균질화기법과 유한요소법을 이용한 복합재료의 등가탄성계수 산정

The Finite Element Analysis for Calculations of Equivalent Elastic Constants Using the Homogenization Method

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요 지

본 논문은 구조물의 미시적 측면에서 유효평균탄성계수를 결정하기 위한 균질화기법인 점근적 방법을 적용하였고, 탄성값을 조사하기 위하여 유한요소법으로 정식화하였다. 수치 예로서 물성치가 각기 다른 등방성 재료를 적층한 부재의 임의 단면에서 단위요소를 해석영역으로 설정하고 산출된 탄성계수를 기존의 해석방법으로부터 산출된 값과 비교하였다. 균질화기법으로 산출된 탄성계수는 과소평가되어 나타나며, 이는 해석영역을 유한요소 정식화하는 과정에서 수정항만큼 차이가 난다는 것을 증명하였다. 기존 해석방법으로는 복합재료의 탄성계수가 단순히 재료의 산술적 평균값으로 계산되는 것과는 달리, 미시적으로 복합재 단위요소의 반복성을 고려함으로써 제안된 해석방법이 보다 유용하다는 것을 보여 주었다.

핵심용어 : 균질화기법, 등가탄성계수, 유한요소정식화, 복합재료

Abstract

This paper discusses the homogenization method to determine effective average elastic constants of a linear structure by considering its microstructure. A detailed description on the homogenization method is given for the linear elastic material and then the finite element approximation is performed for an investigation of elastic properties. An asymptotic expansion is carried out in the cross-section area, or in the unit cell. Two and three lay-up structures made up of individual isotropic constituents are chosen for numerical examples to check discrepancies between results generated by this theoretical development and the conventional approach. Asymptotic characteristics of the process in extracting the stiffness of structure locally formed by spatial repetitions yield underestimated values of stiffness. These discrepancies are detected by the asymptotic corrective term which is ascribed to considerations of microscopic perturbations and proved in the finite element formulation. The asymptotic analysis is the more reasonable in analysing the composite material, rather than the conventional approach to calculate the macroscopic average for elastic properties.

Keywords : *homogenization method, equivalent elastic constants, finite element formulation, composite material*

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1. Introduction

Composite materials play an increasingly important role in industry. Their principal feature provides an excellent ratio of weight to strength suitable for a large variety of engineering applications. It is not simple to predict properties of the material including each microstructure because composites are inhomogeneous. One way of overcoming this difficulty is to find equivalent material model with no consideration of the microscopic mechanics for individual constituents. Such a model should represent the average mechanical behavior as well as the composite material heterogeneities.[1),2)]

Some researches have been done for derivations of eigenstrains of composite materials and determinations of their dependence on different components.[3),4)] Starting from the multiscale asymptotic expansion for the displacement and eigenstrain fields, a closed form expression is needed relating arbitrary eigenstrains to mechanical fields in phases. In homogenization theory the composite material can be assumed to be locally formed by periodic repetitions of very small microscopic cells compared with the overall macroscopic dimensions of the structure. Hence, material properties are periodic functions of the microscopic variable. The homogenization method provides a reasonable solution for some problems where the experimental data is not available or where bounds for equivalent material constants can be found by other theories.[5)] Some cases of the homogenization consider composite materials consisting of elastic media with periodic holes and rigid inclusions and fiber reinforced elastic materials. These may produce slipping between the fiber and the matrix at the slipping boundary with linear, nonlinear or viscous tangential forces.[6)] However, one dimensional expansion is the first essential step for checking the applicability and the generalization of the homogenization method to the three dimensional composite material.

The purpose of the present paper is to obtain the finite element solutions for equivalent material properties of the linear structure which consists of different isotropic materials stacked only in the transverse direction. In doing so, the homogenization method is mathematically formulated and discussed in terms of the finite element approach. Numerical experiments are also performed for a study of its solution accuracy and influence in the overall average solution from a viewpoint of anisotropic structure.

2. Elasticity Homogenization

2.1 Structure Configuration

Consider a structure formed by the spatial repetition in x_2 direction on a base cell made of different materials in the Fig. 1. The cross-section of the structure is represented by a base cell that is very small, of order ε compared with the dimension of the structural body. If the body is subjected to external loads and boundary conditions, the resulting deformations

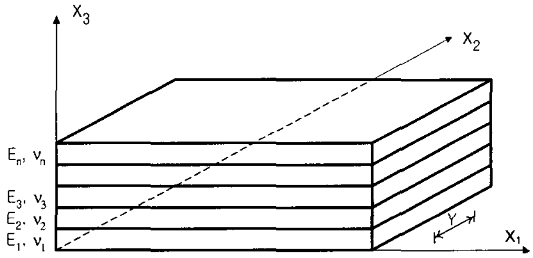

Fig. 1 The configuration of the structure consisting of different isotropic materials stacked in the transverse direction and its unit cell

and stresses rapidly vary from point to point because of repetition of microscopic base cells producing heterogeneity. In other words, these quantities rapidly vary within a small neighborhood ε of a given point x_i(i=1, 2, 3). Thus, it is reasonable to claim that all quantities have two explicit dependences. One is dependent on the macroscopic level x_i, and the other is on the microscopic level x_j/ε, i. e., letting g be a general function, $g=(x_i, x_j/\varepsilon)$. Due to the periodic nature of the microstructure, the dependence of a function on the microscopic variable $y_j=x_j/\varepsilon$ is also periodic.[3]

2.2 Asymptotic Expansions in x_2 Direction

2.2.1 Strain Field

If different isotropic materials are stacked in the transverse direction of the structure, the spatial repetition in the very small neighborhood ε can be considered in the cross-sectional area as well as in one longitudinal direction. The dependence of the displacement u on both macroscopic and microscopic level makes it reasonable to assume that the displacement u can be expressed as an asymptotic expansion with respect to the parameter ε, i. e.,[4]

$$u(x_1, y_2) = u^0(x_1, y_2) + \varepsilon^1 u^1(x_1, y_2) + \varepsilon^2 u^2(x_1, y_2) + \ldots.. \quad (1)$$

A differential operator can be defined as follows:

$$\frac{d}{dx_k} = \partial x_k + \frac{1}{\varepsilon}\partial y_k, \quad (2a)$$

where

$$\partial x_k = \delta_{1k}\frac{\partial}{\partial x_1}, \quad (2b)$$

$$\partial y_k = \delta_{2k}\frac{\partial}{\partial y_2}. \quad (2c)$$

Kronecker delta, δ_{1k} and δ_{2k} (i. e., $\delta_{ik}=1$, $i=k$), is used and the subscript $k(=1, 2)$ denotes a summation index for a asymptotic expansion in terms of the displacement. Thus a symmetric differential operator can expressed in the next equation (3a) by using the differential operator (2a, b, c).

$$e_{ij}(u) = \frac{1}{2}\left(\frac{du_i}{dx_j} + \frac{du_j}{dx_i}\right) = e_{ijx}(u) + \frac{1}{\varepsilon}e_{ijy}(u), \quad (3a)$$

where

$$e_{ijx}(u) = \frac{1}{2}\left(\delta_{1j}\frac{du_i}{dx_1} + \delta_{1i}\frac{du_j}{dx_1}\right) \quad (3b)$$

and

$$e_{ijy}(u) = \frac{1}{2}\left(\delta_{2j}\frac{du_i}{dy_2} + \delta_{2i}\frac{du_j}{dy_2}\right). \quad (3c)$$

By further performing the expansion with equations (1) and (3a), the asymptotic expansion of the strain field is of the following form:

$$e_{ij}(u) = \varepsilon^{-1} e_{ij}^{-1} + \varepsilon^0 e_{ij}^0 + \varepsilon^1 e_{ij}^1 + \ldots, \quad (4a)$$

where

$$e_{ij}^{-1} = e_{ijy}(u^0), \quad (4b)$$

$$e_{ij}^0 = e_{ijx}(u^0) + e_{ijy}(u^1), \quad (4c)$$

and

$$e_{ij}^1 = e_{ijx}(u^1) + e_{ijy}(u^2). \quad (4d)$$

2.2.2 Stress Field

By assuming that the quantity $D_{ijmn}(y_2)$ is the elastic constant in the x_2 direction, the resulting asymptotic expansion for stress is given by

$$\sigma_{ij}(u) = D_{ijmn}(y_2)\, e_{mn}(u). \quad (5)$$

Therefore, the substitution of equation (4a) into equation (5) results in

$$\sigma_{ij}(u)=\varepsilon^{-1}\ \sigma_{ij}^{-1}+\varepsilon^{0}\ \sigma_{ij}^{0}+\varepsilon^{1}\ \sigma_{ij}^{1}+\ldots, \tag{6a}$$

where

$$\sigma_{ij}^{-1}=D_{ijmn}(y_2)\ e_{mn}^{-1}, \tag{6b}$$

$$\sigma_{ij}^{0}=D_{ijmn}(y_2)\ e_{mn}^{0}, \tag{6c}$$

and

$$\sigma_{ij}^{1}=D_{ijmn}(y_2)\ e_{mn}^{1}. \tag{6d}$$

2.2.3 Equilibrium equation

The basic equilibrium equation is of the following form:

$$\frac{d}{d x_k}(\sigma_{ik})+b_i=0, \tag{7}$$

where $b(x_k)$ is the internal body force per unit volume. The differential operator (2a, b, c) and the stress (6a) are applied to equation (7) and then the asymptotic expansion for displacement yields the asymptotic equilibrium equation given by

$$\begin{aligned}&(\partial x_k+\varepsilon^{-1}\partial y_k)(\varepsilon^{-1}\ \sigma_{ik}^{-1}+\varepsilon^{0}\ \sigma_{ik}^{0}+\varepsilon^{1}\ \sigma_{ik}^{1}+..)\\&=\varepsilon^{-2}\frac{\partial\ \sigma_{ik}^{-1}}{\partial y_k}+\varepsilon^{-1}\left(\frac{\partial\ \sigma_{ik}^{0}}{\partial y_k}+\frac{\partial\ \sigma_{ik}^{-1}}{\partial x_k}\right)\\&+\varepsilon^{0}\left(\frac{\partial\ \sigma_{ik}^{0}}{\partial x_k}+\frac{\partial\ \sigma_{ik}^{1}}{\partial y_k}+b_i\right)+\ldots\\&=0.\end{aligned} \tag{8}$$

According to the perturbation theory, coefficients of each power of ε in governing equations must be equal to zero and thus satisfy the followings for each order:

$O(\varepsilon^{-2})$:

This case also holds for a general case k. The coefficient is given by

$$\frac{\partial\ \sigma_{ik}^{-1}}{\partial y_k}=0. \tag{9}$$

The equation (9) has an analogous form to a general equilibrium equation with no term for the body force and can be used to express the strain energy which is represented by the product of strain and stress over the total control volume. In the use of equations (4b) and (6b), the corresponding potential energy is obtained by

$$\begin{aligned}\Pi_{\varepsilon^{-2}}&=\int_0^Y\frac{1}{2}\ e_{mn}^{-1}\ \sigma_{ikmn}^{-1}d y_k\\&=\int_0^Y\frac{1}{2}D_{ijmn}(y_k)\left[e_{mny}(u^0)\right]^2 d y_k.\end{aligned} \tag{10}$$

When the first variation is applied in the equation (10) so as to find out a stationary state, the equation (10) is expressed by

$$\delta\Pi_{\varepsilon^{-2}}=\int_0^Y e_{mny}(u^0)\,D_{ijmn}(y_k)\delta e_{mny}(u^0)d y_k, \tag{11}$$

where $D_{ijmn}(y_k)$ is the symmetric and positive definite matrix, $\delta_{emny}(u^0)$ is arbitrary and thus the equation (11) results in

$$e_{mny}(u^0)=0. \tag{12}$$

This implies that u^0 is independent of the coordinate y_k and a function of only x_k, in other words,

$$u^0=u^0(x_k). \tag{13}$$

In equations (4b) and (6b), the stress σ_{ik}^{-1} shows a sole dependence on y_k and leads to

$$\sigma_{ik}^{-1}=0. \tag{14}$$

$O(\varepsilon^{-1})$:

$$\frac{\partial \sigma_{ik}^{0}}{\partial y_k}+\frac{\partial \sigma_{ik}^{-1}}{\partial x_k}=0. \tag{15}$$

The second term of the equation (15) vanishes due to the equation (14). Therefore, the equation (15) becomes the following with an aid of equations (4c) and (6c):

$$\frac{\partial}{\partial y_k} D_{ijmn}[\, e_{mnx}(u^0)+e_{mny}(u^1)]=0. \tag{16}$$

If the first perturbation term $u_1(x_k,\ y_k)$ can be rewritten via the introduction of variable separations and equation (3b), the displacement u_1 and its associated strain e_{mny} becomes the followings, respectively,

$$u^1(x_k,\ y_k)=H(y_k)\frac{\partial u^0}{\partial x_k} \tag{17}$$

and

$$e_{mny}(u^1)=\frac{\partial H}{\partial y_k}\, e_{mnx}(u^0). \tag{18}$$

The substitution of the equation (18) into (16) yields

$$e_{mnx}(u^0)\frac{\partial}{\partial y_k} D_{ikmn}(\frac{\partial H}{\partial y_k}+1)=0, \tag{19}$$

where the elastic modulus D_{ikmn} is not zero, symmetric and positive, and then the new elastic modulus, or equivalent material stiffness, $\tilde{D}$, is defined by

$$\tilde{D}= D_{ikmn}(\frac{\partial H}{\partial y_k}+1)=constant. \tag{20}$$

Consequently, the equation (15) is reduced to

$$\frac{\partial \sigma_{ik}^{0}}{\partial y_k}= e_{mnx}(u^0)\frac{\partial}{\partial y_k} D_{ikmn}(\frac{\partial H}{\partial y_k}+1)=0. \tag{21}$$

$O(\varepsilon^{0})$:

$$\frac{\partial \sigma_{ik}^{0}}{\partial x_k}+\frac{\partial \sigma_{ik}^{1}}{\partial y_k}+b_i=0. \tag{22}$$

By using equations (4d), (6d) and (21), the equation (22) is of the form

$$\frac{\partial e_{mnx}(u^0)}{\partial x_k}\Big[D_{ikmn}(\frac{\partial H}{\partial y_k}+1)\Big]+b_i + \frac{\partial}{\partial y_k}[\, D_{ikmn}\{ e_{mnx}(u^1)+e_{mny}(u^2)\}]=0. \tag{23}$$

The integration of the equation (23) with respect to y_k along with recalling the equation (2.c) and the displacement u_0 independent of y_k results into

$$\frac{\partial^2 u^0}{\partial x_k^2}\int_0^Y\Big[D_{ikmn}(\frac{\partial H}{\partial y_k}+1)\Big]dy_k+b_iY +[\, D_{ikmn}\{ e_{mnx}(u^1)+e_{mny}(u^2)\}]_0^Y=0. \tag{24}$$

The third term of the equation (24) vanishes due to the periodicity, equal on opposite sides of the unit cell. Accordingly, the equation (24) is reduced to the general form of the equilibrium equation,

$$\frac{\partial^2 u^0}{\partial x_k^2}\frac{1}{Y}\int_0^Y\Big[D_{ikmn}(\frac{\partial H}{\partial y_k}+1)\Big]dy_k+b_i=0, \tag{25}$$

or

$$\tilde{D}\frac{\partial^2 u^0}{\partial x_k^2}+b_i=0, \tag{26}$$

which is also used for computing residual stresses in the macroscopic homogenized problem.

3. Implementation of Homogenization in Finite Element Program

3.1 Finite Element Approach

The equation (19) implies a general form of the equilibrium equation in terms of strain, $\frac{\partial H}{\partial y_k}+1$, with the displacement $e_{mnx}(u^0)$ cancelled out. The similar reasoning in the derivation of the equation (10) can be applied for the derivation of the total potential energy which is represented by[7)]

$$\Pi_{\varepsilon^{-1}} = \int_0^Y \frac{1}{2} D_{ijmn}(y_k)\left(\frac{\partial H(y_k)}{\partial y_k}+1\right)^2 dy_k. \tag{27}$$

If the domain in the unit cell is subdivided into elements, the equation (27) is given by

$$\Pi_{\varepsilon^{-1}} = \sum_{e=1}^{n_{el}} \int_{-1}^{1} \frac{1}{2} D_{ijmn}{}^e\left(\frac{\partial H}{\partial y_k}+1\right)^2 |J| d\xi_k, \tag{28}$$

where the subscript k is equal to 2 for the present problem, the superscript e for summations is the element number, n_{el} the total number of elements, ξ the natural coordinate for an element over the interval [-1, 1], and $|J|$ the Jacobian of the transformation for an element over the interval [0, $\frac{1}{n_{el}}$], in which the dimension Y is selected as the unity because of the domain over the unit cell.

The class of admissible functions for all $H(y_k)$ has continuous derivatives through the first derivative, i. e., C^1 functions, as well as satisfies the essential boundary conditions. Next, a set of degree of freedom for an element $\boldsymbol{d}^e$ is defined and the corresponding displacement is interpolated within the element in terms of $\boldsymbol{d}^e$ such as

$$H(\xi) = \sum_{a=1}^{n_{ed}} N_a(\xi)\, d_a^e = \boldsymbol{N}\,\boldsymbol{d}^e, \tag{29}$$

where n_{ed} is the number of element degree of freedom, $N_a(\xi)$ the shape function associated with degree of freedom d_a^e and the bold face the matrix. The derivative of the displacement $H(\xi)$ allows for the change in variable calculated by

$$\frac{\partial H}{\partial y_k} = \frac{1}{|J|}\frac{\partial N}{\partial \xi}\boldsymbol{d}^e = \boldsymbol{B}^e\,\boldsymbol{d}^e. \tag{30}$$

A set of the global degree of freedom $\boldsymbol{d}$ can be defined for the unit cell continuum and the element degree of freedom $\boldsymbol{d}^e$ is related to $\boldsymbol{d}$ through the Boolean matrix, or the assembly operator $\boldsymbol{A}^e$,

$$\boldsymbol{d}^e = \boldsymbol{A}^e\,\boldsymbol{d}. \tag{31}$$

By substituting the equation (31) into (30), the equation (28) is rewritten as

$$\Pi_{\varepsilon^{-1}} = \sum_{e=1}^{n_{el}} \int_{-1}^{1} \frac{1}{2} \boldsymbol{D}^e\,(\boldsymbol{B}^e\,\boldsymbol{A}^e\,\boldsymbol{d}+1)^2 |J| d\xi_k. \tag{32}$$

The stationary point of the equation (32) is given by taking the first variation,

$$\delta\Pi_{\varepsilon^{-1}} = \left[\sum_{e=1}^{n_{el}} \int_{-1}^{1} |J|\,\boldsymbol{d}^T\,\boldsymbol{A}^{e^T}\,\boldsymbol{B}^{e^T}\,\boldsymbol{D}^e\,\boldsymbol{B}^e\,\boldsymbol{A}^e\,d d\xi_k + \sum_{e=1}^{n_{el}} \int_{-1}^{1} |J|\,\boldsymbol{D}^e\,\boldsymbol{B}^e\,\boldsymbol{A}^e\,d d\xi_k\right]\delta\boldsymbol{d} = 0. \tag{33}$$

The variational displacement $\delta\boldsymbol{d}$ is arbitrary, the bracketed term must be zero. If the assembled stiffness matrix $\boldsymbol{k}$ and the assembled force vector $\boldsymbol{f}$ is denoted by the following, respectively, then the equation (33) has a weak form for the finite element approach.[7)]

$$\boldsymbol{k}\boldsymbol{d} = \boldsymbol{f}, \tag{34}$$

where

$$k= \sum_{e=1}^{n_d} \int_{-1}^{1} |J| \, d^T \; A^{e^T} \; B^{e^T} \, D^e \, B^e \, A^e \, dd\xi_k \tag{35}$$

and

$$f=- \sum_{e=1}^{n_d} \int_{-1}^{1} |J| \, D^e \; B^e \; A^e \; dd\xi_k. \tag{36}$$

For specification of boundary conditions, displacements are fixed at four vertices A, B, C and D in the Fig. 2, i. e., $u_{A,\,B,\,C,\,D} = 0$; displacements are all the same along AB and CD, i. e., $u_i = u_{AB} = u_{CD}$; tractions are free along AC and BC, i. e., $f_{AC} = f_{BD} = 0$. Imposing essential boundary conditions at nodal displacements and natural boundary conditions at nodal forces lead to the global constraint operation which modifies the stiffness matrix from $\boldsymbol{k}$ to $\boldsymbol{K}^H$, the force vector from $\boldsymbol{f}$ to $\boldsymbol{F}^H$ and the displacement vector from $\boldsymbol{d}$ to $\boldsymbol{d}^H$. The equation (34) changes in the form of

$$\boldsymbol{K}^H \; \boldsymbol{d}^H = \boldsymbol{F}^H \tag{37}$$

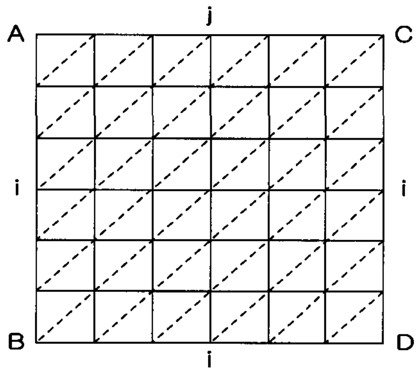

Fig. 2 The configuration of boundary conditions imposed in the unit cell and element grids for square elements and triangle elements of which the dotted-line stands for applications to triangular ones

Equivalent material properties can be derived from the homogenization theory. It can be approximated from the equation (25),

$$\tilde{D} = \int_0^Y D d\, y_k + \int_0^Y D \frac{\partial H}{\partial\, y_k} d\, y_k. \tag{38}$$

The first term of the equation (38) represents the total sum of individual elastic modulus occupied in the unit cell, while the second term corresponds to the force vector $-\boldsymbol{F}^H$ in the equation (37). Accordingly, the homogenized elastic constant is obtained by

$$\tilde{D} = \sum_{i=1}^{n} V_i \, D_i - \boldsymbol{K}^H \; \boldsymbol{d}^H. \tag{39}$$

where V_i is the volume fraction, n the number of constituents, D_i its related stiffness and $\boldsymbol{K}^H$ is the symmetric and positive definite matrix. Thus the second term plays a role of the external energy in a unit cell.

3.2 Pseudo Code

The program is developed to calculate material constants for the finite element analysis based on the homogenized method described above. The basic structure of the program is in Table 1.

Table 1 The pseudo code for the finite element analysis using the homogenized method

```
call preprocessor
call structure (basic cell)
call node (topology)
call element (no. of elements)
call material ( E1, ν1, …, En , νn )
call boundary_condition
call processor
call Gauss_point
call bandwidth
call element_shape (square 4 or 8 nodes,
                    triangle 3 or 6 nodes)
call element_matrix
call constraint (periodic displacement or free traction)
call assembly_matrix
call stiffness
call solve
call postprocessor
call constituent (volume fraction)
call material_constant (homogenized material constants)
```

4. Numerical Experiment

4.1 The Conventional Approach

To compare the homogenized elastic constants with those computed by the conventional approach, micromechanics for this method is based on the properties of the constituent materials.[8] The key feature of the procedure is that assumptions are made with regard to the mechanical behavior of a composite material. The first and the second modulus $\overline{E}_1$ and $\overline{E}_2$, shear modulus $\overline{G}$, major Possion's ratio $\overline{\nu}_{12}$ can be derived as follows:

$$\overline{E}_1 = \sum E_i\ V_i, \tag{40}$$

$$\overline{E}_2 = \frac{1}{\sum \frac{V_i}{E_i}}, \tag{41}$$

$$\overline{\nu}_{12} = \sum V_i\ \nu_i \tag{42}$$

and

$$\overline{G}_{12} = \frac{1}{\sum \frac{V_i}{G_i}}, \tag{43}$$

where E_i is Young's modulus, ν_i Possion's ratio of material i and V_i the volume fraction. The stiffness matrix form is of the form below,

$$\begin{bmatrix} \frac{\overline{E}_1}{1-\overline{\nu}_{12}\,\overline{\nu}_{21}} & \frac{\overline{\nu}_{12}\,\overline{E}_2}{1-\overline{\nu}_{12}\,\overline{\nu}_{21}} & 0 \\ & \frac{\overline{E}_2}{1-\overline{\nu}_{12}\,\overline{\nu}_{21}} & 0 \\ SYM & & \overline{G}_{12} \end{bmatrix} \tag{44}$$

4.2 Numerical Experiments

The material data used for a computational experiments are given in the Table 2. If the axis x_2 and the axis x_3 in the Fig. 1 are replaced simply with the axis 1 and the axis 2, respectively, elastic constants to be evaluated correspond to stiffnesses C_{11}, C_{22}, C_{33} and C_{12}. Table 3 shows a representative comparison of results by the finite element analysis and by the conventional method using 4-node isoparametric square elements. As shown in Fig. 2 other numerical experiments are performed using 8-node isoparametric square elements, and 3-node and 6-node isoparameteric triangle elements.

Table 2 Material data used in the numerical experiment

Material	Elastic Modulus E_1, E_2 [Gpa]	Possion Ratio ν	Shear Modulus G_{12} [Gpa]
1: Aluminium	68.21	0.34	25.45
2: Low Alloy Steel	206.70	0.28	80.74
3: Copper	124.00	0.35	45.93

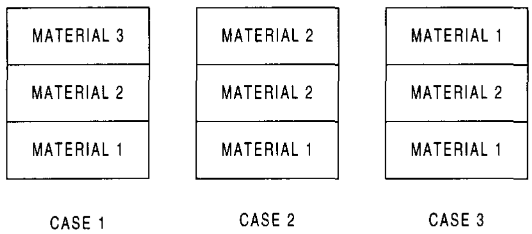

Fig. 3 Two and three lay-up structures for the finite element analysis

Table 3 Comparisons of stiffnesses calculated by the FEM using both the asymptotic approach and the existing approach

	C_{11} [Gpa]		C_{22} [Gpa]		C_{33} [Gpa]		C_{12} [Gpa]	
Case	Asymptotic	Existing	Asymptotic	Existing	Asymptotic	Existing	Asymptotic	Existing
1	143.45	145.45	39.14	122.09	27.28	40.86	36.24	38.52
2	168.81	172.46	44.85	132.43	31.35	46.78	26.25	39.76
3	124.10	124.16	30.94	95.70	21.77	33.00	25.42	30.52

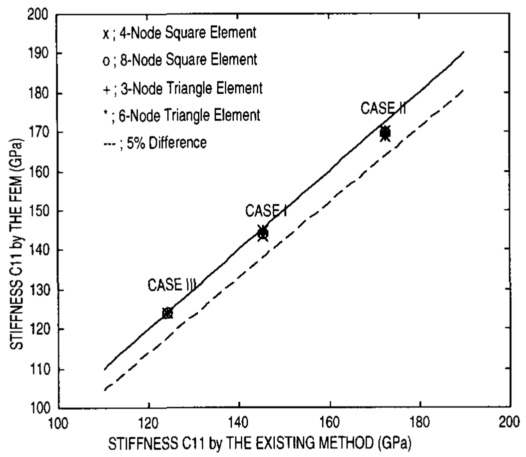

Fig. 4 Comparisons of values of stiffness C_{11} estimated by the finite element method using the asymptotic expansion and by the existing method. The solid line represents a perfect correlation between results from two methods

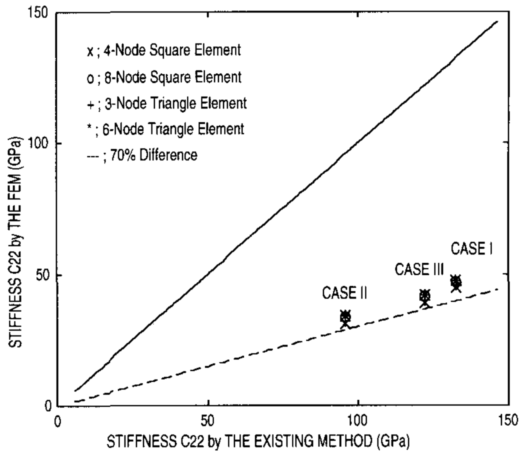

Fig. 5 Comparisons of values of stiffness C_{22} estimated by the finite element method using the asymptotic expansion and by the existing method. The solid line represents a perfect correlation between results from two methods

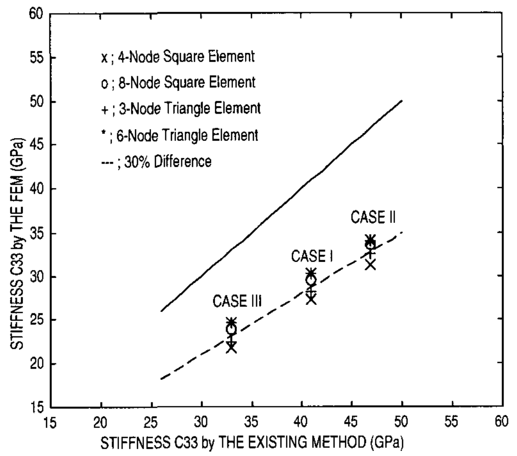

Fig. 6 Comparisons of values of stiffness C_{33} estimated by the finite element method using the asymptotic expansion and by the existing method. The solid line represents a perfect correlation between results from two methods

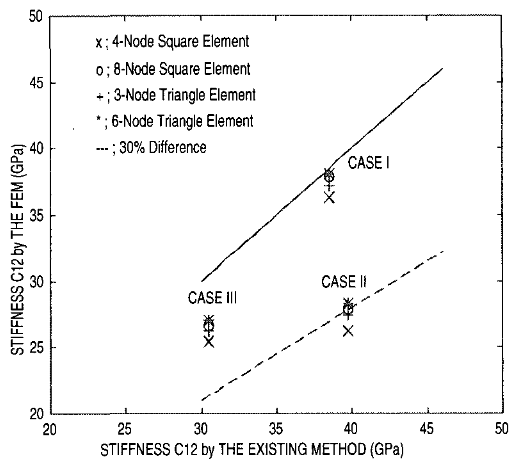

Fig. 7 Comparisons of values of stiffness C_{12} estimated by the finite element method using the asymptotic expansion and by the existing method. The solid line represents a perfect correlation between results from two methods

Three numerical samples are shown in Fig. 3 where each sample has an equivalent volume fraction to all three constituents in the cell.

A little difference is observed between values of C_{11} calculated by both approaches in Fig. 4. It is noted in all figures that the solid line represents a perfect correlation between results from the asymptotic analysis and the existing one, and the dashed a deviation from it. However, the large discrepancies are shown in Fig. 5 and Fig. 6 for values of C_{22} and C_{33} by up to 70% and 30%, respectively. Differences by maximum 30% are resulted in values of C_{12} in Fig. 7 where the stiffness in the case I closely match the value calculated by the conventional method. The value of C_{22} always remains less than that of C_{11} irrespective of the stacking order as well as the constituent from a viewpoint of anisotropic materials. The asymptotic approach with an adoption of up to the second order of strain has a dominant influence on elastic constituent equations. In other words, stiffness results from the conventional approach is based on an assumption that the strain behavior has an average value from a macroscopic viewpoint, whereas the asymptotic approach describes the detailed periodic behavior dependent on microscales in a very small neighborhood of the structure as well as macroscales. The asymptotic analysis is considered the more reasonable in analysing the heterogeneous material because it is locally formed by spatial repetitions. Such characteristics of the process in extracting the stiffness of interest directly yield differences in stiffness. These discrepancies are detected by the asymptotic corrective term which is observed in the finite element formulation. Consequently, values estimated by the asymptotic approach are less than those by the conventional approach which has been known to simply calculate elastic constants on an arithmetic average in composite materials.

5. Concluding Remarks

In this paper the homogenization method is applied for predictions of the equivalent material modulus in the structure stacked by isotropic different materials. The asymptotic approach is presently taken into account perturbations only in one direction depending on characteristics of the structure configuration. Equivalent material properties are computed based on the related finite element formulation where boundary conditions are imposed the same on opposite sides over the domain. Magnitudes of the elastic constant are observed much less than those resulted by the conventional approach. Their discrepancies are accurately estimated by a corrective term from the finite element formulation.

Three cases for numerical experiments are demonstrated for comparisons with the conventional approach in which elastic constants are calculated by the sum of individual fractional elastic constants. It is anticipated that the asymptotic approximation need to be expanded to the three dimensional problem encountered in a general composite material even if here the perturbation is considered in only one direction.

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