

# Computer Simulation of Deformation Behavior of the Rubber Diaphragm

Wan-Doo Kim<sup>†</sup>, Seong-Do-Seong Cho

Korea Institute of Machinery & Materials, 171 Jang-Dong, Yoo-sung-gu, Daejeon, Korea

(Received October 19, 1999)

## 고무 다이어프램의 변형거동 전산해석

김 완 두<sup>†</sup> · 조 성 도 성

한국기계연구원

(1999년 10월 19일 접수)

**ABSTRACT:** A rubber diaphragm is a critical element of accumulators. The material of a diaphragm is nitrile rubber so as to recover and adjust the large deformation under external pressure fluctuation. The performance of accumulators is influenced by the deformation behaviors of the diaphragm. A large deformation behavior of the diaphragm has been investigated using the commercial finite element program MARC K7.1. The several elastic moduli have been used in linear analysis and Ogden's coefficients have been used in non-linear analysis. As a result, it has been shown that the deformation behavior with a elastic modulus of  $0.3 \text{ kg/mm}^2$  is similar to the behavior of non-linear analysis. And, the modified diaphragm shape to reduce the stress concentration has been proposed.

**요약:** 어큐뮬레이터에 사용되는 얇은 막 형태의 고무다이어프램은 외부의 맥동 및 충격압력을 흡수하고 유체의 수송과 압력을 전달하는 역할을 수행하며, 압력 변화에 따른 변형 거동은 어큐뮬레이터의 특성을 결정하는 중요한 설계 변수이다. 고무다이어프램은 고무 자체의 비선형성에 의한 비선형거동과 작은 압력 변화에도 큰 변형을 일으키는 대변형 거동을 나타내며, 임의 압력 이상에서는 고무다이어프램이 변형된 후 용기에 접촉되는 현상이 나타난다. 본 연구에서는 비선형·대변형 해석과 접촉 해석 등이 용이한 구조해석 소프트웨어인 MARC를 이용하여 두 가지 형상에 대한 고무다이어프램의 거동을 해석하여, 고무 물성 변화와 압력 변화에 따른 변형 거동을 예측하고 파손 취약 부위로 예상되는 다이어프램의 고정 부위에서의 응력을 완화하기 위한 설계 개선 방안을 제시하였다.

*Keywords:* computer, simulation, deformation, rubber, diaphragm, accumulator.

## I. Introduction

The accumulator is an important mechan-

ical component that is commonly used to hydraulic and pneumatic machinery and thermal and fluid machinery. The accumulator absorbs and reduces external external pressure

<sup>†</sup>대표저자(e-mail : wdKim@kimm.re.kr)

fluctuation and accumulated hydraulic and pneumatic pressure energy. This function is achieved by the elastic deformation of the diaphragm that is installed at the inside of the accumulator case. The shape of the diaphragm has the excellent elastic characteristic thin membrane of which material is nitrile rubber. The rubber diaphragm has the excellent elastic characteristic that is recovering and adjusting under the external pressure fluctuation. The performance of the accumulator is influenced by the deformation behavior of the diaphragm.<sup>1</sup>

Generally, the new diaphragm accumulators have been developed according to the experience and the method of trial and error. The configuration of new one is modified from the existing model, and the behavior of the deformation can be obtained from prototype test. Recently, the computer simulation tools which are a large deformation non-linear finite element analysis program such as MARC and ABAQUS etc. have been used in prediction and evaluation of behaviors of the non-linear rubber components.<sup>2,3</sup>

In this study, the large deformation behavior of the diaphragm is investigated using the commercial finite element program MARC K7.1.<sup>4</sup> In linear analysis the several elastic modulus are used in non-linear analysis Ogden's coefficients are used. Two types diaphragm are examined to compare the deformed shape under the pressure. The purpose of this study is to estimate the deformation behavior of the rubber diaphragm, which depends on the rubber material properties and the pressure. And, the modified

diaphragm shape to reduce the stress concentration is proposed.

## II. Diaphragm Modelling and Rubber Physical Properties

The diaphragm is installed and fixed at the inside of the globe-shaped accumulator case. The shape of the diaphragm is a bent circle. The cross sections of two typical diaphragms selected by the engineer are shown in Figs. 1 and 2. Type A has larger area than type B. Therefore, type A is more easily deformed to the change of the pressure than type B.

In this study, in order to evaluate approximately the effects of the rubber properties on the deformation the linear analysis using the elastic Young's modulus is performed. In general, the elastic analysis of rubber components for the range of a small strain has shown the good result. The non-linear analysis using the non-linear properties of rubber has to be carried out to obtain the more accurate result. The shear modulus(G) of rubber material is known as  $10 \text{ kg/cm}^2$ , Poisson's ratio is approximately 0.5, and Young's modulus is 3G for small strain range.<sup>5</sup> In this study, the characteristics of deformation behaviors have been investigated with Young's moduli of 30, 100 and  $300 \text{ kg/cm}^2$  and Poisson's ratio of 0.499..

Material tests to characterize the properties of nitrile rubber, which are the tensile test with dumbbell specimen and the compression test with disk specimen, have been carried out. Ogden's coefficients have been obtained from the strain-energy function.<sup>6</sup>

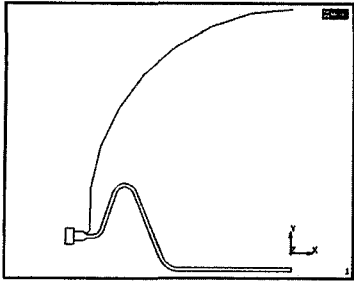


Fig. 1. Type A diaphragm shape.

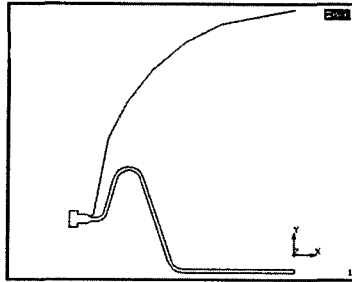


Fig. 2. Type B diaphragm shape.

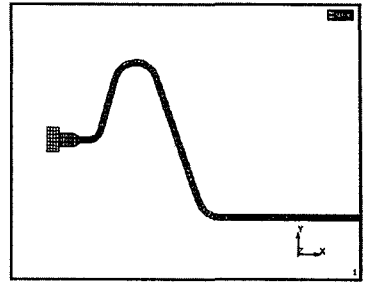


Fig. 3. Finite element model of type A.

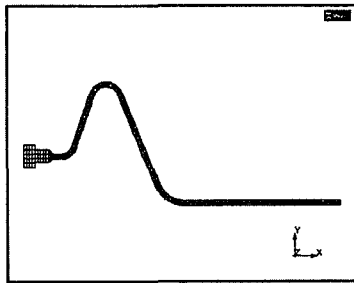


Fig. 4. Finite element model of type B.

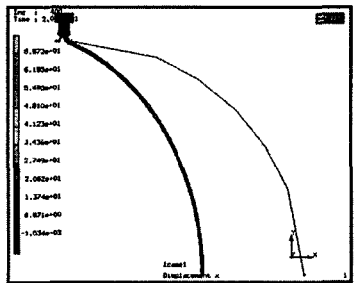


Fig. 5. Deformed shape under 0.2 kg/cm<sup>2</sup>.

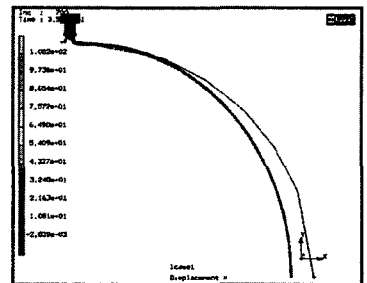


Fig. 6. Deformed shape under 0.35 kg/cm<sup>2</sup>.

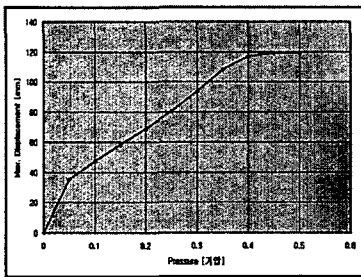


Fig. 7. Pressure vs. max. displacement.

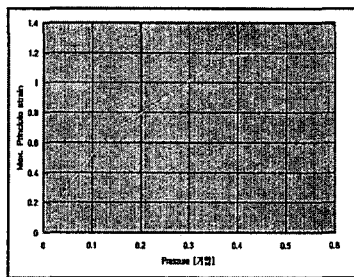


Fig. 8. Pressure vs. max. strain.

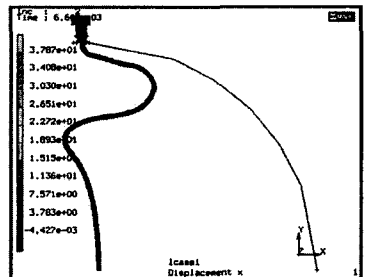


Fig. 9. Deformed shape under 0.007 kg/cm<sup>2</sup> (Type A, E= 30 kg/cm<sup>2</sup>).

Figs. 3 and 4 show the finite element model of type A and B, respectively. Two dimensional axisymmetric hyperelastic 4-node quad full-herrmann formulation elements are used. The thickness of the diaphragm is 2mm uniformly. As the boundary conditions, the

displacements of the side end of the diaphragm are fixed for all directions. Pressure is applied as the edge load up to 1.0 kg/cm<sup>2</sup>, which is the operating pressure of the accumulator. The contact body between the accumulator case and the diaphragm is defined.

The accumulator case is defined as a rigid body and the diaphragm is defined as a deformable body.

### III. Finite Element Analysis

#### 1. Analysis for the flat circular shaped diaphragm

The practical diaphragm has the bent shape in a mid-section as shown Figs. 1 and 2 so as to deform easily. The convergence of the analysis for the diaphragm with the bent area is very deficient. At first, the analysis for the flat circular shaped diaphragm is tried to evaluate the appropriateness of the rubber properties and the finite element model. Three parameters Ogden's coefficients are used to finite element analysis.

Figs. 5 and 6 show the deformed shapes under the pressure of 0.2 and 0.35 kg/cm<sup>2</sup>, respectively. The diaphragm is fully contacted with the accumulator case under 0.35 kg/cm<sup>2</sup>, thus the further deformation is blocked over this pressure. The large deformation and stress and strain concentration is generated at the fixed area. The failure is apt to occur at that position. Figs. 7 and 8 show the non-linear relationships between the applied pressure and the maximum displacement and the maximum strain at mid point.

#### 2. Analysis for Type A

Non-linear finite element analysis with the non-linear properties is necessary to obtain the more accurate result of the behavior of the rubber components. But the linear anal-

ysis using Poisson's ratio and Young's modulus is commonly used for saving the expense and the time in the preliminary design stage. Although the behavior predicted by the linear analysis is different from the practical behavior in the higher strain range, the linear analysis is performed in order to predict the approximate behavior in the small strain range and to evaluate the appropriateness of the rubber properties and the finite element model.

##### 2.1 Linear analysis with Young's modulus of 30 kg/cm<sup>2</sup>

In this case, the deformation is started in low pressure. The contact of the accumulator case is occurred under 0.1 kg/cm<sup>2</sup> that is one-tenth of the operating pressure. The diaphragm is completely contacted to the accumulator case under 0.21 kg/cm<sup>2</sup>. Figs. 9, 10 and 11 show the deformed shapes under 0.007, 0.1 and 0.21 kg/cm<sup>2</sup>, respectively.

##### 2.2 Linear analysis with Young's modulus of 100 kg/cm<sup>2</sup>

When Young's modulus is 100 kg/cm<sup>2</sup>, the deformation is obviously decreased compared with 30 kg/cm<sup>2</sup>. The diaphragm is not contacted with the accumulator case up to 0.1 kg/cm<sup>2</sup> and is contacted under 0.21 kg/cm<sup>2</sup> for the first time. Figs. 12, 13 and 14 show the deformed shapes under 0.007, 0.1 and 0.21 kg/cm<sup>2</sup>, respectively.

##### 2.3 Non-linear analysis with Ogden's coefficients

The Ogden's coefficients for the rubber which are defined as the strain energy func-

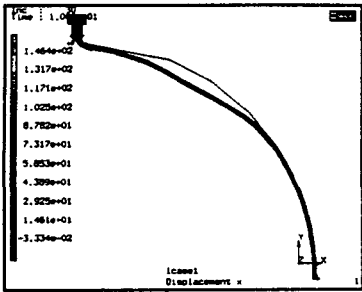


Fig. 10. Deformed shape under 0.1 kg/cm<sup>2</sup> (Type A, E=30 kg/cm<sup>2</sup>).

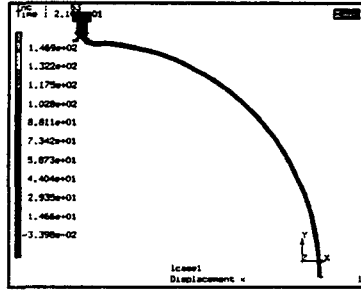


Fig. 11. Deformed shape under 0.21 kg/cm<sup>2</sup> (Type A, E=30 kg/cm<sup>2</sup>).

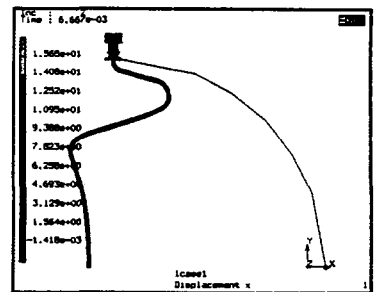


Fig. 12. Deformed shape under 0.007 kg/cm<sup>2</sup> (Type A, E=100 kg/cm<sup>2</sup>).

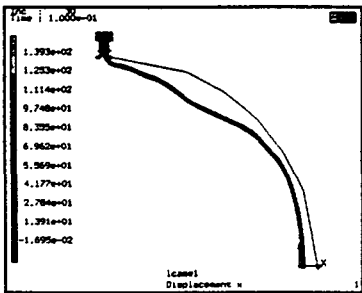


Fig. 13. Deformed shape under 0.1 kg/cm<sup>2</sup> (Type A, E=100 kg/cm<sup>2</sup>).

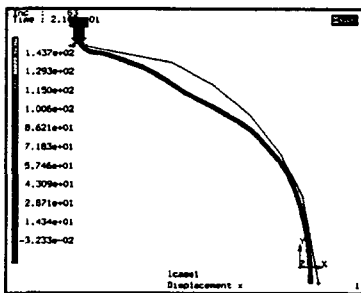


Fig. 14. Deformed shape under 0.21 kg/cm<sup>2</sup> (Type A, E=100 kg/cm<sup>2</sup>).

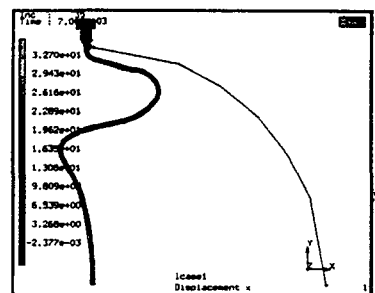


Fig. 15. Deformed shape under 0.007 kg/cm<sup>2</sup> (Type A, Non-linear analysis).

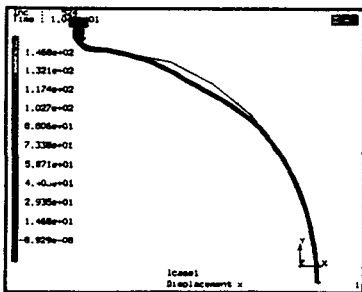


Fig. 16. Deformed shape under 0.1kg/cm<sup>2</sup> (Type A, Non-linear analysis).

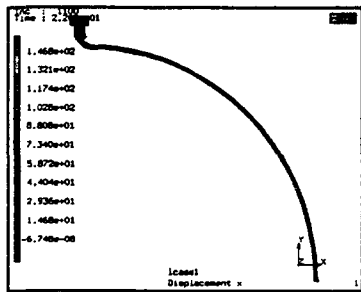


Fig. 17. Deformed shape under 0.21 kg/cm<sup>2</sup> (Type A, Non-linear analysis).

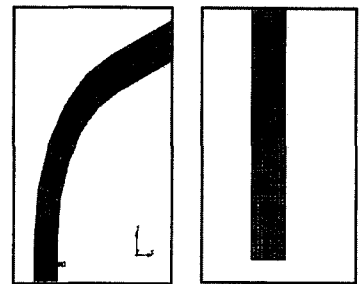


Fig. 18. Adaptive mesh at the bent and the fixed areas

tion, are commonly used in the non-linear analysis. Those are used widely in analysis of o-ring, rubber seal and variable rubber components. The material behavior predicted by

the Ogden's coefficients shows a good agreement with the tensile test data up to 700% strain.<sup>7</sup>

Although the non-linear analysis using the

strain function acquires the accurate solution in comparison with the linear analysis, this analysis is taken a long run time because of the non-linear material properties and it is very difficult to converge a solution. Specially, a lot of efforts are required in order to approach a solution of the diaphragm on account of the non-linear rubber properties and the geometric non-linear behavior. In this study, the time step is minutely divided for approaching a solution to the required pressure range. And an adaptive mesh function for the element at the bent and fixed areas is used for the improvement of the convergence.

Figs. 15, 16 and 17 show the deformed shapes under 0.007, 0.1, 0.21 kg/cm<sup>2</sup>, respectively. The result of the linear analysis with Young's modulus of 30 kg/cm<sup>2</sup> is similar to that of Ogden's coefficients. Fig. 18 shows the minutely divided element mesh endowing with an adaptive mesh function.

### 3. Analysis for Type B

As the total area of type B is geometrically smaller than that of type A, it is expected that the deformation of type B is be smaller than type A.

In order to reduce the stress concentration of the fixed area, the shape of the fixed area should be modified using the linear analysis. The deformed shape with Young's modulus of 100 kg/cm<sup>2</sup> shows the different aspect from that of type A. Type B shows smaller deflection than that of type A as the preliminary expectation. The diaphragm does not contact under 0.21 kg/cm<sup>2</sup> and the con-

tact is started under 0.93 kg/cm<sup>2</sup>. It shows the partial contact under 1.0 kg/cm<sup>2</sup>. Figs. 19, 20, 21 and 22 show the deformed shapes under 0.007, 0.1, 0.21 and 0.93 kg/cm<sup>2</sup>, respectively.

The curvature between the fixed part and the membrane grows large in 5mm from the previous dimension (sharp notch) in order to reduce the maximum strain. Figs. 23 and 24 show the strain distributions of the previous model and the modified model under 0.67 kg/cm<sup>2</sup>. The maximum strain is reduced in 15% from 0.50 to 0.43. This might reduce the potential of a fatigue failure and increase the durability of rubber diaphragm.

Figs. 25 and 26 show the deformed shapes under minus pressures -0.023 and -0.33 kg/cm<sup>2</sup>, respectively. The large deformation is generated in the unfolding of the bent area under the lower pressure, but the small deformation is appeared afterward.

## IV. Discussion and Conclusions

In order to estimate the pressure-deformation characteristics of the rubber diaphragm used in accumulators, the linear analysis and the non-linear analysis have been carried out using MARC that is the commercial non-linear finite element program. The linear analysis using Young's modulus and Poisson's ratio has been carried out to evaluate the effect of Young's modulus on the deformation behavior. The convergence of the non-linear analysis using Ogden's coefficients has been improved by endowing the adaptive mesh with the bent and the fixed areas. Fig. 27 shows the relationship between the maximum

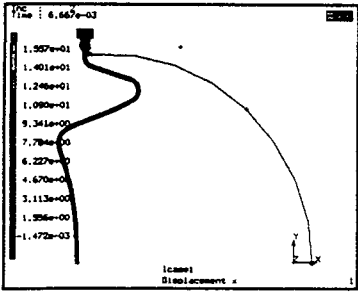


Fig. 19. Deformed shape under 0.007 kg/cm<sup>2</sup> (Type B, E=100 kg/cm<sup>2</sup>).

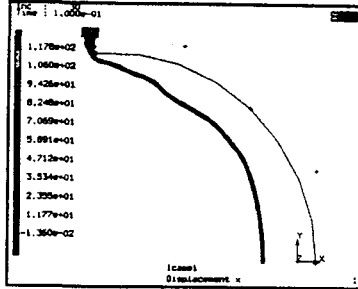


Fig. 20. Deformed shape under 0.1 kg/cm<sup>2</sup> (Type B, E=100 kg/cm<sup>2</sup>).

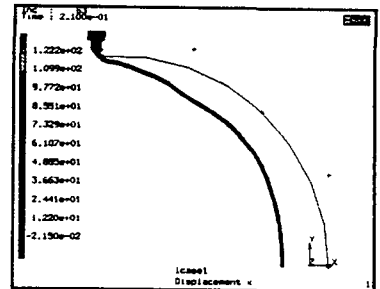


Fig. 21. Deformed shape under 0.21 kg/cm<sup>2</sup> (Type B, E= 100 kg/cm<sup>2</sup>).

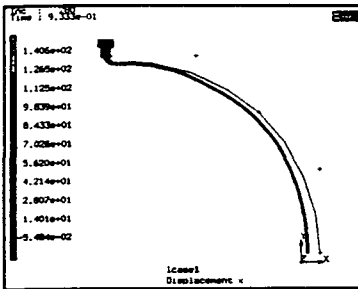


Fig. 22. Deformed shape under 0.93kg/cm<sup>2</sup> (Type B, E=100 kg/cm<sup>2</sup>).

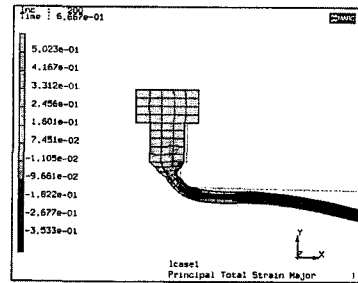


Fig. 23. Strain distribution for old model (Type B, E=100 kg/cm<sup>2</sup>).

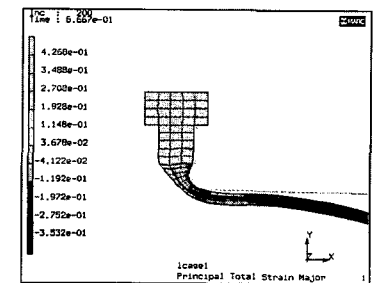


Fig. 24. Strain distribution for old model (Type B, E=100 kg/cm<sup>2</sup>).

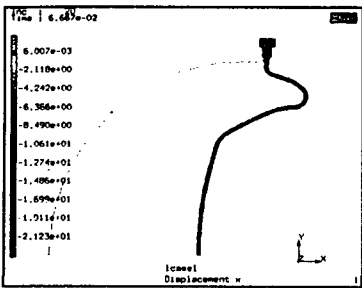


Fig. 25. Deformed shape under -0.023 kg/cm<sup>2</sup>.

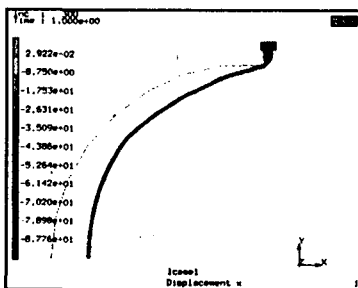


Fig. 26. Deformed shape under -0.33 kg/cm<sup>2</sup>.

displacement at the center of the diaphragm and the pressure for individual cases. The deformation behaviors of the linear analysis using a Young's modulus 30 kg/cm<sup>2</sup> are similar to that of the non-linear analysis using Ogden's

coefficients. Also, it is shown in Figs. 9 and 15 under 0.007 kg/cm<sup>2</sup>, Figs. 10 and 16 under 0.1 kg/cm<sup>2</sup>, and Figs. 11 and 17 under 0.21 kg/cm<sup>2</sup>. It is found out that the linear analysis is useful to predict and evaluate the

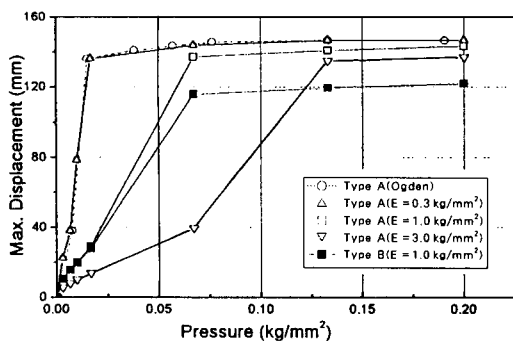


Fig. 27. Relationship between pressure and max. displacement for each case.

deformation behaviors of the rubber diaphragm.

For the purpose of the reducing the stress and strain concentration at fixed area, the curvature between the fixed part and the membrane grows large in 5mm. As a result, the maximum strain is reduced in 15% from 0.50 to 0.43, and this might reduce the potential of fatigue failure and might increase the durability.

### Acknowledgement

This study was supported by a grant from the Critical Technology 21 program (Machinery Design Technology Enhancement).

### References

1. E. H. Smith, Mechanical engineer's Reference Book, 12th ed., 1994, p. 15/8.
2. W. D. Kim, "Application of Finite Element Analysis Technology for the Mechanical Rubber Component," Proceedings of KSME Solid and Structures Division, 1998. 6. 26. pp. 97~104.
3. Rubber Mechanics, Proceedings of KSME Solid and Structure Mechanics Division, 1997. 9. 24. pp. 97~104.
4. Mentat 3.1, MARC K7.1 New Features Users Guide, 1997, MARC.
5. Anti-Vibration Rubber, Japan Railroad Industry, 1975, p. 7.
6. W. D. Kim, "The Specimen Design for the Rubber Material Test using the Finite Element Analysis," KSME 98R712, 1998, pp. 81. 87.
7. Non-linear Finite Element Analysis of Elastomers, MARC Analysis Research Corporation, 1996, p. 9.