{1}{2}$, and $0 < \beta < 1$

$$S_T(\theta, \beta) = \frac{1}{T}\sum_{t=1}^{T} \varphi_\beta(y_t - f(x_t, \theta)), \qquad (1.5)$$

where $\varphi_\beta(\lambda)$ is called a check function which is defined

$$\varphi_\beta(\lambda) = \begin{cases} \beta\lambda, & \lambda \geq 0, \\ (\beta - 1)\lambda, & \lambda < 0. \end{cases}$$

For a given an observation y_t, any vector $\tilde{\theta}_T(\beta)$ in Θ which minimizing the objective function $S_T(\theta, \beta)$ be called a Regression Quantile Estimators (RQE) of θ_o based on $\{y_t\}_{t=1}^{T}$.

Lastly we think the another new objective function in (1.4) such as

$$D_T(\theta, \beta) = \frac{1}{T}\sum_{t=1}^{T} \phi_\beta(y_t - f(x_t, \theta)), \qquad (1.6)$$

where $\phi_\beta(\lambda)$ is also called a check function which is defined

$$\phi_\beta(\lambda) = \begin{cases} \beta\lambda^2, & \lambda \geq 0, \\ (1 - \beta)\lambda^2, & \lambda < 0. \end{cases}$$

For a given an observation y_t, any vector $\widehat{\theta_T}(\beta)$ in Θ which minimizing $D_T(\theta, \beta)$ be called an Asymmetric Least Squares Estimators (ALS) of θ_o based on $\{y_t\}_{t=1}^{T}$.

Kim T.S. and Kim H.G(2000) and Kim T.S., Kim H.G. and Hur S.(2000) proved the asymptotic properties of RQE. The asymptotic normality of ALS is discussed by Jang S.E.(1999).

The estimators which are defined in the above has the proper properties in the asymptotic sense. But in the practical phenomenon, we deal with the finite data set. So they are invalid and not adjusted in a small sample size. The general objective of the research is to verify the validation of the above different estimators using the Monte Carlo simulation when the sample size are a finite small size.

The four different asymptotic results are discussed in section 2. And then, for the asymptotic state which means the large sample size, we are able to check the relative efficiency of the four estimators. It is given in section 3. And in section 4, we will check the properties of four estimates by the Monte Carlo simulation and compare each other estimates.

2. The Asymptotic Properties

2.1 The Least Squares Estimators

Theorem 2.1 : Let

$$y_t = \sum_{r=1}^{q} \{A_{ro}\cos(\omega_{ro}t) + B_{ro}\sin(\omega_{ro}t)\} + \varepsilon_t,$$

where $0 < \omega_{ro} < \pi$, for $r = 1, 2, \cdots, q$, and ε_t are distributed independently and identically with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t^2) = \sigma^2 < \infty$. And assume that

$$\lim_{T\to\infty} \min_{1\leq r\neq s\leq q} (T \mid \omega_{ro} - \omega_{so} \mid) = \infty.$$

Then we have the LSE is a strongly consistent estimator of θ_o and

$$(P_1(\widehat{\theta_{1T}}), P_2(\widehat{\theta_{2T}}), \cdots, P_q(\widehat{\theta_{qT}})),$$

where for $r = 1, 2, \cdots, q$, $P_r(\widehat{\theta_{rT}}) =$

$$(\sqrt{T}(\widehat{A_{rT}} - A_{ro}), \sqrt{T}(\widehat{B_{rT}} - B_{ro}), \sqrt{T^3}(\widehat{\omega_{rT}} - \omega_{ro}))$$

converges in law $N(0_{3q\times 1}, \sigma^2\Sigma^{-1})$, where $\Sigma = (\Sigma_{rs})_{3q\times 3q}$, for $r, s = 1, 2, \cdots, q$, and

$$\Sigma_{rs}=\begin{cases} 0\ , & \text{if } r\neq s \\ \begin{pmatrix} \frac{1}{2} & 0 & \frac{B_{ro}}{4} \\ 0 & \frac{1}{2} & \frac{-A_{ro}}{4} \\ \frac{B_{ro}}{4} & \frac{-A_{ro}}{4} & \frac{A_{ro}^2+B_{ro}^2}{6} \end{pmatrix}, & \text{if } r=s. \end{cases}$$

proof] For the detailed proof, see Walker(1971) and Kundu and Mitra(1996).

2.2 The Least Absolute Deviation Estimators

Theorem 2.2 : Under the same conditions of Theorem 2.1, the LAD is a strongly consistent estimator of θ_o and

$$(P_1(\widetilde{\theta_{1T}}), P_2(\widetilde{\theta_{2T}}), \cdots, P_q(\widetilde{\theta_{qT}})),$$

where for $r=1,2,\cdots,q$,

$$P_r(\widetilde{\theta_{rT}}) = (\sqrt{T}(\widetilde{A_{rT}}-A_{ro}),\ \sqrt{T}(\widetilde{B_{rT}}-B_{ro}), \sqrt{T^3}(\widetilde{\omega_{rT}}-\omega_{ro}))$$

converges in law $N(0_{3q\times 1}, \frac{1}{\{2g(0)\}^2}\Sigma^{-1})$, where $g(\varepsilon_t)$ is a continuous probability density function of ε_t.

proof] For the detailed proof, see Kim T.S., Kim H.K. and Choi S.H.(2000).

2.3 The Asymmetric Least Squares Estimators

Theorem 2.3 : Under the same conditions of Theorem 2.1 except $E(\varepsilon_t)=0$ and with the additional condition $G(0)=\beta$, $(0<\beta<1, \beta\neq 0.5)$, where $G(\varepsilon_t)$ is a distribution function of the error term ε_t, the ALS is a strongly consistent estimator of θ_o and

$$(P_1(\widehat{\theta_{1T}}(\beta)), P_2(\widehat{\theta_{2T}}(\beta)), \cdots, P_q(\widehat{\theta_{qT}}(\beta)),$$

where for $r=1,2,\cdots,q$,

$$P_r(\widehat{\theta_{rT}}(\beta)) = (\sqrt{T}(\widehat{A_{rT}}(\beta)-A_{ro}),\ \sqrt{T}(\widehat{B_{rT}}(\beta)-B_{ro}),\ \sqrt{T^3}(\widehat{\omega_{rT}}(\beta)-\omega_{ro}))$$

converges in law

$$N\left(0_{3q\times 1}, \frac{(1-2\tau)d+\tau^2(\sigma^2+\mu^2)}{[\beta+\tau(1-2\beta)]^2}\Sigma^{-1}\right),$$

where $g(\varepsilon_t)$ is a continuous probability density function of ε_t, and $\tau=\frac{b}{2b-\mu}$, $\mu=E(\varepsilon_t)$, $b=\int_{-\infty}^{0} xg(x)dx$, $d=\int_{-\infty}^{0} x^2 g(x)dx$.

proof] For the detailed proof, see Jang S.E.(1999).

2.4. The Regression Quantile Estimators

Theorem 2.4 : Under the same conditions of Theorem 2.3, The RQE is a strongly consistent estimator of θ_o and

$$(P_1(\widetilde{\theta_{1T}}(\beta)), P_2(\widetilde{\theta_{2T}}(\beta)), \cdots, P_q(\widetilde{\theta_{qT}}(\beta)),$$

where for $r=1,2,\cdots,q$,

$$P_r(\widetilde{\theta_{rT}}(\beta)) = (\sqrt{T}(\widetilde{A_{rT}}(\beta)-A_{ro}),\ \sqrt{T}(\widetilde{B_{rT}}(\beta)-B_{ro}),\ \sqrt{T^3}(\widetilde{\omega_{rT}}(\beta)-\omega_{ro}))$$

converges in law $N(0_{3q\times 1}, \frac{\beta(1-\beta)}{\{g(0)\}^2}\Sigma^{-1})$, where $g(\varepsilon_t)$ is a continuous probability density function of ε_t.

proof] For the detailed proof, see Kim T.S., Kim H.G. and Hur S.(2000).

3. The Asymptotic Relative Efficiency

Since the assumptions of the LSE are equal to the LAD, so the asymptotic efficiency of LSE relative to LAD estimators is

$$\lim_{T\to\infty} eff\,(\widehat{\theta_T} | \widetilde{\theta_T}) = \lim_{T\to\infty} \frac{Var(\widehat{\theta_T})}{Var(\widetilde{\theta_T})} = \{2g(0)\}^2\sigma^2.$$

It does implies that the LSE is asymptotically more efficient than LAD in the sinusoidal model for the normal distribution, but for the heavy-tailed error distributions more than the normal distribution likewise the Laplace distribution, t-distribution and logistic distribution, etc, the LAD is asymptotically more efficient than the LSE. On the other hand, for $\beta \neq 0.5$, the asymptotic efficiency of ALS relative to RQE is

$$\lim_{T\to\infty} eff\,(\widehat{\theta_T}(\beta) | \widetilde{\theta_T}(\beta)) = \frac{g^2(0)[(1-2\tau)d+\tau^2(\sigma^2+\mu^2)]}{\beta(1-\beta)[\beta+\tau(1-2\beta)]^2}.$$

Likewise the asymptotic efficiency of LSE relative to LAD estimators, the asymptotic efficiency of ALS relative to RQE is less than 1 for the normal error distribution and larger than 1 for the heavy-tailed error distribution than the normal distribution.

4. Monte Carlo Simulation

We performed some Monte Carlo simulations to compare the four different estimators. We will check the behaviour of the estimators for small sample sizes since they are asymptotically founded the exact relations. Numerical results are reported for $T=10,15,25$ and $\omega=0.25\pi(\approx 0.785398)$, $0.5\pi(\approx 1.570796)$, $0.75\pi(\approx 2.356194)$.

	T=10		T=15		T=25	
	LSE	LAD	LSE	LAD	LSE	LAD
.25 π	.7843	.7810	.7852	.7852	.7842	.7833
	.0052	.0098	.0016	.0028	.0004	.0006
	.1352	.1395	.0699	.0715	.0361	.0371
	.901	.842	.932	.887	.932	.854
.50 π	1.5693	1.5718	1.5700	1.5690	1.5705	1.5701
	.0052	.0099	.0016	.0028	.0003	.0006
	.1284	.1335	.0732	.0751	.0345	.0350
	.904	.863	.928	.883	.941	.882
.75 π	2.3555	2.3583	2.3566	2.3576	2.3563	2.3586
	.0051	.0096	.0016	.0028	.0003	.0006
	.1213	.1260	.0767	.0779	.0339	.0350
	.922	.877	.918	.859	.947	.888

Table 3.1 The distribution of error is $N(0,1)$

We studied the four cases as follows. the first case is the errors are assumed to be independent and normally distributed random variables with mean zero and variance one. At the second case, the distribution of error is the Laplace distribution with a parameter $\phi=3$ and at the third case, the distribution of error is normally distributed random variables with mean one and variance one. The last case is the distribution of error is the Laplace distribution with a parameter $\phi=3$ and $\beta=0.4$. For a particular T and ω, 1000's different sets of data were generated. The two parameters A and B are taken as 1.5 all. Under the given each data set, we estimated the nonlinear parameter ω by the four methods, and described the average estimates in the first rows, average mean squared error(MSE) in the second rows and the average length of 95% confidence interval over 1000 simulations in the third rows respectively. And the last rows in each table mean the coverage probabilities.

	T=10		T=15		T=25	
	LSE	LAD	LSE	LAD	LSE	LAD
.25 π	.7535	.7674	.7576	.7744	.7741	.7742
	.1041	.0550	.0293	.0146	.0062	.0032
	.5535	.5773	.3060	.3057	.1468	.1495
	.881	.890	.853	.907	.849	.902
.50 π	1.5346	1.5732	1.5505	1.5593	1.5554	1.5563
	.1018	.0549	.0289	.0150	.0063	.0032
	.5469	.5714	.3051	.3115	.1478	.1495
	.848	.925	.879	.912	.859	.883
.75 π	2.3507	2.3494	2.3322	2.3499	2.3483	2.3519
	.1047	.0522	.0296	.0146	.0063	.0031
	.5507	.5550	.3080	.3087	.1465	.1455
	.847	.910	.872	.922	.897	.937

Table 3.2 The distribution of error is the Laplace with a parameter $\phi=3$

	T=10		T=15		T=25	
	ALS	RQE	ALS	RQE	ALS	RQE
.25 π	.7844	.7854	.7854	.7819	.7857	.7857
	.0090	.0135	.0025	.0036	.0005	.0007
	.1583	.1631	.0790	.0815	.0411	.0414
	.933	.897	.938	.900	.950	.885
.50 π	1.5694	1.5662	1.5712	1.5703	1.5708	1.5704
	.0091	.0135	.0025	.0037	.0005	.0008
	.1509	.1549	.0832	.0848	.0395	.0400
	.948	.905	.939	.893	.951	.892
.75 π	2.3553	2.3551	2.3566	2.3551	2.3562	2.3563
	.0091	.0135	.0025	.0036	.0005	.0008
	.1447	.1503	.0857	.0870	.0384	.0390
	.946	.908	.931	.870	.960	.898

Table 3.3 The distribution of error is $N(1,1)$

From Table 3.1, when the distributions of error is the normal distribution with mean zero and variance one, we see that for the most parts, the average estimate values using least squares method is closer to the true

	T=10		T=15		T=25	
	ALS	RQE	ALS	RQE	ALS	RQE
0.25 π	.7699	.7792	.7666	.7718	.7117	.7801
	.1163	.1045	.0327	.0285	.0073	.0055
	.5494	.56619	.3045	.3097	.1519	.1513
	.860	.893	.784	.916	.566	.915
0.50 π	1.5748	1.5672	1.5580	1.5675	1.5315	1.5636
	.1216	.1071	.0335	.0286	.0071	.0055
	.5637	.5745	.3115	.3155	.1499	.1487
	.841	.896	.803	.920	.601	.938
0.75 π	2.3621	2.3557	2.3432	2.3531	2.3324	2.3557
	.1190	.1046	.0329	.0294	.0071	.0058
	.5528	.5706	.3084	.3151	.1491	.1483
	.812	.892	.836	.928	.633	.934

Table 3.4 The distribution of error is the Laplace with a parameter $\phi=3$ and $\beta=0.4$

parameter than the average estimates using the least absolute deviation method, and then the average MSE and the average length of 95% confidence intervals in LSE are smaller than the LAD, but the coverage probabilities in LSE is larger than the LAD. Hence we obtain the results the LSE is superior to the LAD likewise in the asymptotic sense. Also according to Table 3.2, Table 3.3 and Table 3.4 respectively we conclude for the symmetric heavy-tailed distribution the LAD is superior to the LSE, for the asymmetric normal distribution the ALS is superior to the RQE and for the asymmetric heavy-tailed distribution the RQE is superior to the ALS.

5. Conclusions

Likewise the asymptotic cases, we have a similar results under the small sample size for the most part. It is also observed that, for the four cases, as T increases the absolute difference between true parameter values and estimates and the average mean squared errors and the average length of confidence interval decreases and the coverage probability increases.

Therefore, in order to estimate the true parameters, do not depend only the least squares methods, but also we should determine the use of the estimating method among the four different ways by the results that is the determination of the error's distribution using the general statistical analysis.

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