Theoretical Development and Design Aids for Expansion Joint Spacings

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ABSTRACT

It has been a well known fact that buildings having inappropriate expansion joints in their spacings may be subject to exterior damages due to extensive cracks on the outer walls under service loads and structural damages due to excessive moment induced by temperature changes at ultimate load conditions. Unfortunately, consistent code provisions are unavailable regarding spacings of expansion joints from different foreign structural codes. And a more serious problem is that no quantitative measurements on spacings is given in our codes for building structures.

In order to establish a rational guideline on the spacing of expansion joints, theoretical approaches are taken in this study. The developed theoretical formula is, then, converted to a design chart for structural designers' convenience in its use. The chart considers both service and ultimate load stages.

Keywords : design aids, expansion joint, temperature, theory, shrinkage, strength criterion, serviceability criterion

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1. Introduction

Expansion or contraction due to the temperature changes or concrete shrinkage may damage structures during their service life. It is generally acknowledged by structural engineers that placing proper expansion joints can avoid structural damages or nonstructural cracks induced by temperature changes or concrete shrinkage. Different expansion joint spacings are reguired a different foreign codes and a consistent specification on spacings of expansion joints is not available. Since guidelines for the spacings of expansion joints are not provided in a domestic reinforced concrete code, it becomes a usual practice of placing expansion joints based on structural designers′ personal experience or intuition.

This practice sometimes results in inappropriate expansion joint spacings and buildings suffer from extensive cracks in their exterior walls or from improperly designed structural components with insufficient reservation for additional flexural strengths due to temperature changes and shrinkage, especially at the exterior columns. The objective of this research is to theoretically define the spacing of expansion joints and to present design aids useful for determining the spacinqs without temperature or shrinkage effect computer analysis. Although the scope of this research is restricted to one story reinforced concrete frames, more general frames can be similarly modeled as discussed in this paper.

2. Theoretical Development of Expansion Joint Spacings

2.1 Expansion Joint Spacing-Strength Criterion

The structural code[7] reguires the ultimate strength of a reinforced concrete member subject to dead load(L_D), live load(L_L), and temperature load(L_T) to withstand the factored load combinations of

$$U = 0.75(1.4L_D+1.7L_L+1.5L_T) \qquad (1)$$

If proper spacings of expansion joints are provided in the building and the building is free from the negative effects of temperature and shrinkage on the structural performance, the members can be designed with the factored load combinations of dead and live loads without temperature-induced loads:

$$U = 1.4L_D+1.7L_L\,. \qquad (2)$$

The above two equations (Eq.(1) and Eq.(2)) state that the following relationship to be true if a member can be designed without consideration of the temperature effects:

$$1.4L_D+1.7L_L \geq 0.75(1.4L_D+1.7L_L+1.5L_T) \qquad (3a)$$

or

$$L_T \leq 0.311L_D+0.378L_L \qquad (3b)$$

where,

L_T = internal forces at a member due to temperature changes.

Eqs.(3a)and (3b) imply that the load combinations including temperature effect (Eq.(1)) can be neglected as long as forces occuring at a section due to temperature change is less than or equal to the sum of fractional dead and live loads($0.311L_D+0.378L_L$).

$$\frac{L_T}{U} \leq \frac{0.311L_D + 0.378L_L}{1.4L_D + 1.7L_L} \approx 0.222 \quad (4)$$

Dividing Eq.(3b) by Eq.(2) leads to for most representative values of L_D and L_L. more specifically, if the temperature effects is neglected in the analysis, the internal forces induced by temperature changes need to be limited within $0.222M_u$ where M_u is the external moment by the load combinations of Eq.(2).

Shrinkage is a long term effect as concrete loses its moistures contained in the cementitious matrix. Shrinkage strain depends on various influential factors and can be expressed as the following equations according to the code[7]

$$\varepsilon_{sh}(t, t_s) = \varepsilon_{sho}\,\beta_s(t - t_s) \quad (5)$$

where,

$\varepsilon_{sh}(t, t_s)$ = nondimensional shrinkage strain [cm/cm] at time(t)

t = number of days after concrete casting

t_s = number of days exposed to exterior air after concrete casting.

In the above equation, ε_{sho} and $\beta_s(t - t_s)$ can be estimated by

$$\varepsilon_{sho} = \varepsilon_s(f_{cu})\,\beta_{RH} \quad (6)$$

$$\varepsilon_s(f_{cu}) = [160 + 10\,\beta_{sc}(9 - f_{cu}/100)] \times 10^{-6} \quad (7)$$

$$\beta_{RH} = \begin{cases} -1.55[1 - (RH/100)^3] & (40\% \leq RH \leq 99\%) \\ 0.25 & (RH \geq 99\%) \end{cases} \quad (8)$$

$$\beta_{sc} = \begin{cases} 4 : \text{type 2 cement} \\ 5 : \text{type 1 and 5 cement} \\ 6 : \text{type 3 cement} \end{cases}$$

If external temperature is different than 20℃, β_{RH} and $\beta_s(t - t_s)$ need to be modified as

$$\beta_{RH} = [1 + (\frac{8}{103 - RH})(\frac{T - 20}{40})] \times (Eq.(8))] \quad (9)$$

$$\beta_s(t - t_s) = \sqrt{\frac{(t - t_s)}{3.5\,h^2 \exp[-0.06(T - 20)] + (t - t_s)}} \quad (10)$$

where,

$f_{cu}(t)$ = compressive strength of concrete at time t (kgf/cm^2)

h = average thickness of the part of the member nuder consideration. (cm)

RH = relative humidity in percent(%)

T = exterior or curing temperature(℃).

From the above equations, shrinkage strain can be obtained if relative humidity, member dimensions, concrete strength, type of cements, and elapsed time after completion of a building construction are known. The shrinkage effect can be converted to the temperature effect by using the following equation

$$\alpha \cdot T_{sh} = \varepsilon_{sh}(t, t_s) \quad (11)$$

where,

α = coefficient of linear thermal expansion of concrete = 1.0 ×10-5(1/℃)

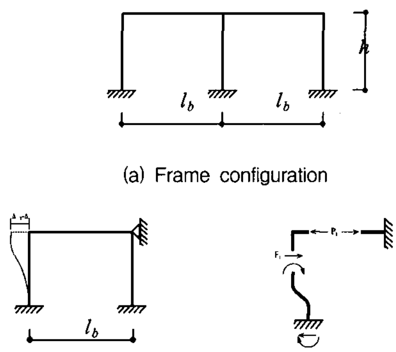
T_{sh} = equivalent temperature change due to shrinkage.

The generally accepted value of shrinkage strain varies between 0.0002 and 0.0007. The value of 0.00015 is used in this study as a representative shrinkage strain based on the concrete specification[8] that the shrinkage strain of 0.00015 can be used for statically indeterminate structures. If this value for shrinkage strain is adopted, the equivalent temperature change due to shrinkage can be found as $T_{sh} = 15\,^{\circ}C$. This equivalent temperature change T_{sh} can be added to atmospherical temperature change T_t and consequently the internal forces L_T in Eq(1) which can be regard as being induced by sum of those changes.

Therefore, the temperature effect term, should be regarded as a combinational effect of both temperature and shrinkage which is converted using Eq.(4).

Theoretical expressions for the expansion joint spacings are developed for one story frames in the subsequent sections. Since columns are subjected to shear forces and bending moments when they are being pushed outward or pulled inward by the elongation or shortening of the beams or slabs with the temperate changes, columns are considered as critical members[2]. The frames are modeled using the assumptions similar to the ones made for shear building, but beams and slabs are treated as flexible instead of infinitely rigid. From the analytical model, governing factors influencing internal flexural moment as well as lateral displacement subject to the temperature changes are found. These factors are then included in design equations with slight modification in the design charts as the main parameters.

i) One Story Two Bay Frame Model



(a) Frame configuration

(b) Displacement (c) Free body diagram

Fig. 1 One story two bay frame

Fig. 1(a) shows the one story, two bay frame subjected to a lateral displacement due to temperature changes and Fig. 1(b) illustrates the half of the frame using symmetry. Fig. 1(c) represents the free body diagram for the frame shown in Fig. 1(b). From Fig. 1(c), it can be shown that

$$P_1 = F_1 \tag{12a}$$

where,

$$P_1 = \frac{EA_b}{l_b}(\Delta_T - \Delta) \tag{12b}$$

$$F_1 = \frac{3EI_c}{h^3}\Delta \tag{12c}$$

Δ_T = maximum lateral displacement of the exterior column if restraining columns are absent

$= \dfrac{\alpha \cdot T \cdot L_j}{2n}$

n = number of spans

T = temperature changes including equivalent value converted from shrinkage effects

$= T_t + T_{sh}$

T_t = atmospherical temperature changes
T_{sh} = equivalent temperature changes due to shrinkage
I_c = column moment of inertia [cm^4]
A_b = sectional area of a beam[cm^2]
h = column height[cm]
l_b = beam length[cm].

Solving Eq.(12) for the maximum lateral displacement Δ, we can find internal moment M_T generated at the exterior column from the shear building assumption

$$\Delta = \frac{A_b h^3}{3I_c l_b + A_b h^3} \Delta_T \quad (13a)$$

$$M_c = \frac{3EI_c A_b h}{3I_c l_b + A_b h^3} \Delta_T \quad (13b)$$

ii) One Story 5 Bay Frame

Fig. 2(a) represent one story, 5 bay frame and Fig. 2(b) illustrates the model for this frame. Only a half of the frame is taken for the analysis. The free body diagram representing the frame of Fig. 2(b) is shown in Fig. 2(c) from which the following equilibrium equations can be obtained

$$P_1 = F_1 + P_2 \quad (14a)$$

$$P_2 = F_2 + P_3 \quad (14b)$$

$$P_3 = F_3 \quad (14c)$$

where,

$$P_1 = \frac{2EA_b}{l}(\Delta_T/2 - \Delta_1) \quad (14d)$$

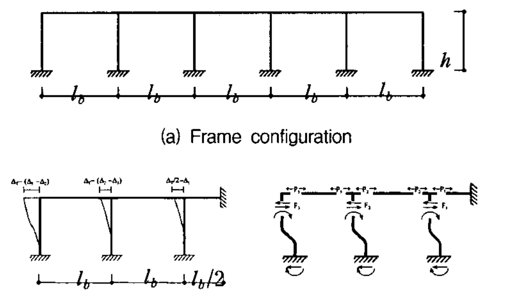
$$P_2 = \frac{EA_b}{l}(\Delta_T - (\Delta_2 - \Delta_1)) \quad (14e)$$

$$P_3 = \frac{EA_b}{l}(\Delta_T - (\Delta_3 - \Delta_2)) \quad (14f)$$

$$F_1 = \frac{3EI_c}{h^3}\Delta_1 \quad (14g)$$

$$F_2 = \frac{3EI_c}{h^3}\Delta_2 \quad (14h)$$

$$F_3 = \frac{3EI_c}{h^3}\Delta_3 \quad (14i)$$



(a) Frame configuration

(b) Displacement (c) Free body diagram

Fig. 2 One story 5 bay frame

By solving three equations (Eq.(14a) through (14c)) for three unknowns (Δ_1, Δ_2, and Δ_3) , the lateral displacement and moment at exterior column due to the temperature changes can be found as

$$\Delta = \frac{A_b h^3}{3I_c l_b + A_b h^3}\Delta_T \times \left[1 + \frac{1}{\left(\frac{3I_c l_b}{A_b h^3}\right)^3 + 6\left(\frac{3I_c l_b}{A_b h^3}\right)^2 + 9\left(\frac{3I_c l_b}{A_b h^3}\right) + 2}\right] \quad (15a)$$

$$M_T = \frac{3EI_c A_b h}{3I_c l_b + A_b h^3}\Delta_T \times \left[1 + \frac{1}{\left(\frac{3I_c l_b}{A_b h^3}\right)^3 + 6\left(\frac{3I_c l_b}{A_b h^3}\right)^2 + 9\left(\frac{3I_c l_b}{A_b h^3}\right) + 2}\right] \quad (15b)$$

It is important to note that Eq.(13) and Eq.(15) possess the common factors for lateral displacement and moment at exterior column. It can be shown that, as the number of spans increases, the above procedure of finding lateral displacement and moment at exterior column results in the following general expressions.

For the one story frame with n spans

$$\Delta = \frac{\alpha TL_j}{2n} \times \frac{A_b h^3}{3I_c l_b + A_b h^3}[1 + \frac{1}{f(k;n)}] \quad (16a)$$

$$M_T = \frac{\alpha TL_j}{2n} \times \frac{3EI_c A_b h}{3I_c l_b + A_b h^3}[1 + \frac{1}{f(k;n)}] \quad (16b)$$

where,

$f(k;n)$ = function of $(\frac{3I_c l_b}{A_b h^3})$ and the number of span.

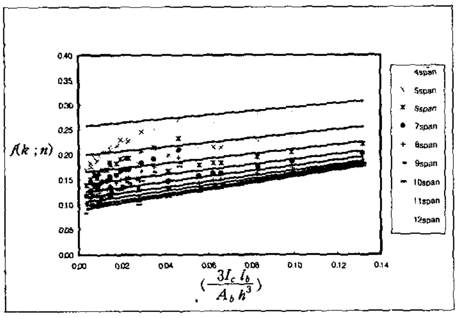


Fig. 3 Regression results for f(k;n)

The function $f(k;n)$ is determined by interpolating approximate estimates from Eq.(16) with exact results from a finite element based computer analysis. Fig. 3 represents $f(k;n)$ fitted with exact results from computer analyses for various types of one story frames. Coefficients of fitted linear equations with exact results are tabulated in Table 1. Fig.4 illustrates the comparisons between M_T values predicted by Eq.(16b) and corresponding values from exact analyses.

Table 1 Regression coefficients for f(k;n)

Span Number	$f(k;n) = (\frac{3I_c l_b}{A_b h^3})a + b$	
	a	b
4 Span	0.6557	0.2552
5 Span	0.6330	0.1975
6 Span	0.6319	0.1632
7 Span	0.6550	0.1397
8 Span	0.6767	0.1235
9 Span	0.6969	0.1119
10 Span	0.7242	0.1027
11 Span	0.7425	0.0958
12 Span	0.7693	0.0903

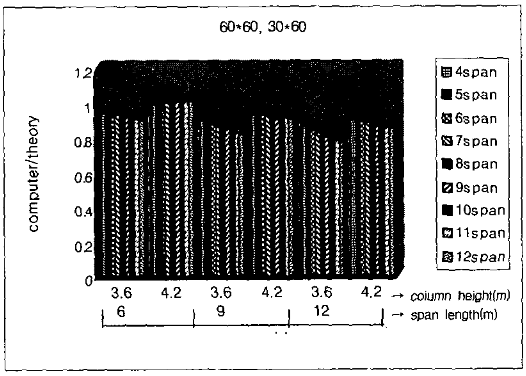


Fig. 4 Comparisons of M_T values with computer analysis

Maximum 20% relative differences are observed but mostly the relative differences are bound within 10%.

If proper expansion joints are used, the bending moment induced by temperature changes is less than or equal to 2.22% of the moment by load combinations of $1.4L_D + 1.7L_L$ as shown in Eq.(4)

$$M_T \leq 0.222 M_u \quad (17a)$$

Substituting Eq.(16b) for M_T above, we get

$$\frac{\alpha TL_j A_b h^3}{2n(3I_c l_b + A_b h^3)}[1 + \frac{1}{f(k;n)}] \leq 0.222M_u. \quad (17b)$$

Solving for expansion joint spacing L_j, we obtain

$$L_j \le 0.444n\frac{K \cdot M_u}{\alpha \cdot E \cdot T}\left[\frac{f(k;n)}{1+f(k;n)}\right]\ (m) \quad (18)$$

where,

$$K = \frac{h^2}{3I_c} + \frac{l_b}{A_b h}\ [1/\mathrm{cm}^2];$$

M_u = moment at exterior column by $1.4L_D + 1.7L_L$ [t · m]

E = concrete modulus of elasticity [kgf/cm^2]

$T = T_t + T_{sh}$ [℃]

$T_t = \beta \Delta T$ [℃]

β = 1.0 for buildings not provide with temperature control, 0.70 for buildings with temperature control and without air conditioning, 0.55 for without both temperature control and air conditioning.

2.2 Expansion Joint Spacing-Serviceability Criterion

In order to prevent curtain walls from forming undesirable cracks by the lateral movement of structural members due to temperature changes, maximum allowable displacement $h/300$ suggested in BJERRUM code[4] is adopted in this study. The three hundredth of the story height(h) as the maximum limit for lateral displacement is based the assumption that a curtain wall may start cracking at and beyond this limit value. Introducing this limit to the Eq.16(a), we have

$$\frac{\alpha T L_j A_b h^3}{2n(3I_c l_b + A_b h^3)}\left[1+\frac{1}{f(k,n)}\right] \le \frac{h}{300} \quad (19)$$

Solving for L_j, we get

$$L_j \le \frac{n \cdot K'}{150\alpha \cdot T}\left[\frac{f(k,n)}{1+f(k,n)}\right]\ (m) \quad (20)$$

where,

$$K' = \frac{3I_c\, l_b}{h^2 A_b} + h.$$

3. Design Aids for Expansion Joint Spacings

Design aids are developed for the simple application of finding expansion joint spacings for structural designers. From the previous theoretical development, the spacings of expansion joint can be represented by the following two equations

strength criterion:

$$L_j \le 0.444n\frac{K \cdot M_u}{\alpha \cdot E \cdot T}\left[\frac{f(k;n)}{1+f(k;n)}\right]\ (m) \quad (18)$$

where,

$$K = \frac{h^2}{3I_c} + \frac{l_b}{A_b h}$$

serviceability criterion:

$$L_j \le \frac{n \cdot K'}{150\alpha \cdot T}\left[\frac{f(k,n)}{1+f(k,n)}\right]\ (m) \quad (20)$$

where,

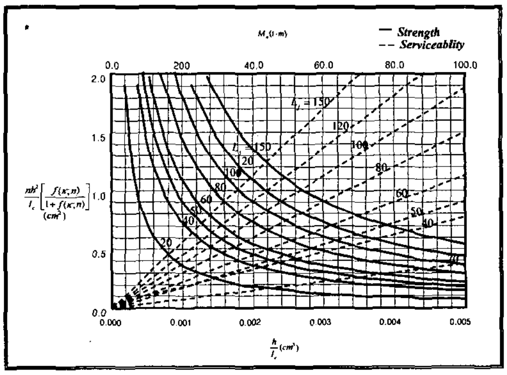
$$K' = \frac{3I_c\, l_b}{h^2 A_b} + h(1/\mathrm{cm}^2)$$

In Eq(18), $\alpha = 1.0\times10^5$, $E = 15{,}000\sqrt{f_{ck}}$ and ΔT are known quantities if the material properties and the regional area of a building location are given. The independent parameters are, K, M_u, n, and $f(k;n)$. $f(k;n)$ can be found from Fig. 3 for the given number of spans(n). For the case of K in Eq.(18), the second term($\frac{l_b}{A_b h}$) is much less than the first term($\frac{h^2}{3I_c}$) in most cases and is dropped in the subsequent development of design aids. This simplification slightly alters the expansion joint spacings and the expansion joints spacings(L_j) drawn on the graph with an externally applied moment as an abscissa and $\frac{nh^2}{I_c}\left[\frac{f(k\,;n)}{1+f(k\,;n)}\right]$ as an ordinate.

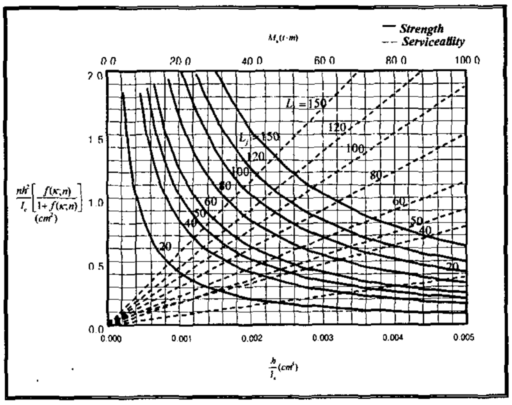
The expansion joint spacings from serviceability criterion (Eq.(20)) are dependent on $f(k\,;n)$, K', and n for given α and T. K' can be expressed as

$$K' = \left(\frac{h^2}{I_c}\right)\cdot\left[\frac{3\,I_c^{\,2}l_b}{A_b h^4}+\frac{I_c}{h}\right] \qquad (21)$$

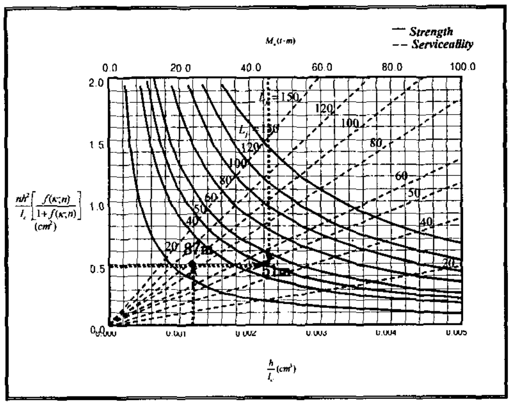
In the above equation, the first term in the bracket $\frac{3\,I_c^{\,2}l_b}{A_b h^4}$ is usually much less than the second term $\frac{I_c}{h}$. Therefore, if can be dropped without a loss of any significance. The expansion joint spacings on the basis of serviceability criterion can, therefore, be drawn overlapped on the coordinate system for the strength design criterion if $\frac{I_c}{h}$ is taken as an abscissa and $\frac{nh^2}{I_c}\left[\frac{f(k\,;n)}{1+f(k\,;n)}\right]$ as an ordinate. Fig. 5 shows some of typical design aids developed in this study. Note that each design aid is developed for different concrete strength and temperature change.



(a) $T=25°C.,\ f_{ck} = 210kgf/cm^2$



(b) $T=25°C.,\ f_{ck} = 270kgf/cm^2$



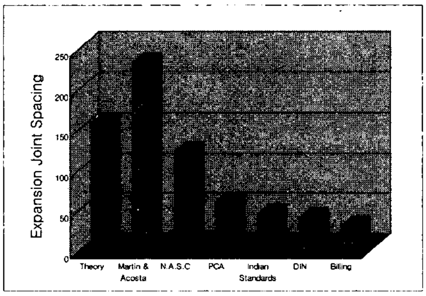
(c) $T=30°C,\ f_{ck} = 210kgf/cm^2$

Fig. 5 Some examples of design aids for expansion joints

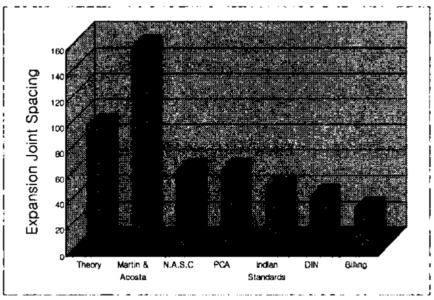
4. Comparison Between Different Provisions on Expansion Joint Spacings

The developed equations for the expansion joint spacing are compared with different code equations and expressions from other researchers. Table 2 summarizes the frame used for this comparison. Fig. 6(a) through

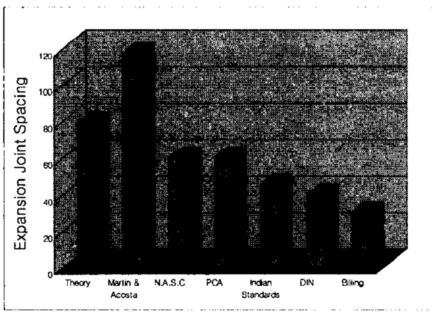
Fig. 6(c) show that for different temperature changes, except Martin and Acosta equation[1], the developed equation gives larger spacings for expansion joint. It is worth mentioning that all the expansion joint spacings from PCA[10], Indian Standards[6], DIN[9], and Billing[5] do not consider in their equations those factors of temperature changes, sectional dimensions and geometric properties of the members, concrete strength, and moments by external loads.



(a) T = 20℃



(b) T = 30℃



(c) T = 40℃

Fig. 6 Comparisons of expansion joint spacings with different codes or equations

Table 2 Sectional and geometrical properties of the frame used for comparison in Fig. 6

Items		Dimensions
Column	Sectional dimensions	45x45cm
	height	420cm
	Moment of inertia	341,719 cm^4
Beam	Sectional dimensions	30x50cm
	Sectional area	1,500cm^2
span		1,200cm

NASC[2] takes into account the building materials, geometrical configurations of a building, and temperature changes, but all other important parameters such as beam and column sectional dimensions, their geometrical properties, and the number of spans are omitted. The equation suggested by Martin and Acosta includes most of influential parameters except the number of spans compared to the developed equation.

5. Illustrative Example

A simple example for the use of design aids is given in this section. The frame considered in this example has one story reinforced frame with 8 equal spans. Sectional and geometrical characteristics of the frame are summarized in table 3. The frame is assumed to be made of concrete strength of $f_{ck} = 210 kgf/cm^2$ and is subject to uniform dead load of $D = 4.5\, tf/m$ and live load of $L = 2.0 tf/m$. Temperature change of $30^{o}C$ is expected. First, from Fig.3, $f(k,n)$ can be read as 0.13 following the evaluation of $\frac{3I_c l_b}{A_b h^3}$ =0.0083. Since the applied factored load combination of $1.4L_D + 1.7L_L$ results in 45 $tf \cdot m$ at the exterior column and, $n \cdot h^2/I_c \cdot [f(k,n)/(1 + f(k,n))] = 0.475\, cm^2$, we get L_j from the strength criterion equal to 51m from Fig. 5(c). For serviceability criterion, we evaluate $\frac{h}{I_c}$ equal to 0.0012 $[1/cm^3]$ and get L_j equal to 87m from Fig. 5(c). Since 51m is less than 87m, the frame needs expansion joint spacing less than or equal to 51m.

Table 3 Sectional and geometrical properties of the example frame

Items		Dimensions
Column	Sectional dimensions	45x45cm
	height	420cm
	Moment of inertia	341,719 cm^4
Beam	Sectional dimensions	30x50cm
	Sectional area	1,500cm^2
span		900cm

6. Conclusion

Since there are confusing code regulations regarding expansion joint spacings, a reasonable approach is needed to quantitatively evaluate expansion joint spacings.

Theoretical approaches are taken in this study to find the influential factors in determining expansion joint spacings. For the analytical modeling, assumptions similar to shear building are made with flexible lateral displacements allowed for beams and slabs. The expansion joint spacings determined based on strength criterion and serviceability criterion are then converted into design aids for the simple applications in practical structural designs. The following conclusions can be drawn from this study.

(1) The developed equation for the expansion joint spacing considers main parameters affecting strength and serviceability of a frame such as temperature changes, relative humidity, elapsed time after construction, concrete strengths, degree of shrinkage depending on the type of cements, sectional properties of beams and columns, and their length.

(2) According to the developed equation, the expansion joint spacing derived with respect to the strength criterion becomes smaller either for larger temperature changes or for increased $f(k;n)$ and becomes larger as column flexural strength increases.

(3) The values for the expansion joint spacings specified at different foreign codes are not similar and no expansion joint spacing is given in our domestic code. Compared with the values given from different codes and researchers, the developed spacing allowed larger values except those suggested by Martin and Acosta[1]. It seems that the values from the different codes are conservatively specified. Most of code provisions or expressions from other researchers also have a lack of influential factors in their expressions.

(4) The developed equation is converted to the design aids for the practical engineers. The developed design aids include the effects of external moment, sectional dimensions and lengths of beams and columns, number of spans, temperature changes, and concrete strengths.

(5) The developed equation as well as design aids for determining expansion joint spacings are for the one story buildings only and can only be used when designers attempt to structurally design the frame without considering temperature effects.

More exact analysis based on finite element method is needed if the building possesses irregular shapes and/or different spans.

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