유한요소법을 이용한 정현상으로 taper진 부재의 고유치 산정

Determination of Eigenvalues of Sinusoidally Tapered Members by Finite Element Method

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요 지

양단이 단순지지 상태이고, 정현상(正弦狀)으로 taper진 부재의 두 고유치(탄성임계하중과 횡방향 고유진동수)를 유한요소법으로 산정하였다. 그 결과로 얻어진 고유치의 계수들을 실무에 종사하는 구조 기술자들이 쉽게 이용할 수 있도록 각각의 경우에 대하여 간단한 대수 방정식으로 표시하였다. 유한요소법으로 구한 고유치 계수값과 대수 방정식에 의한 추정 계수값들의 상관계수는 어느 경우에나 거의 단위치(1)에 가까운 값으로 대수 방정식의 정당성을 뒷받침 하였다. 횡방향 진동수에 미치는 압축하중의 영향도 검토하였다. 이를 위하여 부재에 작용하는 축방향력을 차례로 증가시키면서 여기에 대응하는 진동수를 산정하였다. 그 결과 압축하중비 (P/P_{cr})와 진동수비의 제곱 $((\omega/\omega_0)^2)$의 합은 어느 경우에나 거의 단위치(1)로 나타났다.

핵심용어 : 정현상으로 taper진 부재, taper 매개 변수, 단면 성질 매개 변수, 탄성 임계 하중, 횡방향 고유 진동수, 회귀분석

Abstract

The two eigenvalues (elastic critical load and natural frequency of lateral vibration) of sinusoidally tapered bars with simply supported ends were determined by the finite element method. For the convenience of structural engineers who are engaged in the structural design or vibration analysis of tapered beam-columns, eigenvalue coefficients were expressed by simple algebraic equations. The validity of each algebraic equation was confirmed by the value of unity for each correlation coefficient. The influence of axial thrust on the lateral vibration frequency was also investigated. For this purpose, the axial thrust was increased successively and the corresponding frequency was calculated. The approximate linear relationship between the axial thrust and the square of the frequency was confirmed for each of the tapered members.

Keywords : *sinusoidally tapered member, taper parameter, sectional property parameter, elastic critical load, natural frequency of lateral vibration, regression analysis*

1. Introduction

The vibrating tapered members under axial thrust are often seen in several fields of engineering problems. A tapered column under the ground motion or shock (impact) is such

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an example. In this case, the dynamic behavior of a member is governed by the two eigenvalues of that member, that is, the elastic critical load($=P_{cr}$) and the natural frequency of lateral vibration ($=\omega_0$). The two eigenvalues of a prismatic bar can be easily determined either by analytical methods or by numerical methods.

For the cases of tapered members(especially when the cases of sinusoidally tapered members), however, the determination of eigenvalues becomes possible only when one relies on the numerical methods. Furthermore, the axial force *vs* frequency relationship such as given by Eq. 1 is hard to determine because of several geometric factors, for example, the sectional property parameter and the taper parameter. (see Fig. 1) The first combination of (0,2) represents the case of a built-up member consisting of four angles connected by diagonals. The sectional properties about the strong axis of a rectangular member with variable thickness can be represented by (1,3). The combination of (2,4) represents the properties of a solid truncated cone or pyramid.

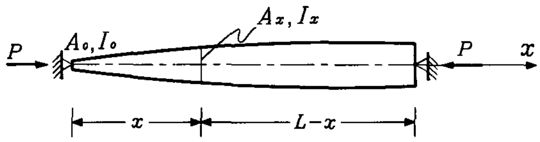
In this paper, the two eigenvalues of the simply supported tapered members shown in Fig. 1 were first determined by the finite element method. Next, the results were expressed by simple algebraic equations for the convenience of structural engineers. To investigate the relationship between axial thrust and frequency of a member, the axial force was successively increased from zero to the elastic critical load of that member. Next, the corresponding frequencies were calculated. It turned out that Eq. 1 is also applicable to the tapered members.

2. Formulation of the Problem

For the single prismatic members, the axial thrust, P and the reduced lateral vibration frequency, ω due to P, are generally related by the following equation. (see Fig. 5)[1]

$$\frac{P}{P_{cr}}+\left(\frac{\omega}{\omega_0}\right)^2=1.0 \qquad (1)$$

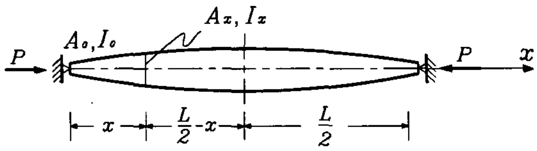
In Eq. 1, P_{cr} and ω_0 denote the elastic critical load and the natural frequency of lateral vibration, respectively. Experimental tests on rigid rectangular frames[1),2)] also showed that the relationship given by Eq. 1 is also applicable with minimal error.



$$A(x)=A_0\ (1+\alpha\sin\frac{\pi x}{2L})^m$$

$$I(x)=I_0\ \ (1+\alpha\sin\frac{\pi x}{2L})^n$$

(a) Non-symmetrically Tapered Member



$$A(x)=A_0\ (1+\alpha\sin\frac{\pi x}{L})^m$$

$$I(x)=I_0\ \ (1+\alpha\sin\frac{\pi x}{L})^n$$

(b) Symmetrically Tapered Member

taper parameter, α

: α=0.0, 0.1,⋯, 2.0

sectional property parameter, (m,n)

:(0,2), (1,3), (2,4)

Fig. 1 Sinusoidally tapered members

The two eigenvalues of a single prismatic member are easily determinated even by analytical methods. For the rectangular frames, however, only the numerical method[3),8)] makes the determination of eigenvalues possible. As stated earlier, this paper aims to prove that Eq. 1 is also applicable even in the cases of sinusoidally tapered members. (see Fig. 1) Here in this paper, the finite element method was adopted to determine the eigenvalues. The formulation of element matrices are outlined briefly in the following[4)~6)].

The flexural strain energy of the element is given by

$$U=\frac{1}{2}\int_0^l EI(x)\left(\frac{d^2v}{dx^2}\right)^2 dx \qquad (2\text{-a})$$

If the element is vibrating harmonically with frequency ω, the external work of the element is given by

$$W=\frac{\omega^2}{2}\int_0^l v^2\cdot\frac{\rho}{g}A(x)dx+\frac{P}{2}\int_0^l\left(\frac{dv}{dx}\right)^2dx + \frac{1}{2}\{\delta\}^T[k]\{\delta\} \qquad (2\text{-b})$$

where the first term denotes the work done by inertia force(ρ : unit weight of material). The remaining two terms are the work due to constant axial force P and nodal forces $\{q\}$ with $\{q\}=[k]\{\delta\}$ respectively.

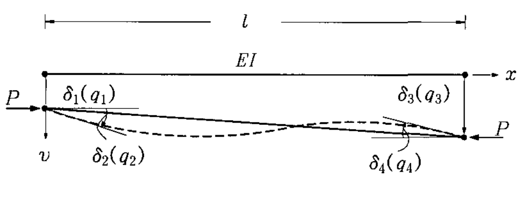


Fig. 2 Typical element with four degrees of freedom

The displacement function, $v(x)$ in Eq. 2 can be expressed by the following matrix form.

$$v=[N]\{\delta\},\quad [N]=[\,N_1\quad N_2\quad N_3\quad N_4\,] \qquad (3)$$

where $\{\delta\}$ is the nodal displacement vector and shape function component, N_i $(i=1,2,3,4)$ is given by

$$N_1=1-3\left(\frac{x}{l}\right)^2+2\left(\frac{x}{l}\right)^3$$

$$N_2=x\left(1-\frac{x}{l}\right)^2$$

$$N_3=3\left(\frac{x}{l}\right)^2-2\left(\frac{x}{l}\right)^3$$

$$N_4=\frac{x^2}{l}\left(\frac{x}{l}-1\right) \qquad (4)$$

Equating strain energy for the element to the external work and rearranging terms, one can obtains.

$$[k]=[k]_b-P[k]_g-\omega^2[m]_c \qquad (5)$$

The matrices appearing in Eq. 5 are given by[3),9)]

$[k]_b$ (=flexural stiffness matrix)

$$=\int_0^l EI(x)\left\{\frac{d^2N}{dx^2}\right\}^T\left\{\frac{d^2N}{dx^2}\right\}dx$$

$$=\frac{EI(e)}{l^3}\begin{bmatrix} 12 & & & symm. \\ 6l & 4l^2 & & \\ -12 & -6l & 12 & \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \qquad (6\text{-a})$$

$[k]_g$ (=geometric stiffness matrix)

$$=\int_0^l\left\{\frac{dN}{dx}\right\}^T\left\{\frac{dN}{dx}\right\}dx$$

$$=\frac{1}{30l}\begin{bmatrix} 36 & & & symm. \\ 3l & 4l^2 & & \\ -36 & -3l & 36 & \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \qquad (6\text{-b})$$

$[m]_c$ (=consistent mass matrix)

$$= \int_0^l \frac{\rho}{g} A(x)\{N\}^T\{N\}\,dx$$

$$= \frac{\rho A(e)l}{420g}\begin{bmatrix} 156 & & & symm. \\ 22l & 4l^2 & & \\ 54 & 13l & 156 & \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \quad (6\text{-c})$$

where $I(e)$ and $A(e)$ denote the values of $I(x)$ and $A(x)$ computed at mid-length of each element. The errors produced by these replacements are insignificant. For example, when the member of Fig. 1 (b) is subdivided into 20 equal elements (l=L/20), the exact $(k_{11})_b$ is given by $960\,EI_0/l^3$ when (m,n)=(2,4) and $\alpha=2.0$. Meanwhile, $(k_{11})_b$ is $964\,EI_0/l^3$ when $I(x)$ is replaced by $I(e)$. In the same way, the exact $(m_{11})_c$ is given by $3.309\,\rho A_0 l/g$ and $(m_{11})_c$ is $3.329\,\rho A_0 l/g$ when $A(x)$ is replaced by $A(e)$.

3. Eigenvalue Calculation

When the element matrices are assembled for the whole member, the external force vector, $\{Q\}$ and the external displacement, $\{\Delta\}$ are related by the following equation.

$$\{Q\}=([K]_b-P[K]_g-\omega^2[M]_c)\{\Delta\},\ \{Q\}=\{0\} \quad (7)$$

where $[K]_b$, $[K]_g$ and $[M]_c$ are the assembled matrices of whole member, respectively.

To obtain the critical load, one lets ω^2 be zero in Eq. 7. In the same way, one lets P be zero determine the natural frequency of lateral vibration. Due to the size of assembled matrices, eigenvalues were determined by computer-aided iteration method. The first eigenvalue calculation by iteration method became possible when Eq. 7. was transformed into the following form[3].

(Critical Load)

$$([K]_b^{-1}[K]_g-\frac{1}{P}[I])\{\Delta\}=\{0\} \quad (8\text{-a})$$

(Natural Frequency)

$$([K]_b^{-1}[M]_c-\frac{1}{\omega^2}[I])\{\Delta\}=\{0\} \quad (8\text{-b})$$

where $[I]$ is the unit matrix.

It is generally known that the results obtained by numerical methods converge to the exact values rapidly by mesh refinement. Fig. 3 shows some examples of convergency related to the present eigenvalues. As can be seen in these graphs, eigenvalues converge to certain values after N=16.

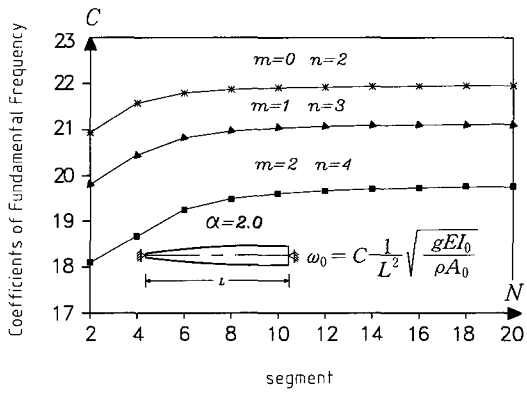


Fig. 3 Convergency of fundamental frequency

In this paper, tapered members were divided into 20 equal segments (N=20) and the eigenvalues were determined by the procedures given by Eq. 8. Table 1 shows the critical load coefficients and Table 2 shows those of natural frequency. The columns "C_{fem}" in these two tables indicate the eigenvalue coefficients determined by finite element method. The curves depicted in Fig. 4 represent the critical load coefficients C with respect to parameter α and n.

Table 1 Critical load coefficient

$$P_{cr} = C\frac{EI_0}{L^2}$$

(a) Non-symmetrically tapered bar

α	n = 2		n = 3		n = 4	
	C_{fem}	C_{est}	C_{fem}	C_{est}	C_{fem}	C_{est}
0.0	9.8693	9.8053	9.8693	9.8258	9.8693	9.9547
0.1	11.2334	11.2146	11.9793	11.9596	12.7663	12.7769
0.2	12.6577	12.6642	14.2884	14.2943	16.1134	16.0756
0.3	14.1459	14.1560	16.8162	16.8300	19.9055	19.8509
0.4	15.6760	15.6893	19.5269	19.5666	24.1479	24.1028
0.5	17.2604	17.2641	22.4459	22.5041	28.8742	28.8312
0.6	18.8970	18.8803	25.5887	25.6426	34.0767	34.0362
0.7	20.5705	20.5381	28.8993	28.9820	39.7644	39.7177
0.8	22.3103	22.2374	32.4012	32.5224	45.9174	45.8758
0.9	24.0996	23.9782	36.1387	36.2636	52.5316	52.5104
1.0	25.9174	25.7606	40.0381	40.2059	59.6213	59.6216
1.1	27.8065	27.5844	44.1282	44.3490	67.2307	67.2093
1.2	29.7224	29.4497	48.4779	48.6931	75.3165	75.2736
1.3	31.6634	31.3565	52.9919	53.2381	83.9358	83.8144
1.4	33.6854	33.3049	57.6590	57.9840	92.9372	92.8318
1.5	35.7387	35.2948	62.4939	62.9309	102.4208	102.3257
1.6	37.8592	37.3261	67.6152	68.9309	112.4648	112.2962
1.7	40.0030	39.3990	72.8792	73.4275	122.9639	122.7432
1.8	42.2168	41.5134	78.3146	78.9772	134.0065	133.6668
1.9	44.3655	43.6693	83.9940	84.7278	145.2971	145.0670
2.0	46.6824	45.8667	89.7409	98.6794	157.2013	156.9737

(b) Symmetrically tapered bar

α	n = 2		n = 3		n = 4	
	C_{fem}	C_{est}	C_{fem}	C_{est}	C_{fem}	C_{est}
0.0	9.8693	9.6282	9.8693	10.6451	9.8693	12.9940
0.1	11.6079	11.5271	12.5768	12.8581	13.6348	14.7225
0.2	13.4586	13.4906	15.6868	15.6225	18.2482	17.9377
0.3	15.4344	15.5185	19.2161	18.9473	23.8603	22.6307
0.4	17.5174	17.6109	23.1678	22.8235	30.5056	28.8045
0.5	19.7034	19.7677	27.5799	27.2541	38.2950	36.4590
0.6	22.0243	21.9890	32.4422	32.2391	47.2418	45.5942
0.7	24.4493	24.2747	37.8027	37.7785	57.5867	56.2102
0.8	26.9626	26.6249	43.6334	43.8723	69.2625	68.3069
0.9	29.6192	29.0395	50.0039	50.5204	82.3064	81.8844
1.0	32.3613	31.5187	56.8047	57.7230	97.0227	96.9427
1.1	35.2118	34.0662	64.2012	65.4799	113.1292	113.4816
1.2	38.1732	36.6702	72.0452	73.7912	130.6720	131.5013
1.3	41.2212	39.3427	80.4965	82.6569	150.0019	151.0018
1.4	44.3751	42.0797	89.4541	92.0770	171.2479	171.9830
1.5	47.6505	44.8811	99.0309	102.0515	193.6790	194.4450
1.6	50.9614	47.7469	109.0645	112.5804	218.4140	218.3877
1.7	54.4866	50.6773	119.7284	123.6637	244.5312	243.8111
1.8	58.0626	53.6720	131.0469	135.3014	272.5061	270.7153
1.9	61.7016	56.7313	142.6699	147.4934	302.1071	299.1003
2.0	65.4916	59.8550	155.0469	160.2399	332.8216	328.9660

Table 2 Natural frequency coefficient

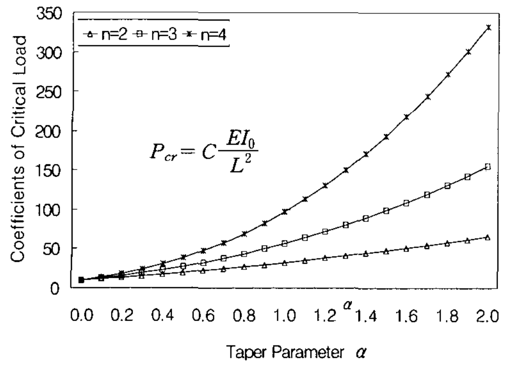
$$\omega_0 = C\frac{1}{L^2}\sqrt{\frac{gEI_0}{\rho A_0}}$$

(a) Non-symmetrically tapered bar

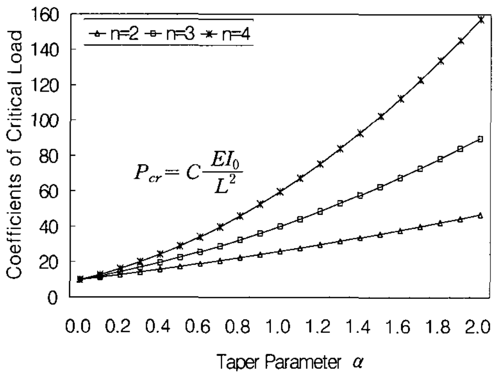
α	(m, n) = (0, 2)		(m, n) = (1, 3)		(m, n) = (2, 4)	
	C_{fem}	C_{est}	C_{fem}	C_{est}	C_{fem}	C_{est}
0.0	9.8695	9.8958	9.8695	9.9117	9.8695	9.9348
0.1	10.5314	10.5416	10.5292	10.5438	10.5246	10.5463
0.2	11.1845	11.1828	11.1705	11.1683	11.1549	11.1450
0.3	11.8321	11.8195	11.8028	11.7852	11.7576	11.7309
0.4	12.4665	12.4515	12.4149	12.3945	12.3356	12.3039
0.5	13.0945	13.0790	13.0176	12.9962	12.8981	12.8640
0.6	13.7165	13.7018	13.6160	13.5904	13.4435	13.4113
0.7	14.3278	14.3201	14.1955	14.1769	13.9735	13.9458
0.8	14.9399	14.9338	14.7647	14.7558	14.4855	14.4673
0.9	15.5473	15.5429	15.3343	15.3272	14.9804	14.9761
1.0	16.1440	16.1474	15.8883	15.8910	15.4623	15.4720
1.1	16.7443	16.7472	16.4344	16.4471	15.9376	15.9550
1.2	17.3348	17.3425	16.9859	16.9957	16.4007	16.4251
1.3	17.9158	17.9332	17.5250	17.5367	16.8590	16.8825
1.4	18.5043	18.5194	18.0516	18.0700	17.2959	17.3269
1.5	19.0857	19.1009	18.5689	18.5958	17.7246	17.7585
1.6	19.6705	19.6778	19.0955	19.1140	18.1522	18.1773
1.7	20.2475	20.2501	19.6101	19.6246	18.5698	18.5832
1.8	20.8280	20.8179	20.1166	20.1276	18.9845	18.9762
1.9	21.3798	21.3810	20.6264	20.6230	19.3733	19.3564
2.0	21.9607	21.9396	21.1631	21.1109	19.7681	19.7238

(b) Symmetrically tapered bar

α	(m, n) = (0, 2)		(m, n) = (1, 3)		(m, n) = (2, 4)	
	C_{fem}	C_{est}	C_{fem}	C_{est}	C_{fem}	C_{est}
0.0	9.8695	9.8896	9.8695	9.9012	9.8695	9.9214
0.1	10.7042	10.7093	10.6994	10.7090	10.6974	10.7099
.2	11.5278	11.5256	11.5149	11.5095	11.4898	11.4846
0.3	12.3479	12.3383	12.3175	12.3028	12.2696	12.2456
0.4	13.1587	13.1476	13.1047	13.0887	13.0223	12.9927
0.5	13.9604	13.9534	13.8847	13.8674	13.7569	13.7260
0.6	14.7654	14.7557	14.6520	14.6388	14.4640	14.4455
0.7	15.5636	15.5545	15.4153	15.4028	15.1709	15.1511
0.8	16.3511	16.3499	16.1665	16.1596	15.8565	15.8430
0.9	17.1457	17.1417	16.9171	16.9091	16.5208	16.5210
1.0	17.9303	17.9301	17.6470	17.6514	17.1893	17.1853
1.1	18.7124	18.7149	18.3821	18.3863	17.8296	17.8357
1.2	19.4931	19.4963	19.0993	19.1139	18.4467	18.4723
1.3	20.2667	20.2741	19.8202	19.8343	19.0648	19.0950
1.4	21.0383	21.0485	20.5303	20.5473	19.6870	19.7040
1.5	21.8122	21.8194	21.2426	21.2531	20.2677	20.2992
1.6	22.5688	22.5868	21.9386	21.9516	20.8698	20.8805
1.7	23.3486	23.3507	22.6368	22.6427	21.4431	21.4480
1.8	24.1152	24.1112	23.3373	23.3266	22.0119	22.0018
1.9	24.8722	24.8681	24.0091	24.0032	22.5661	22.5417
2.0	25.6382	25.6216	24.6921	24.6726	23.0875	23.0678



(a) Non-symmetrically tapered bar



(b) Symmetrically tapered bar

Fig. 4 Changes of critical load coefficients

In general, the numerical analysis results are valid only for the particular values of parameters that govern a certain engineering problem. To generalize the eigenvalue coefficients obtained by finite element method, the following second order algebraic equations was proposed.

$$C_{est} = (A_0 + A_1\alpha + A_2\alpha^2) + (B_0 + B_1\alpha + B_2\alpha^2)n + (C_0 + C_1\alpha + C_2\alpha^2)n^2 = C_{fem} \quad (9)$$

The constants $A_0, A_1, \cdots, C_1, C_2$ were determined by the regression technique[7]. The regression results are summarized in Table 3.

To confirm the validity of the proposed algebraic equations, the correlation coefficients defined by Eq. 10 was also calculated.

$$r = \frac{\sum\{(C_{fem} - \overline{C}_{fem}) \cdot (C_{est} - \overline{C}_{est})\}}{\sqrt{\{\sum(C_{fem} - \overline{C}_{fem})^2\}\{\sum(C_{est} - \overline{C}_{est})^2\}}} \quad (10)$$

where C_{est} denotes the eigenvalue coefficients estimated by the proposed method and $\overline{C}_{fem}$, $\overline{C}_{est}$ are the mean values of C_{fem}, C_{est}, respectively. Correlation coefficients calculated by Eq. 10 equals nearly one($r \doteqdot 1.0$) in any case of Table 3.

Table 3 Regression constants

(a) Critical load, (Eq. 9)

Non-symm. $r \doteqdot 1.0$		
A_0= 9.8856	A_1= 6.5555	A_2=-0.2687
B_0=-0.0021	B_1= 0.0197	B_2= 0.1325
C_0= 0.0036	C_1=-0.0284	C_2=-0.0565
Symm. $r \doteqdot 1.0$		
A_0= 9.8922	A_1= 8.2362	A_2=-0.2031
B_0=-0.0099	B_1= 0.0492	B_2= 0.1505
C_0= 0.0043	C_1=-0.0299	C_2=-0.0681

(b) Natural frequency, (Eq. 9)

Non-symm. $r \doteqdot 1.0$		
A_0= 10.0895	A_1= -1.8699	A_2= 3.5616
B_0= -0.2505	B_1= 8.8225	B_2= -6.5527
C_0= 0.0542	C_1= -0.4738	C_2= 2.9048
Symm. $r \doteqdot 1.0$		
A_0= 11.5904	A_1=-13.1274	A_2= 19.6933
B_0= -2.3131	B_1= 26.0353	B_2=-30.0564
C_0= 0.6606	C_1= -5.0689	C_2= 10.9106

Table 4 Axial thrust vs frequency square relation

(a) Non-symmetrically tapered bar $\left(R=\frac{P}{P_{cr}},\ \Omega^2=(\frac{\omega}{\omega_0})^2\right)$

	R	Ω^2										
		α=0.0	α=0.2	α=0.4	α=0.6	α=0.8	α=1.0	α=1.2	α=1.4	α=1.6	α=1.8	α=2.0
	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.2	0.8002	0.7999	0.8002	0.8004	0.8003	0.8012	0.8009	0.8007	0.8002	0.8010	0.8019
(m,n)	0.4	0.5998	0.6001	0.6002	0.6006	0.6009	0.6012	0.6014	0.6018	0.6017	0.6019	0.6025
=(0,2)	0.6	0.3997	0.4010	0.4006	0.4002	0.4006	0.4017	0.4012	0.4019	0.4021	0.4018	0.4023
	0.8	0.1997	0.2006	0.2005	0.2000	0.2007	0.2010	0.2014	0.2020	0.2004	0.2012	0.2013
	1.0	0.0	0.0007	-0.0004	0.0001	0.0004	0.0	0.0003	0.0003	0.0003	-0.0018	-0.0004
	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.2	0.8002	0.8013	0.8011	0.8007	0.8021	0.8028	0.8030	0.8041	0.8050	0.8068	0.8081
(m,n)	0.4	0.5998	0.6004	0.6013	0.6011	0.6034	0.6045	0.6044	0.6067	0.6078	0.6097	0.6130
=(1,3)	0.6	0.3997	0.4022	0.4016	0.4021	0.4034	0.4046	0.4063	0.4068	0.4090	0.4109	0.4127
	0.8	0.1997	0.2001	0.2012	0.1998	0.2028	0.2034	0.2035	0.2050	0.2060	0.2071	0.2097
	1.0	0.0	0.0003	0.0003	-0.0012	0.0008	-0.0006	-0.0009	0.0002	-0.0010	0.0008	0.0004
	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.2	0.8002	0.7996	0.8019	0.8028	0.8040	0.8056	0.8085	0.8104	0.8120	0.8121	0.8160
(m,n)	0.4	0.5998	0.6011	0.6024	0.6045	0.6065	0.6109	0.6130	0.6159	0.6187	0.6209	0.6245
=(2,4)	0.6	0.3997	0.4001	0.4024	0.4051	0.4069	0.4109	0.4140	0.4166	0.4194	0.4226	0.4262
	0.8	0.1997	0.1996	0.2013	0.2033	0.2046	0.2066	0.2099	0.2111	0.2141	0.2151	0.2198
	1.0	0.0	0.0	-0.0002	-0.0009	-0.0007	0.0005	-0.0002	0.0006	-0.0001	-0.0008	0.0018

(b) Symmetrically tapered bar $\left(R=\frac{P}{P_{cr}},\ \Omega^2=(\frac{\omega}{\omega_0})^2\right)$

	R	Ω^2										
		α=0.0	α=0.2	α=0.4	α=0.6	α=0.8	α=1.0	α=1.2	α=1.4	α=1.6	α=1.8	α=2.0
	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.2	0.8002	0.8002	0.8001	0.8000	0.8006	0.8001	0.8005	0.8008	0.8016	0.8008	0.8001
(m,n)	0.4	0.5998	0.6007	0.5998	0.6002	0.6002	0.6005	0.6006	0.6011	0.6017	0.6006	0.6004
=(0,2)	0.6	0.3997	0.4000	0.3993	0.3999	0.4011	0.4000	0.4004	0.4007	0.4022	0.4005	0.4003
	0.8	0.1997	0.1996	0.1997	0.1995	0.2002	0.2005	0.2007	0.2007	0.2014	0.1996	0.2003
	1.0	0.0	-0.0002	0.0003	-0.0004	0.0002	0.0003	0.0	0.0007	0.0010	-0.0008	0.0002
	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.2	0.8002	0.7996	0.8001	0.7999	0.8002	0.8004	0.8008	0.8020	0.8016	0.8006	0.8029
(m,n)	0.4	0.5998	0.5987	0.6000	0.6013	0.6014	0.6022	0.6016	0.6029	0.6026	0.6017	0.6039
=(1,3)	0.6	0.3997	0.4001	0.4005	0.4005	0.4011	0.4016	0.4017	0.4015	0.4026	0.4024	0.4050
	0.8	0.1997	0.1998	0.2002	0.2008	0.2001	0.2009	0.2016	0.2009	0.2016	0.2016	0.2023
	1.0	0.0	-0.0012	-0.0002	0.0007	-0.0003	0.0004	0.0002	0.0006	0.0006	-0.0009	0.0008
	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.2	0.8002	0.8003	0.8007	0.8021	0.8008	0.8004	0.8040	0.8024	0.8043	0.8045	0.8071
(m,n)	0.4	0.5998	0.6000	0.6006	0.6015	0.6023	0.6024	0.6035	0.6051	0.6061	0.6089	0.6118
=(2,4)	0.6	0.3997	0.3999	0.3998	0.4031	0.4014	0.4022	0.4056	0.4052	0.4075	0.4076	0.4136
	0.8	0.1997	0.2005	0.1995	0.2019	0.2017	0.2016	0.2040	0.2031	0.2043	0.2066	0.2104
	1.0	0.0	0.0006	-0.0006	0.0006	-0.0008	-0.0012	0.0014	-0.0010	0.0	-0.0005	0.0028

4. Axial Thrust *vs* Frequency Relation

To obtain the lateral frequencies of the member under the variable thrust, P Eq. 7 was transformed into the following form

$$([K]_b - RP_{cr}[K]_g - \Omega^2\omega_0^2[M]_c)\{\Delta\} = \{0\} \qquad (11)$$

where R is the load ratio ($R = P/P_{cr}$). By changing R from 0.0 to 1.0 (R=0.0, 0.2, ⋯, 1.0) the corresponding frequencies were calculated by Eq. 8. Table 4. shows square of frequencies under axial thrusts. As can be seen in this table, $R+\Omega^2$ is near unity in any case. (see Fig. 5) Thus, it can be concluded that Eq. 1 is also applicable to the tapered members with relatively small error.

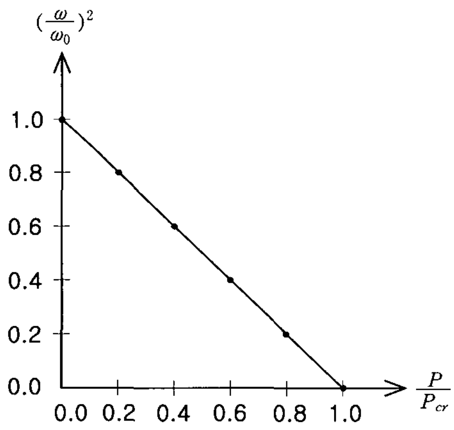


Fig. 5 Variation of frequency with axial thrust

5. Conclusion

The two eigenvalues (elastic critical load and natural frequency of lateral vibration) of sinusoidally tapered members with simply supported ends were determined by the finite element method. The parameters considered in this paper were taper parameter, α and sectional property parameter (m, n). The taper parameter, α, is assumed to change from zero to two. Sectional property parameter (m, n) is assumed to have the combination of (0, 2), (1, 3), and (2, 4). With the increasing values of the sectional property parameter (m, n), the natural frequencies of the members showed decreasing phenomena, which are opposite to the elastic critical loads of those members. Two eigenvalue coefficients were represented by simple algebraic equations. The eigenvalue coefficients estimated by proposed algebraic equations coincides with those obtained by the finite element method.

To obtain the axial thrust and frequency relationship, the axial thrust was increased step by step and the corresponding frequency was calculated. The axial thrust to elastic critical load ratio and the square of the frequency to natural frequency ratio can be approximately represented in any case by a straight line.

The eigenvalues of sinusoidally tapered members can not be determined by any of analytical methods. And so in the future the proposed algebraic equations based on the results of finite element method should be confirmed by some of experimental means.

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