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Probabilistic Approach of Stability Analysis for Rock Wedge Failure

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ABSTRACT: Probabilistic analysis is a powerful method to quantify variability and uncertainty common in engineering geology fields. In rock slope engineering, the uncertainty and variation may be in the form of scatter in orientations and geometries of discontinuities, and also test results. However, in the deterministic analysis, the factor of safety which is used to ensure stability of rock slopes, is based on the fixed representative values for each parameter without a consideration of the scattering in data. For comparison, in the probabilistic analysis, these discontinuity parameters are considered as random variables, and therefore, the reliability and probability theories are utilized to evaluate the possibility of slope failure. Therefore, in the probabilistic analysis, the factor of safety is considered as a random variable and replaced by the probability of failure to measure the level of slope stability. In this study, the stochastic properties of discontinuity parameters are evaluated and the stability of rock slope is analyzed based on the random properties of discontinuity parameters. Then, the results between the deterministic analysis and the probabilistic analysis are compared and the differences between the two analysis methods are explained.

INTRODUCTION

Most engineering geology problems involve uncertainty and variability, which are inevitably difficult to establish and predict. Uncertainty and variability are caused by insufficient information of site conditions and incomplete understanding of a failure mechanism. In rock slope stability analysis, the uncertainty and variability may be in the form of a large scatter in attitude data and the geometry of joint and also test results. Therefore, one of the most difficult jobs in rock slope engineering is the selection of the single representative values from widely varied data. Therefore, many engineers and researchers have attempted to limit and quantify the variation and uncertainty in their data and have adopted various methods to indicate the uncertainty and variation in the results of analysis. Casagrande (1965) noted the nature and importance of 'the calculated risk' in geotechnical engineering. In several examples, he showed how the unknown risks affected the stability of projects. Peck (1969) suggested the observational method to maintain control over uncertainty by revising estimates of site conditions and parameters when additional information becomes available. However, as Mostyn and Small (1987) have discussed, traditionally most engineers have taken the variation of their data into account by a selection of i) the appropriate values for input parameters into analysis and ii) an appropriate factor of safety.

Probability theory and statistical techniques, have been applied to engineering geology field to deal properly with variability and uncertainty. Application of probabilistic analysis has provided an objective tool for quantifying and modeling variability and uncertainty. However, although several probabilistic analysis methods have been proposed to consider and quantify the uncertainty and variability for rock slope stability, the techniques have been limited mostly to purely theoretical investigation. In this paper, an application of the probabilistic method to practical problems in rock slope stability analysis will be introduced. For this objective, a rock cut in North Carolina is selected and the probabilistic approach is applied to analyze the stability of wedge failure in the area. Then the results of the probabilistic analysis are compared to the results of the deterministic analysis. In

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addition, the stochastic properties of discontinuity geometry and strength parameters which are defined to use in the probabilistic analysis will be discussed.

DETERMINISTIC METHOD

Most input parameters (e.g. material strength, joint geometry and pore water pressures) in the factor of safety calculation are precisely unknown because of the uncertainty and variations in testing, modeling, and spatial variation. Thus, each of these should be considered as a random variable. The analysis with different values for each of these parameters can result in different values of factor of safety. Consequently, the factor of safety itself is also a random variable depending on many input variables.

However, in the deterministic analysis, the factor of safety calculation requires fixed values for parameters that actually exhibit a degree of uncertainty. Therefore, the conventional factor of safety does not reflect the degree of uncertainty of these parameters. In most cases, the mean value of these parameters is assigned as a fixed value but some engineers tend to select values higher or lower than the mean, due to uncertainty and variation in input parameters. This can yield very different factor of safety values for the same project. Consequently, inconsistency is likely to exist among engineers and between applications by the same engineer. In addition, the same factor of safety (FS) can be associated with a large range of reliability level and thus FS is not a consistent measure of safety. As Harr (1987) pointed out, the probability of failure can vary over many order of magnitude for the same deterministic factor of safety in the same project. Another disadvantage in the deterministic analysis is that the factor of safety values cannot be compared for different modes of failure. According to Einstein and Baecher (1982), the mechanically equivalent definitions of failure have different factor of safety values. Therefore, as Tabba (1984) pointed out, there are shortcomings of the deterministic analysis; 1) inability to account for variations in properties and conditions, 2) difficulty in portraying the relative importance of various sets of data in the overall stability condition, and 3) inability to predict failure in cases where failure has actually occurred.

PROBABILISTIC METHOD

The probabilistic analysis, as an alternative to the deterministic approach, has been introduced to consider and quantify the uncertainty and variability in parameters and the analytical model. In this analysis, the factor of safety is considered as a random variable and can be replaced by the probability of failure to measure the level of slope stability. The probability of failure is simply defined as the probability of having FS<1 under the probability density function (PDF) of factor of safety. The physical meaning of probability of failure can be considered as the percentage of slopes that slide compared to a certain number of geometrically similar slopes (Shuk, 1970). However, because we consider only one specific slope, the probability of failure is interpreted as a measure of relative likelihood of occurrence of failure (Coates, 1981).

In general, the probabilistic analysis is performed in two steps: The first step consists of analysis of available geotechnical data to determine the basic statistical parameters, such as mean and variance, and probability density function which enables us to represent and predict the random property of geotechnical parameters. The mean value of the PDF represents the best estimate of the random variable and the standard deviation or coefficient of variation (c.o.v.) of the PDF represents an assessment of the uncertainty.

In the second step, risk analysis of slope stability is accomplished using the basic statistical parameters and probability density function determined in the previous step. Two methods of risk analysis are commonly used, the Monte Carlo simulation and First Order Second Moment method (FOSM). In this study, the Monte Carlo simulation method is used to evaluate the probability of failure. This simulation method can be used when the PDFs of each component variable are completely prescribed. In this procedure, values of each component are generated randomly by its respective PDFs and then these values are used to evaluate the factors of safety. By repeating this calculation, the probability of failure can be estimated by the proportion of calculations where the safety factor is less than one. This calculation is reasonably accurate only if the number of simulations is very large.

SITE INTRODUCTION

The study area is the rock cut along Interstate highway 26 in North Carolina U.S.A., which is under construction. This area is located within the Blue Ridge Belt, a geologic province bounded to the west by the Great Smoky thrust fault and to the east by the Brevard Fault zone. This province is one of the several physiographic provinces which comprise the Appalachian Highlands. This province consists of high metamorphic grade, Middle Proterozoic basement to early Paleozoic, off-shelf cover sediments and Paleozoic igneous intrusives. Rocks underlying this study area are Pre-Cambrian in age (950~1250 m.y.). Granite gneiss, biotite gneiss and schist, quartz monzonite, and migmatitic gneiss make up the bulk of these probably metasedimentary rocks that have been intruded by younger granitic rocks and are referred to as the Cranberry Granite Gneiss (Fig. 1).

The Cranberry granite gneiss is the major lithology in this area and is a medium-grained, even-textured rock that varies from light to dark gray in color. It is composed of quartz, orthoclase, muscovite, biotite, and occasionally hornblende. The gneiss contains many small portions of mica gneiss, mica schist, hornblende gneiss, schistose basalt, and pegmatite. The unit also contains a moderate amount of quartz veins. A large number of discontinuity orientations and geometries were measured from the field in this study and their stochastic properties were analyzed by author (Park, 1999). In addition, several core samples for estimation of discontinuity strength parameters were obtained and used for direct shear tests.

ANALYSIS FOR STOCHASTIC PROPERTIES OF DISCONTINUITY PARAMETERS

As discussed previously, the random or stochastic properties of geological and geotechnical parameters for slope stability should be defined first in the probabilistic analysis. Using the statistical inference, the probabilistic distribution and statistical parameters in the geological and geotechnical environment is defined. The statistical inference is the procedure that information obtained from sampled data is used to make generalizations about the populations from which the samples were obtained (Ang and Tang, 1975). This is an important objective in this study and also an important procedure used to obtain accurate and proper analysis results. This is because as noted by Kulatilake *et al.* (1985), a different choice of density distribution affects significantly the probability of slope failure. In the current study, discontinuity orientation, length, spacing

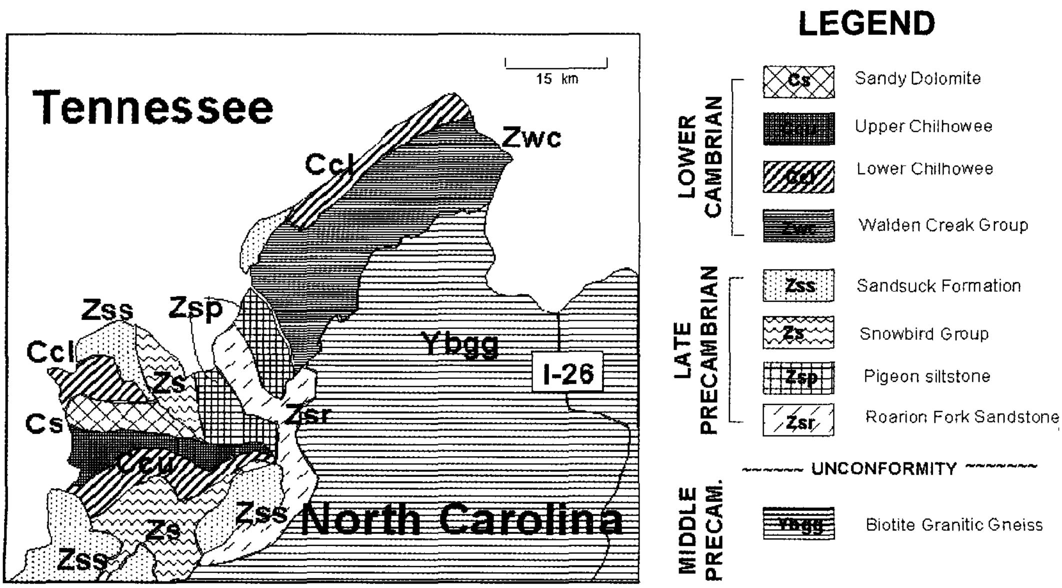

Fig. 1. Geology map of study area.

and strength parameters are considered as random variable and defined their stochastic properties.

Discontinuity orientation

One of the major reasons to carry out a statistical analysis for discontinuity properties is to find a proper probability distribution representing discontinuity parameters and their random properties. There have been a number of studies to determine the appropriate probability density distribution for a discontinuity orientation distribution. Fisher (1953) proposed a distribution on the basis of the assumption that a population of orientation values was distributed about a true value. This assumption is similar to the idea of discontinuity normals being distributed about a true value within a set. He assumed that the probability, $P(\theta)$, that an orientation value selected randomly from the population makes an angle of between θ and $d\theta$ with the true orientation is given by

$$P(\theta) = \eta e^{k\cos\theta}\ d\theta \qquad (1)$$

where k is a constant controlling the shape of the distribution and is commonly referred to as Fisher's constant, which is a measure of the degree of clustering within the population. This constant is a variable expressed as follow;

$$\eta = \frac{k\sin\theta}{e^{k} - e^{-k}} \qquad (2)$$

Combining the two equations above, the following probability density distribution is obtained

$$f(\theta) = \frac{k\sin\theta\ e^{K\cos\theta}}{e^{k} - e^{-k}} \qquad (3)$$

In view of its simplicity and flexibility, the Fisher distribution provides a valuable model for discontinuity orientation data (Priest, 1993). However, one disadvantage of this distribution is that the distribution provides only an approximation for asymmetric data because this distribution is a symmetric distribution. Therefore, many researchers have proposed a number of models that can provide better fits for asymmetric orientation data. However, these models are more complex in their parameter estimation. Futhermore, because of their complexity, a generation of random values from those asymmetric

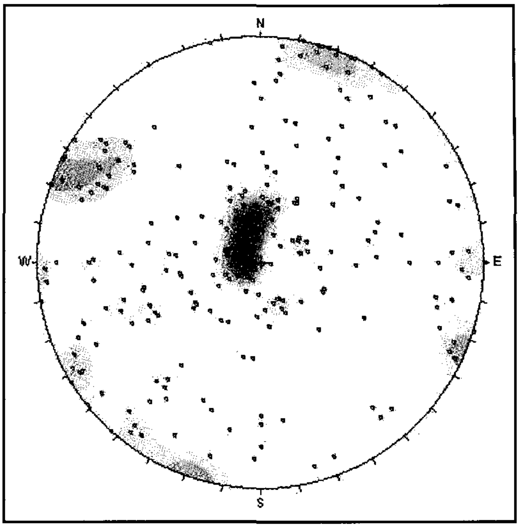

Fig. 2. Lower hemisphere stereographic projection of discontinuity normals in the study area.

orientation distributions is very difficult to accomplish and subsequently the analyses based on this probabilistic density function are difficult to perform. Therefore, the author decided to use this distribution for discontinuity orientation in this study.

Total 279 discontinuity orientations were measured from the scanline method on outcrops and from borehole for this study (Fig. 2). However, according to the author's field survey, the critical discontinuities affecting rock slope stability in this area are the foliations. Therefore, the foliations are mainly dealt with in this study. After the measurement in the field, the discontinuity orientations were corrected for sampling bias using the weighting factor suggested by Terzaghi (1965) and Priest (1985). The sampling bias was caused by linear sampling method such as scanline sampling method and borehole sampling method because the sampling line tends to intersect preferentially those discontinuities whose normals make small angles to the sampling line (Priest, 1985). Using the corrected orientation data, the orientation sets were identified by applying algorithm proposed by Mahtab and Yegulp (1982). In the study, CANDO program that has been developed by Priest and his colleagues at University of South Australia on the basis of Mahtab and Yegulp (1982)'s algorithm, was used because this program uniquely provides information about degree of clustering, that is Fisher constant.

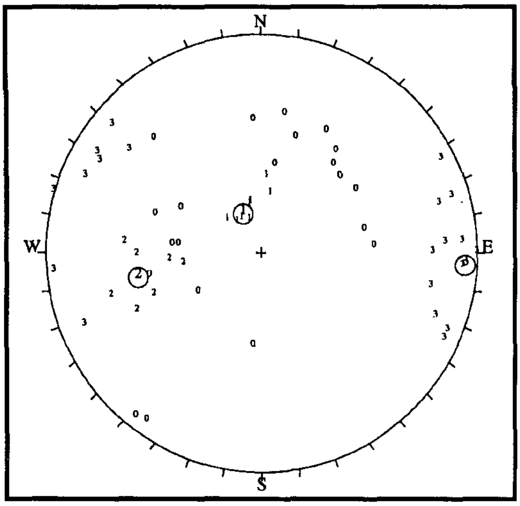

Fig. 3. Lower hemisphere stereographic projection of clustered foliation normals in 1-26 Area.

The clustering results of foliation orientation measured in this study area were plotted in Fig. 3. Total three different foliation sets were clustered in this area and their mean orientation are 155/22, 079/60 and 273/87 for J1, J2 and J3, respectively. Fisher constants are 72, 29 and 20 for J1, J2 and J3, respectively.

Discontinuity length

The length of a discontinuity is defined as the distance over which the tensile and cohesive strength of the rock substance has been reduced or lost. Knowledge of the length of discontinuities in a rock mass is important in the prediction of rock mass behavior and the analysis of rock slopes because the discontinuity lengths influence the size of blocks that may be formed. However, in the deterministic analysis such as limit equilibrium method, the length of discontinuity is not considered in procedure of stability analysis. That is, the length data is not accounted for the calculation of FS either for kinematic analysis. In comparison, in the proba- bilistic analysis, the discontinuity lengths can be involved in many different ways. Especially in this study, the length is considered as the concept of persistence. That is, the probability that the joint length is long enough to form a block able to slide is evaluated and then this probability is multiplied by the probability of slope failure which was commonly evaluated on the basis of the assumption that the joint is fully persistent. Consequently, this approach can overcome the limitation of the conservative and deterministic analysis which assumes a 100% persistent joint. Therefore, this new approach offers advancement beyond one of the disadvantages in the deterministic analyses.

Because the borehole sampling method by which the most of discontinuity data was obtained in the study does not provide information on discontinuity size due to small diameter of cores, the limited amount of discontinuity length data were provided. Therefore, the probability density function for discontinuity length is decided from the previous researches and literatures.

A lognormal distribution has been proposed as a representative distribution model by many different researchers (McMahon, 1971; Bridge, 1976; Barton, 1978; Einstein *et al.*, 1980). However, according to Priest and Hudson (1981), the lognormal distribution is a biased distribution caused by scanline sampling. They have attempted to derive the form of uncorrected distribution from the corrected distributions, when the negative exponential, uniform and normal distributions are considered as possible correct distributions. Their results show that when the corrected distribution has a negative exponential distribution, the lognormal distribution is obtained as an uncorrected distribution using graphical and analytical methods. Therefore, the negative exponential distribution is an appropriate distribution and it also has an advantage that sampling bias caused by scanline method can be canceled out by adopting this distribution. Consequently, the exponential distribution is commonly used in probabilistic analysis to represent a stochastic property of the discontinuity length and also in this study. This is because sufficient theoretical and practical grounds have been provided in the literature and the assignment of a negative exponential density distribution is the most efficient way to remove the sampling bias.

The mean length of foliation obtained from field survey was 2.4 m. However, due to the limitation of the number of data, all three sets of foliation were assigned the same mean value.

Discontinuity spacing

The purpose of discontinuity spacing measurement is to obtain the size of the blocks which compose a rock mass. Stability analysis and design are strongly dependent on the block size because weight forces, forces due to water pressure, and failure mechanism depend on the block size. Although mean discontinuity spacing value provides a direct measure of spacing data, several previous studies have tried to represent the distribution of measured data by statistical analysis and description because the spacing data is considered as a random variable.

Approximately 100 spacing values were collected in field by author (Fig. 4) and then in order to determine the appropriate distribution of spacing in the study, Chi-square goodness-of-fit tests were performed. The lognormal and negative exponential distributions, which are two candidate distribution models for spacing, were tested in this study (Table 1). This is because the previous researchers proposed and utilized the distribution, and those theoretical distributions are bounded at zero and are skewed to the right and therefore, those characteristics are similar to the properties of the spacing distribution. On the basis of field measurements and theoretical considerations, Priest and Hudson (1976) concluded that the distribution of discontinuity spacing for various sedimentary rock types could be modeled by the negative exponential probability density function. This conclusion has been supported by others, such as Wallis and King (1980) and Baecher (1983). However, some publications such as Rouleau

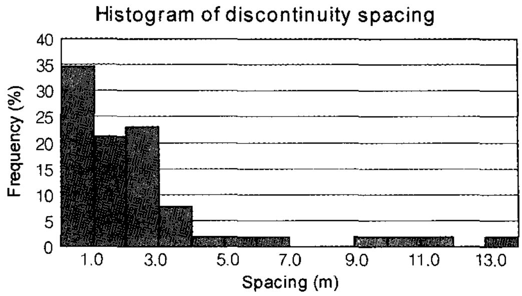

Fig. 4. Histogram of discontinuity spacing.

Table 1. Chi-square test results for relative goodness of fit in spacing data.

Interval	Observed Frequency (n_i)	Theoretical Frequency (e_i)		$(n_i-e_i)^2/e_i$	
		Exponential	Lognormal	Exponential	Lognormal
0-0.15	0	2.58	0.55	2.59	0.55
0.15-1.00	18	12.40	15.16	2.53	0.53
1.00-1.85	5	9.28	11.57	1.98	3.73
1.85-2.70	7	6.96	7.15	0.001	0.003
2.70-3.55	11	4.72	4.60	8.34	8.89
3.55-4.40	4	3.90	3.10	0.002	0.26
4.40-5.25	1	2.92	2.18	1.27	0.64
5.25-6.10	1	2.19	1.58	0.65	0.21
6.10-6.95	1	1.64	1.17	0.25	0.03
6.95-7.80	0	1.22	0.89	1.23	0.89
7.80-8.65	0	0.92	0.69	0.92	0.69
8.65-9.50	0	0.69	0.54	0.69	0.54
9.50-10.35	2	0.51	0.43	4.26	5.66
10.35-11.20	1	0.39	0.35	0.97	1.21
11.20-12.05	0	0.29	0.29	0.29	0.29
12.05-12.90	1	0.22	0.24	2.83	2.47
12.90-13.75	0	0.16	0.20	0.16	0.19
13.75-14.60	1	0.12	0.16	0.12	0.17
14.60-15.45	0	0.09	0.14	0.09	0.14
15.45-16.30	0	0.07	0.12	0.07	0.12
16.30-17.15	0	0.05	0.10	0.05	0.10
17.15-18.00	0	0.04	0.09	0.04	0.09
>18.00	0	0.03	0.08	0.03	0.08
		51.43	51.37	29.35	27.47

and Gale (1985) and Sen and Kazi (1984) proposed the lognormal probability distribution for discontinuity spacing.

A total of 52 data points in the same set were used to evaluate the goodness of fit. The calculated $\Sigma(n_i-e_i)^2/e_i$ values of both distributions are smaller than $C_{0.95,\ 20}$, 31.4, obtained from the Chi-Square distribution table at 5% significant level with 20 degree of freedom (Table 1). Therefore, both the lognormal distribution and the exponential distribution appear to be valid for spacing at the significant level of 5%. However, because the calculated $\Sigma(n_i-e_i)^2/e_i$ value for the lognormal distribution, 27.4 is smaller than that for the exponential distribution, 29.4, the lognormal distribution is better than the exponential distribution. Therefore, the lognormal distribution is used to represent the random property of discontinuity spacing in the study. The mean value of foliation spacing is 2.4 m and the same value is assigned for all three sets.

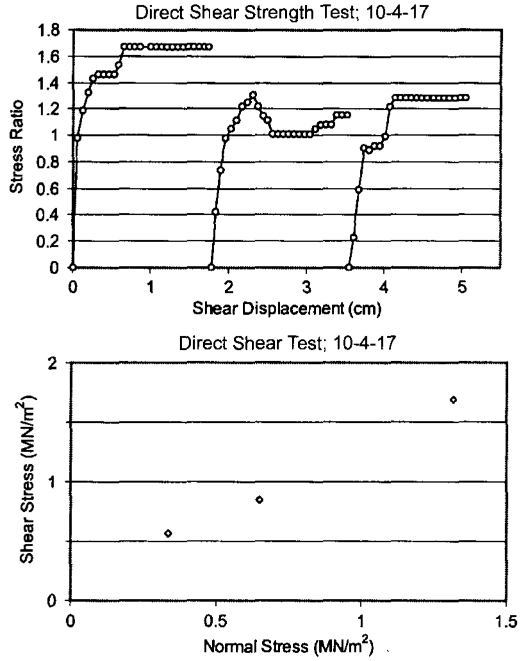

Fig. 5. Plots of direct shear test results.

Discontinuity strength parameters

Total 6 cores were obtained for the estimation of shear strength parameters for foliation and tested by direct shear test. All normal and shear stress data have been corrected for the elliptical influence of the shear surface and the shear sample was reset to its original position at the start of each test. Samples were subjected to the highest normal stresses during the first test of the four shear tests that were run on each sample. The second and third tests were conducted at lower normal stresses and the fourth one was run at an intermediate normal stress. Fig. 5 is one example of direct shear test plots carried out in this study. For the evaluation of the residual friction angles, the first test of direct shear tests was removed in this plot. From the shear and normal stress plot, cohesion and friction angle can be obtained for each sample. However, as Hoek (1997) pointed out, in rock mechanics, cohesion is a mathematical quantity related to surface roughness. Therefore, in this study, the cohesion is considered as zero and the friction angle is only considered as discontinuity strength parameter. The friction angles ranged from 23.1° to 30.8° and the mean value and standard deviation of these data are respectively 27.03° and 2.94. However, the number of direct shear test for foliation is too small to carry out statistical inference. Therefore, the direct shear tests for joints were used in the determination of appropriate PDF for discontinuity strength parameter.

Compared to other discontinuity parameters, limited researches have been accomplished previously regarding statistical evaluation of discontinuity strength parameters. However, although limited work has been accomplished, two different distributions are suggested for shear strength parameters. Mostyn and Li (1993) considered ϕ as normally distributed. However, in the paper by Muralha and Trunk (1993), a lognormal distribution is adopted for ϕ. They insisted that the lognormal distribution had an advantage of assuming that the shear strength will not yield negative values. Therefore, in the current study, both normal and lognormal distributions are considered as possible distribution models to represent random properties of strength parameters and both distributions are tested for their validity. According to author's test (Park, 1999), both distributions appear to be valid models for internal friction angle, but the normal distribution model is superior to the lognormal model according to the test. Table 2 shows the results of Chi-square goodness-of-fit test for internal friction angle.

Table 2. Chi-square test results for relative goodness of fit in friction angle.

Interval	Observed Frequency (n_i)	Theoretical Frequency (e_i)		$(n_i-e_i)^2/e_i$	
		Normal	Lognormal	Normal	Lognormal
<30.0	0	0.025	0.014	0.025	0.014
30.0-32.5	1	0.152	0.141	4.735	5.258
32.5-35.0	0	0.602	0.668	0.602	0.668
35.0-37.5	1	1.550	1.706	0.195	0.292
37.5-40.0	2	2.595	2.609	0.136	0.142
40.0-42.5	3	2.823	2.599	0.011	0.062
42.5-45.0	3	1.997	1.808	0.504	0.787
45.0-47.5	1	0.918	0.928	0.007	0.006
47.5-50.0	0	0.274	0.369	0.274	0.369
>50.0	0	0.053	0.118	0.053	0.118
		10.989	10.959	6.543	7.715

When the significant level is considered as 5% and the degrees of freedom equals 7, the $C_{0.95,7}$ value determined from Chi-square table equals 14.1. Both distributions appears to be valid models because the calculated values are less than 14.1. However, $\Sigma(n_i-e_i)^2/e_i$ value of normal distribution is less than $\Sigma(n_i-e_i)^2/e_i$ value of lognormal distribution. Therefore, in the probabilistic analysis of the stochastic procedure, the normal distribution is used for shear strength as a probability density function to simulate the random characteristics of shear strength.

PROBABILISTIC ANALYSIS FOR ROCK WEDGE STABILITY

Monte carlo simulation

The Monte Carlo simulation is commonly used to evaluate the failure probability of a mechanical system, in particular, when direct integration is not practical or when the equation to integrate is difficult to obtain. This simulation is the most widely used among the probabilistic analysis methods and many others applied it to evaluate slope stability (Kulatilake *et al.*, 1985; Muralha and Trunk. 1993). In this research, the Monte Carlo method is employed because the deterministic model for rock wedge failure is not easy to solve by any other risk analysis methods. The Monte Carlo simulation approach is to assume that for a given stability analysis, each variable takes a single value selected randomly from its measured distribution, independent of the other variables. The group of randomly selected parameters is combined with the fixed input data to generate a single random value for the factor of safety. This process is repeated many times to generate a large number of different factor of safety. The simulation procedure used in this study is expressed in a flowchart in Fig. 6. This simulation procedure can be divided into two steps. The first step is the kinematic analysis and this analysis is to examine the kinematic feasibility. That is, based on the discontinuity orientation data, whether the rock body defined by discontinuities can move or not will be checked. The second step is the kinetic analysis. If the kinematic analysis indicates that the structural condition is potentially unstable, the kinetic stability is assessed by the limit equilibrium method. Therefore, from the two steps in simulation procedure, we can obtain the probability of kinematic instability and the probability of kinetic instability.

Probabilistic assessment

The Monte Carlo simulation performed in the previous simulation procedure yields a list of factors of safety for every possible kinematically unstable rock block. Absolute values of the factor of safety of less than one indicate that blocks will fail. However, the definition of the probability of failure is sometimes quite vague in previous publications. According to Quek and Lèung (1995), the probability of failure is expressed as

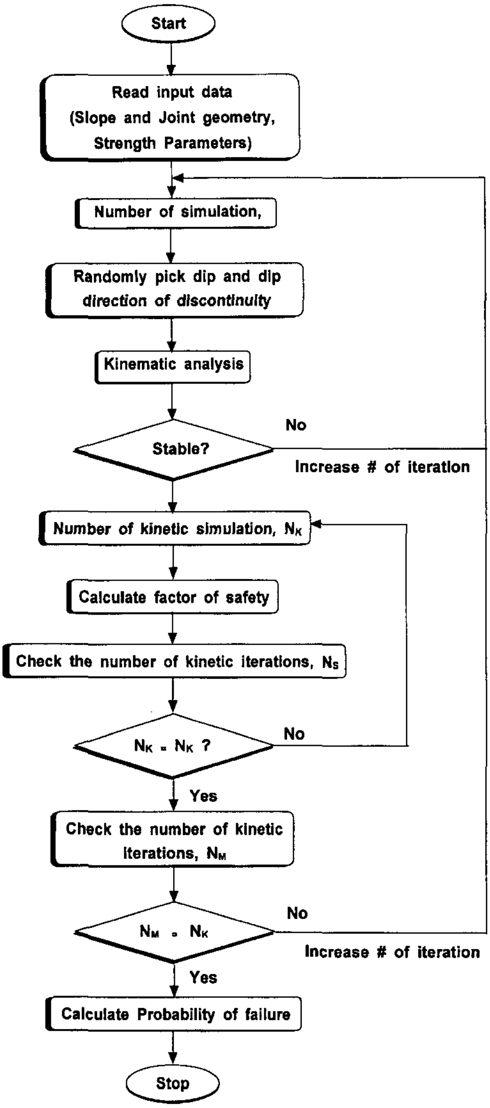

Fig. 6. Flowchart of probabilistic analysis in the study.

$$P_f=\frac{N_F}{N_T} \tag{3}$$

where N_F is the number of iterations that the blocks are failed, that is, factors of safety are less than 1, and N_T is the total number of iterations that the blocks analyzed. However, as Feng and Lajtai (1998) pointed out, N_T can be interpreted in two different ways; N_T is either the total number of iteration performed or only those iterations that form kinematically unstable blocks. Depending on the definitions, the probability of failure will change.

Therefore, for clear definition, the probability of failure is defined as the multiplication of the probability of kinematic instability by the probability of kinetic instability in this study. This multiplication is based on the concept of composite models. The concept of composite models is that all the individual components that affect probability of failure must be compiled to calculate probability of failure. The probability of kinematic instability (P_m) is evaluated as the ratio of the number of iterations (or the number of wedges formed in each iteration) that are determined as kinematically unstable to the number of total iterations.

$$P_m=\frac{N_m}{N_T} \tag{4}$$

where N_m is the number of iterations that a block is kinematically unstable, and N_T is the total number of iterations.

The probability of kinetic instability is the ratio of the number of the iterations that the factor of safety is less than 1 to the number of iterations that the factor of safety is calculated. However, the kinetic analysis is performed only when the wedge is kinematically unstable, the probability of kinetic instability (P_n) is

$$P_n=\frac{N_f}{N_m} \tag{5}$$

where N_f is the number of iterations that a block has factor of safety less than one.

Therefore, on the basis of the concept of composite model, the probability of failure is

$$P_f=P_m\times P_n=\frac{N_m}{N_T}\times\frac{N_f}{N_m} \tag{6}$$

This is possible because the probability of kinetic instability is a conditional probability based on the premise that the block is kinematically unstable. Therefore, based on this probabilistic theory, the probability of failure is defined as the ratio of the number of iterations that factor of safety is less than one, which is based on premise that the wedge is kinematically unstable, to the number of total iterations. This method provides a clear definition based on probability theory and simplifies the evaluation of factor of safety. In addition, this concept overcomes the limitation of the previous

Table 3. Input values for discontinuity properties

Discontinuity Parameters		Set I. D. J1	J2	J3
Orientation	Mean	155/22	079/60	273/87
	Fisher Constant	72	29	20
	PDF	Fisher	Fisher	Fisher
Friction Angle	Mean	27.03	27.03	27.03
	Standard Deviation	2.94	2.94	2.94
	PDF	Normal	Normal	Normal
Length	Mean	2.4	2.4	2.4
	PDF	Negative Exponential	Negative Exponential	Negative Exponential
Spacing	Mean	2.4	2.4	2.4
	PDF	Lognormal	Lognormal	Lognormal

researches without confusion.

RESULTS OF PROBABILISTIC ANALYSIS

The input parameters, discussed previously and used in this analysis, are listed in Table 3. The discontinuity parameters such as orientation, length, spacing and strength parameters of discontinuity are mainly accounted for as probabilistic parameters. On the other hand, the slope geometry parameters are commonly considered as deterministic parameters. The height of slope cut analyzed its stability in the study is approximately 60 m and orientation of the slope is 140°/45°, and these parameters are considered in the deterministic way. As mentioned previously, three major foliation sets were identified from the field and borehole data and their mean orientations (155/22 for J1, 079/60 for J2 and 237/87 for J3) and Fisher constants (72, 29 and 20 for J1, J2 and J3 respectively) were listed in the table. In addition, the shear strength and geometries for each foliation set were also listed in Table 3. However, due to the limitation of the amount of data, the same values for each set were assigned for friction angle, length and spacing of foliation. Based on input values and the simulation procedure discussed previously, the computer algorithm was developed to analyze wedge stability. This computer algorithm is developed using Microsoft Excel and its built-in Visual Basic (VBA) programming environment because those have several advantages. One advantage is that the Monte Carlo simulation can be implemented easily using spreadsheet and its built-in Visual Basic programming environment. This is because most spreadsheets have their built-in random number generator and this makes the complicated and repeated procedure in Monte Carlo simulation easy. Another advantage is its statistical function. Because a spreadsheet has built-in basic statistical functions, such as normal, lognormal and exponential distribution, we can use these functions easily without considering the PDF of each distribution function which is not easy to formulate. Consequently, using input values and the computer algorithm, the probabilistic analyses were carried out and the results of the analysis for wedge failure are listed in Table 4. In this simulation procedure, approximately 20,000 cycles of Monte Carlo simulation is performed. Then, in order to compare the probability of slope failure with the deterministic analysis results, the factors of safety for each wedge generated randomly were evaluated and then also listed in Table 4.

According to the deterministic analysis results in Table 4, no discontinuity combinations indicate the unstable condition. In other words, the deterministic analysis, evaluated using single representative values for discontinuity parameters indicates a stable condition for wedge failures in all discontinuity combinations. However, on the basis of probabilistic analysis results in Table 4, the probabilities of kinematic instability for J1 & J2, J1 & J3 and J2 & J3 combinations are 21.5%, 11.7% and 0.9%, respectively. Therefore, there are the possibilities of kinematic instability for all combinations of discontinuity sets on the probabilistic analysis. That is, especially in case for J1 & J2 combination, the

Table 4. Results of wedge failure for deterministic analysis and probabilistic analysis.

Set No. 1	Set No. 2	Factor of Safety	Probability of Failure		Total Probability of Failure
			Kinematic	Kinetic	
J1	J2	Stable	0.215	0.183	0.039
J1	J3	Stable	0.117	0.641	0.075
J2	J3	Stable	0.009	0.583	0.005

intersection line of the combination evaluated by each single representative orientation for two discontinuity sets is located in a stable area of stereograph. However, if we consider the scattering of orientations, 21.5% of intersection lines, formed from two randomly picked discontinuity orientations in two discontinuity sets, are indicated as unstable kinematically. The probability of kinetic instability are 18.3%, 64.1% and 58.3% for J1 & J2, J1 & J3, and J2 & J3, respectively, and therefore, these combinations have quite high probabilities of kinetic instability. The total probabilities of wedge failure for each combination evaluated by the multiplication of the two previous probabilities are 3.9%, 7.5% and 0.5%, respectively. Therefore, there is a big difference between the results of the deterministic analysis and the probabilistic analysis for J1 & J3 combination especially because the deterministic analysis indicates as stable but the probabilistic analysis shows 7.5% probability of failure. Based on the 1% of the acceptable failure probability for rock slope suggested by Priest and Brown (1982), J1 & J3 combination is interpreted as unstable in a probabilistic analysis but it is analyzed as stable in the deterministic analysis. This results of the probabilistic analysis coincides with Golder associates report in this area (Roberds and Wyllie, 19996). According to the report, the possibility that the foliation may form narrow wedges in this area is observed in the field. Consequently, the deterministic analysis based on a fixed representative value of discontinuity parameters fails to indicate the possibility of failure. This is because the deterministic analysis can not consider the scatter of discontinuity parameters. That is, even though the representative value of discontinuity parameters does not indicate unstable condition, many other scattered data could show the instability and the slope can be in an unstable condition. Therefore, it can be said that there is a possibility that the deterministic analysis based on the single representative value of discontinuity parameters can lead to misinterpretation of rock slope stability.

CONCLUSIONS

The result of comparison between deterministic analysis and probabilistic analysis in the study area indicates that the analysis result of probabilistic analysis could be quite different from that of the deterministic analysis. The deterministic analysis based on a single value of discontinuity parameters fails to indicate the possibility of slope failure. Consequently, the deterministic analysis is unable to represent the actual condition of rock slope because this analysis does not consider random properties of parameters and therefore, this misinterpretation can cause serious problems. By contrast, the probabilistic analysis is more representative of the actual behavior of parameters and provides analysis results. Therefore, it is recommended that the probabilistic analysis should be used especially in cases when significant scatter in the parameters is observed. In addition, the stochastic properties of discontinuity parameters were discussed in this study. The appropriate probability density functions for each parameter were proposed on the basis of goodness-of-fit test for field data and theoretical reasons. These properties were used in this study and also could be utilized in future studies.

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확률론적 해석방법을 이용한 쐐기파괴의 안정성 해석

박 혁 진

요 약 : 확률론적 해석방법 (probabilistic analysis)은 현장으로부터 획득한 자료들에서 일반적으로 발생하는 가변성과 불확실성을 효과적으로 정량화하여 해석에 이용할 수 있는 방법 중의 하나로 제안되었다. 특히 암반사면공학에서는 이러한 가변성과 불확실성이 불연속면의 방향 및 기하학적 특성, 그리고 실내실험 결과의 분산으로 나타난다. 확률론적 해석방법은 불연속면의 기하학적 특성과 강도 특성을 확률변수 (random variable)로 취급하여 신뢰성이론 (reliability theory)과 확률이론 (probability theory)을 근거로 분석하였으며 이를 기초로 하여 Monte Carlo Simulation과 같은 해석법을 이용, 구조물의 붕괴가능성을 확률로 표현하였다. 확률론적 해석 방법은 기존의 안전율을 대체하여 구조물의 안정성을 붕괴확률 (probability of failure)로 제안하였으며 이 붕괴확률은 안전율의 확률분포함수 (probability density function)에서 안전율이 1보다 작을 가능성을 확률로 나타낸 수치이다. 이 방법은 안전율의 개념을 기초로 하여 자료의 분산을 고려하지 않은 채 단일 대표 값만을 이용하여 구조물의 안정성을 판단하는 전통적인 결정론적 해석방법 (deterministic analysis)과 비교되어진다. 본 논문에서는 확률론적 해석방법을 이용하여 불연속면 특성들의 확률특성을 고찰하였으며 이를 기초로 하여 암반사면의 안정성 해석에 응용했다. 또한 확률론적 해석과 결정론적인 해석의 결과를 비교, 그 차이점을 설명하고자 하였다.